Criteria for Initiation of Tokamak Disruptions

K. I. Hopcraft
Blackett Laboratory, Imperial College of Science and Technology, London SW7 2BZ, England

and

M. F. Turner
Culham Laboratory, Abingdon, Oxon, OX14 3DB, England
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The process by which a tokamak plasma evolves from an equilibrium state containing a saturated magnetic island to one which is disruptively unstable is discussed and illustrated by numerical simulation of a resistive magnetoplasma. Those elements which are required to initiate a disruption are delineated.

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Of the many plasma processes limiting the performance of the tokamak configuration of magnetic confinement devices, the disruption is potentially the most damaging. Consequently, the disruption has received much attention and several models purport to explain it as a consequence of resistive magnetohydrodynamic activity within the plasma.[1–6]

In Refs. 1 and 2, the magnetic island associated with the $m=2$ tearing mode couples to and drives higher m modes on different resonant surfaces once a threshold in the amplitude of the $m=2$ mode is attained. This occurs when magnetic islands of different helicity overlap, leading to a form of turbulence and ergodicity of the field lines within the region of overlap. It is suggested that this then leads to a dispuptive loss of plasma and current. In Ref. 3, it is found that a single $m=2$ island touching a limiter or cool-gas region at the plasma edge has a strong destabilizing effect, while in Refs. 4 and 5, the $m=2$ mode is found to grow spontaneously if the safety factor $q(r)=rB_z/RB_\theta$ at the plasma periphery falls below a threshold. In Ref. 6 it was shown that for broad current profiles with $q(0)\geqslant 1.5$, the saturation width of the $m=2$ island is large enough for it to nearly cover the entire minor radius of the plasma. Such states are related to the "vacuum bubbles" of Kadomtsev and Pogutse.[7]

In Refs. 1, 2, and 6, the calculations are initiated from idealized distributions of the current with no magnetic islands present, and the subsequent evolution then followed. In this Letter, we initiate calculations from resistively torn equilibrium states containing saturated $m=2$ islands and follow the evolution to a state which is disruptively unstable by slowly changing a global plasma parameter, in this case the total current. The adiabatic evolution enables us to delineate those elements which both trigger the disruption and allow it to proceed. The results are independent of the choice of equilibrium since the initial state is always a saturated one.

The equations solved are those appropriate for a large–aspect-ratio cylindrical tokamak. We solve Maxwell's equations, the tearing-mode stability equation, and a single-fluid energy equation for the temperature in one dimension. Growth rates of each individual mode are calculated from an equation similar to that derived by White *et al.*[8] The electromagnetic fields are determined by Maxwell's equations and Ohm's law:

$$\partial B_{\theta 0}(r,t)/\partial t=\partial E_{z0}(r,t)/\partial r, \tag{1}$$

$$j_{z0}(r,t)=(1/\mu_0 r)\partial(rB_{\theta 0})/\partial r, \tag{2}$$

$$E_{z0}(r,t)=\eta j_{z0}(r,t), \tag{3}$$

where η is the classical resistivity, proportional to $T^{-3/2}$. The values of $B_{\theta 0}$ and j_{z0} are used to solve the tearing-mode stability equation $\nabla\times(\mathbf{j}\times\mathbf{B})\cdot\mathbf{z}=0$,

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\psi_{mn}}{dr}\right)-\frac{m^2}{r^2}\psi_{mn}-\frac{dj_{z0}/dr}{B_{\theta 0}(1-nq/m)}\psi_{mn}=0, \tag{4}$$

for $m\geqslant 2$, where ψ_{mn} is the poloidal flux associated with the mode with poloidal and toroidal mode numbers m and n. The quantities with the subscript "0" refer to the equilibrium, that is the components with $m=n=0$. Equation (4) is solved in the regions $r<r_{mn}$ and $r>r_{mn}$, where $q(r_{mn})=m/n$. Solutions are used to calculate the quasilinear stability parameter,

$$\Delta'(w_{mn})=\frac{1}{\psi_{mn}(r_{mn})}\left.\frac{d\psi_{mn}}{dr}\right|_{r_{mn}-(1/2)w_{mn}}^{r_{mn}+(1/2)w_{mn}},$$

and the magnetic island width w_{mn} is calculated from[8]

$$\frac{dw_{mn}}{dt}=\frac{1.66}{\mu_0}\eta(r_{mn})\Delta'(w_{mn}). \tag{5}$$

The energy equation is

$$\frac{\partial T(r,t)}{\partial t}=\frac{1}{3n_0}\left[E_{z0}j_{z0}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\kappa\frac{\partial T}{\partial r}\right)\right]+R(r,t), \tag{6}$$

where the number density n_0 and the thermal conductivity κ are assumed constant except that κ is enhanced within a magnetic island, consistent with the field lines being isothermal. The function $R(r,t)$ is a source or sink term and the solution of (6) enables the resistivity to be self-consistently evolved. If a $q=1$ surface exists within the plasma, the resistivity is enhanced in such a way as to maintain $q(0)\sim 1$. The enhanced resistivity is given by

$$\eta^* = [1-q(0)][1-\tfrac{3}{4}(r/r_1)^2]^2/\tau$$

for $r \le 2r_1/\sqrt{3}$, where r_1 is the radius of the $q=1$ surface and τ is the relaxation time of the resistive internal kink.[4] The thermal conductivity is also enhanced according to $\kappa_1 = \kappa_0 + \kappa_I[1-\tfrac{3}{4}(r/r_1)^2]^2$, where $\kappa_0 = 7\times 10^{19}\ \mathrm{ms^{-1}}$ is the background conductivity and $\kappa_I = 100\kappa_0$. For an island with $m\neq 1$, $\kappa = \kappa_0 + \kappa_I[1-(x_0/2w_{mn})^2]$, where x_0 is the distance from the island center. Since $\kappa_I \gg \kappa_0$, the functional form of the conductivity within the island is not crucial, the net effect being to flatten the temperature profile across the region of instability. The effect of the $m=1$ mode is to flatten the temperature and also to restrict the amount of current able to flow about the axis. An important point to stress is that although direct mode coupling is excluded by the model, modes can influence each other's behavior by deformation of the current profile. Because of this, it must be borne in mind that the results that we will present are quasilinear rather than fully nonlinear.

The boundary conditions used to solve Eqs. (1)–(6) prescribe the time dependence of the plasma current $B_{\theta 0} = B_{\theta 0}(t)$ at $r=a$ together with a constant wall temperature $T(a,t) = \mathrm{const}$, and a free plasma boundary so that $d\psi_{mn}(a)/dr = -m\psi_{mn}(a)/a$. The boundary conditions at the origin are $B_{\theta 0}=0$, $\partial T/\partial r = 0$, $\partial E_{z0}/\partial r = 0$, and $\psi_{mn} \propto r^m$.

The first calculation is initiated from the equilibrium state depicted in Fig. 1 which shows the current and temperature profiles across the minor radius. The saturated $m/n = 2/1$ island width is $0.09a$, the position of the $q=1$ surface is at $r=0.2a$ and $q(a)=3.125$. The source-sink term R in Eq. (6) is set equal to zero so that Ohmic heating is balanced by thermal conduction losses. The equilibrium state is altered by the application of a linear current ramp $B_\theta(t) = B_\theta(0) + \dot{B}_\theta t$ with $\dot{B}_\theta = \mathrm{const}$. The safety factor at the plasma edge changes according to $\dot{q}_a \approx -Rq_a^2\dot{B}_\theta/aB_{z0}$ causing the resonant surfaces within the plasma to move towards the edge.

Figure 2 shows the total current $I(a)$ and the current within the $q=2$ and $q=\tfrac{3}{2}$ surfaces, $I(q=2)$ and $I(q=\tfrac{3}{2})$, plotted as functions of time. The monotonic rise in $I(a)$ is reflected in $I(q=2)$ and $I(q=\tfrac{3}{2})$, which are, however, modulated, there being a distinct phase lag between relative maxima, suggesting that there is an inward propagation of current den-

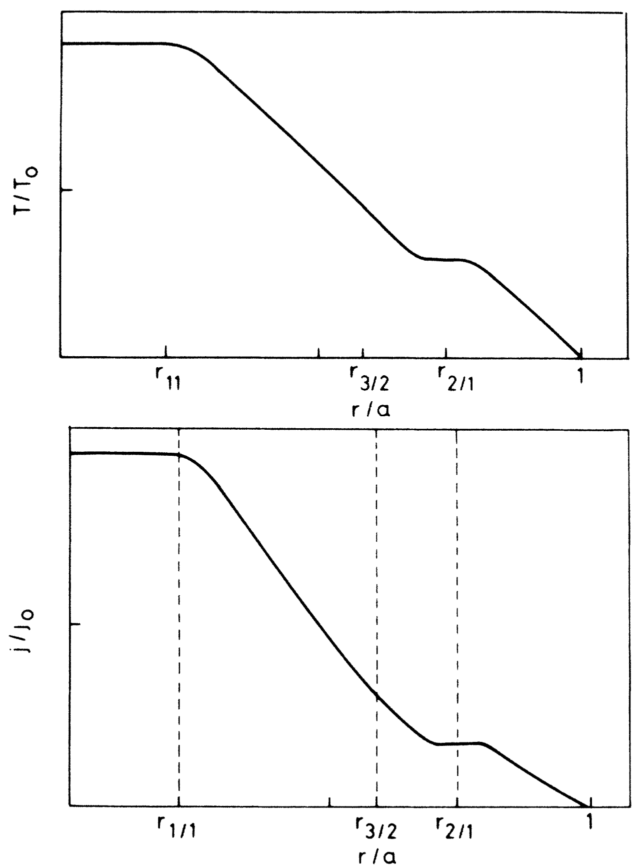

FIG. 1. Equilibrium current and temperature profiles used to initiate the calculation. The positions of the $q=1$, $\tfrac{3}{2}$, and 2 surfaces are shown.

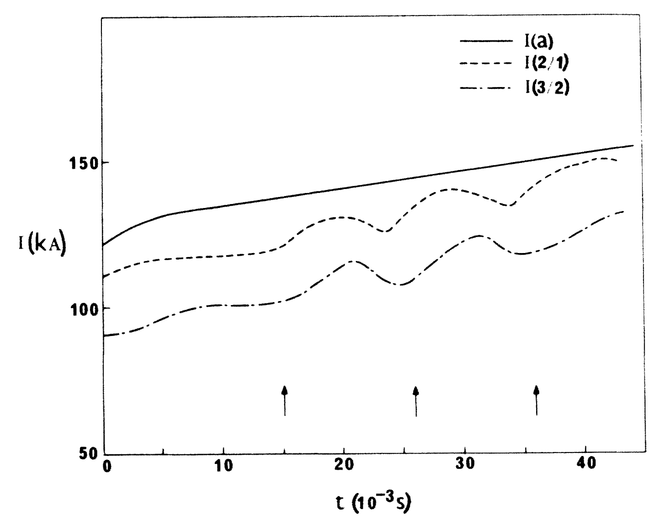

FIG. 2. Total current $I(a)$ and the current within the $q=\tfrac{3}{2}$ and 2 surfaces, $I(3/2)$ and $I(2/1)$. The arrows denote the times at which minor disruptions occur.

sity on a time scale faster than the characteristic diffusion time. Figure 3 shows the 2/1 and 3/2 island widths together with the position of the $q = 1$ surface plotted at their relative positions across the minor radius against time; also plotted is the central temperature. The phase lag between increased instability of the 2/1, 3/2, and $m = 1$ modes is again evident and suggests that the current is forced towards the axis on the time scale of growth of each successive island. At each instant when the $q = 1$ surface moves away from the axis, the enhanced thermal conductivity in the region $r \leq 2r_1/\sqrt{3}$ causes the central temperature to fall, fall, indicative of a minor disruption having occurred; the times at which this occurs are marked by the arrows in Fig. 2.

The following sequence of events occurs during the evolution between each increase in the 2/1 activity and the drop in the temperature on axis. (i) The increasing poloidal flux at the plasma edge causes the current to rise. This propagates inwards, steepening the current density gradient within r_{21}. (ii) The steepening of the current gradient destabilizes the 2/1 island which grows, modifies the resistivity profile, and thereby advects the current inwards on the time scale of the island growth. (iii) The current density gradient within r_{32} steepens, destabilizing the $\frac{3}{2}$ island which grows and advects current inwards in a similar fashion. (iv) The rise in the total current on axis together with the restriction on the amount able to flow due to the $m = 1$ instability forces the $q = 1$ surface outwards, transporting energy from the central core and causing the temperature to drop. (v) The cooling of the central core dissipates the current around the axis and causes the $q = 1$ surface to contract, though at a greater radius than prior to the disruption.

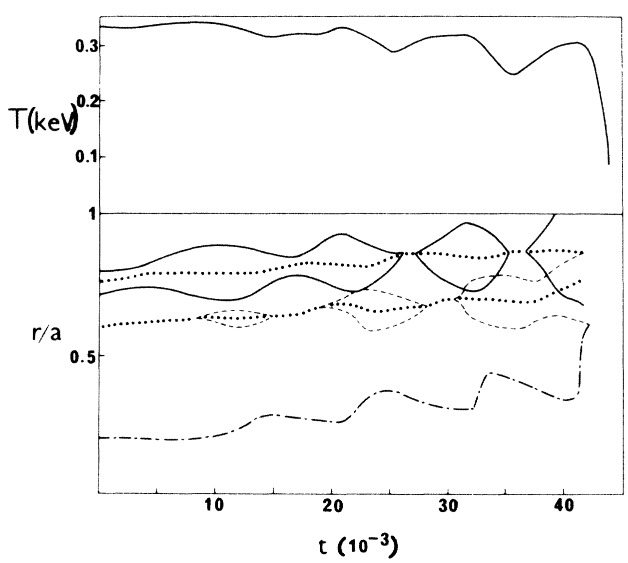

FIG. 3. Time evolution of the 2/1 (solid line) and 3/2 (dashed line) magnetic islands, together with the position of the $q = 1$ surface (dot-dashed line) and the central temperature.

Circumstances causing the "major" disruption at $t \sim 40$ ms from the start are rather different from those described previously. It is triggered by the 2/1 island intersecting the limiter at $r = a$. The loss of the stabilizing current gradient in $r > r_{21}$ causes $\Delta_{21}' >> 1$, so that w_{21} increases rapidly, intersecting the 3/2 island. The plasma in the combined 2/1-3/2 island cools to the limiter temperature thereby causing the current channel to contract and forcing the current inwards. This in turn causes the spontaneous outward movement of the $q = 1$ surface which connects with the 2/1 island, creating a thermal "short circuit" across the entire minor radius and leading to a disruptive energy quench, as can be seen by the rapid fall in the temperature. The role of the 3/2 island is merely to hasten the time for the connection between the hot core and the cool periphery. It will be noted that at $t \sim 24$ ms the growth of the 3/2 mode actually stabilizes the 2/1 mode by removing the current gradient in $r < r_{21}$! We are led to the conclusion that two prerequisites for a major disruption to proceed are (1) that there be a magnetic island close to the plasma edge and (2) that there be a region close to the axis where the $m = 1$ mode is unstable. It is known, however, that disruptions occur without sawteeth oscillations having been observed.[9]

To illustrate how a disruption can be initiated without a $q = 1$ surface being present in the plasma, we introduce a loss term in Eq. (6) of the form $R(r) = -A\exp(-r/b)^2$ in order to model an accumulation of impurities near the axis. The saturated 2/1 island width characteristic of this new equiliblrium is $0.1a$ while $q(0)$ and $q(a)$ are 1.34 and 3.78, respectively. The equilibrium state is evolved as before by application of the linear current ramp. Figure 4 shows

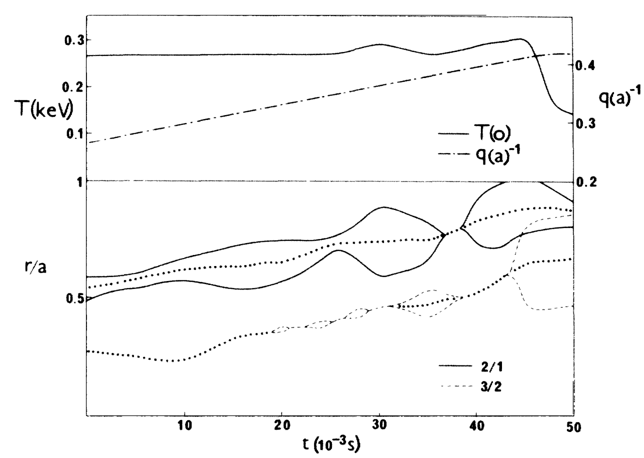

FIG. 4. The central temperature and $1/q(a) \propto I(a)$, together with the 2/1 and 3/2 magnetic islands, plotted as functions of time. There is no $q = 1$ surface in the plasma.

the time evolution of the 2/1 and 3/2 islands together with the central temperature and $1/q(a)$ which is proportional to the total current. Since the rate at which the impurities radiate is assumed to be constant and is a function of r alone, the impurities restrict the current less severely than does the $m=1$ mode. However, the net effect is to flatten the current profile close to the axis and thus steepen the current density gradient in the vicinity of the $q=\frac{3}{2}$ and 2 surfaces. The trigger to the disruption is again due to the intersection of the 2/1 island with the limiter. The loss of a stabilizing current gradient in $r > r_{32}$ causes $\Delta'_{32} >> 1$ and hence the loss of equilibrium of the 3/2 island. At the time of the disruption at $t \sim 43$ ms from the start, $q(0)=1.1$ and $q > q(0)$ everywhere.

As a result of these calculations of the quasilinear evolution of a resistive magnetoplasma, we conjecture that the following criteria must be fulfilled in order to initiate a current threshold major disruption.

(1) A large magnetic island intersecting a limiter or cold plasma mantle: Present tokamaks run at high currents, implying that $q(a) \sim 3$; the obvious candidate for this island is the $m/n=2/1$.

(2) An intermediate large magnetic island resonant between the $q=2$ surface and axis: The presence of such an island ensures connection across the entire minor radius. However, this condition is sufficient though not necessary in order to allow the disruption to proceed.

(3) A method of restricting the amount of current flowing around the magnetic axis: The $m=1$ instability maintains $q(0) \approx 1$, and so limits the amount of current able to flow thereby increasing the destabilizing current gradient within the $q=2$ surface. The effect of a concentration of impurities on axis has a similar effect.

In Ref. 6 it is suggested that the vacuum bubble states might be avoided by heating the plasma center and driving $q(0) \rightarrow 1$. We have found that current restriction near the axis is an important prerequisite for initiating a disruption and so such measures must be performed with care.

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