Some Quantum Corrections to Calabi-Yau Compactification

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(Received 24 February 1986)

The counterterms constructed by Green and Schwarz in the field-theory limit of superstrings are put into a form of Calabi-Yau compactification. Dimension-six operators are explicitly extracted. The modifications of the gauge kinetic terms and of the Kähler potential are obtained. Axion couplings and some *CP*-nonconserving interactions are given.

PACS numbers: 12.10.Gq, 11.17.+y

There has been much interest in the unified theories of superstrings[1] since Green and Schwarz sparked the field by the discovery of anomaly-free string theories.[2] The subsequent developments of heterotic construction[3] of superstrings and the compactification on certain Ricci-flat Kähler manifolds[4,5] seem to have demonstrated that one can argue a realistic unification and its phenomenological properties, based on $E_8\times E_8$ superstring theory, within certain *Ansätze* and approximations. Certainly, one of the central issues is to extract dynamics involving massless particles in some convincing manner.

Much of the recent phenomenological developments along these lines are, in practice, based on the field-theory (or zero slope) limit of superstrings and a few results,[5,6] on complete tree string amplitudes: One may naturally wonder what we obtain from quantum corrections of superstrings. At the present, a workable approach to Calabi-Yau compactification in full-fledged string theory does not appear to exist. There is, however, a relatively simple way to give some of the characteristic terms among these corrections, coming back to the original work by Green and Schwarz[2]: The "counterterms" S_1', S_2, and S_3' necessary to have an anomaly-free $d=10$, $N=1$ supergravity do come from string quantum corrections.

Superstring theories are one-loop finite and believed to be finite to all orders. The anomaly-free properties are closely tied to this fact[2] and may be viewed as a consequence of finiteness. Upon truncation to the massless sector (field-theory limit), finiteness is lost, but the anomaly-free property is maintained with the addition of those counterterms. The point of view[2] pursued here is that they represent some of the contributions coming from massive modes, thereby implementing constraints coming from anomaly-free theory.

The aim of this Letter is to investigate the significance of the counterterms in the context of Calabi-Yau compactification. Below, we explicitly perform the decomposition of ten-dimensional fields, with its basis on Calabi-Yau space. We mostly use the language of differential forms.

The starting point is a truncation of the E_8 vector potential one-form in $M_4\times$ (Calabi-Yau) space following the recipe of Witten[7]:

$$A(x,y) = 1\otimes\hat{\alpha}_6(y)+A_4(x)\otimes 1+1\otimes A_6(x,y), \tag{1}$$

where $A_4\equiv A_4^{(1,78)}$ is the E_6 gauge field in M_4, $A_6\equiv A^{(3,27)+(3^*,27^*)}$ is a massless scalar field in M_4 arising from the zero modes in Calabi-Yau space[8] and is expanded in terms of the available harmonic forms[7] (see below). $\hat{\alpha}_6\equiv\alpha^{(8,1)}$ is a *c*-number background configuration; it must be a holomorphic, stable one-form. The corresponding field-strength two-form is

$$F=1\otimes\hat{F}_6+F_4\otimes 1+1\otimes\{\hat{D}_6,A_6\}+\{D_4,A_6\}+1\otimes A_6^2, \tag{2}$$

where $D_4\equiv d_4+A_4$, $\hat{D}_6\equiv d_6+\hat{\alpha}_6$, and $\hat{F}_6\equiv d_6\hat{\alpha}_6+\hat{\alpha}_6^2$. The last quantity must satisfy $(\hat{F}_6)_{mn}=0$ and $g^{m\bar{n}}(\hat{F}_6)_{m\hat{n}}=0$ because of the properties of $\hat{\alpha}_6$.[4]

The gauge field for the second E_8 (E_8') undergoes a trivial truncation $A'\equiv A_4'\otimes 1$ according to the above recipe. The antisymmetric tensor field decomposes like

$$B=B_4\otimes 1+1\otimes B_6. \tag{3}$$

The background spin connection $\hat{\omega}_6$, obtained from truncation $\omega=\hat{\omega}_6+\omega_4$ (and $R=\hat{R}_6+R_4$), is related to the background gauge field through $\mathrm{tr}_1\hat{F}_6^2=30\,\mathrm{tr}_1\hat{R}_6^2$.[4] The truncated expressions we obtain for S_1', S_2, and S_3' are respectively

$$\begin{aligned} S_1' = -\tfrac{1}{108\,000}c\Big(&30\int B_4([4\,\mathrm{tr}_1F_4A_6^2+2\,\mathrm{tr}_1\{D_4,A_6\}^2]G+4[\mathrm{tr}_1\{D_4,A_6\}\{\hat{D}_6,A_6\}+\tfrac{1}{3}d_4\,\mathrm{tr}_1A_6^3]^2) \\ &+30\int B_6(2(\mathrm{tr}_1F_4^2)G+[\mathrm{tr}_1\{D_4,A_6\}^2+2\,\mathrm{tr}_1F_4A_6^2]^2-(\mathrm{tr}_2F_4'^2)G)-\int \mathrm{tr}_1[\{D_4,A_6\}A_6]G\omega_{3Y_2}^{(d=4)} \\ &-2\int(\hat{\omega}_{3Y_1}^{(d=6)}+\mathrm{tr}_1[A_6\{\hat{D}_6,A_6\}]+\tfrac{2}{3}\mathrm{tr}_1A_6^3)[\mathrm{tr}_1\{D_4,A_6\}\{\hat{D}_6,A_6\}+\tfrac{1}{3}d_4\,\mathrm{tr}_1A_6^3]\omega_{3Y_2}^{(d=4)}\Big), \end{aligned} \tag{4}$$

$$S_2=-\tfrac{1}{16}c\int(\mathrm{tr}R_4^2)(\mathrm{tr}\hat{R}_6^2)B_6, \tag{5}$$

and

$$S_3' = \tfrac{1}{7200} c \Big\{ 30 \int B_6 [(\mathrm{tr} R_4^2) G + \mathrm{tr} \hat{R}_6^2 \, \mathrm{tr} F_4^2] + 30 \int B_4 \, \mathrm{tr} \hat{R}_6^2 \, \mathrm{tr}_1 [\{D_4, A_6\}^2 + 2 F_4 A_6^2]$$

$$+ 5 \int \hat{\omega}_{3L}^{(d=6)} [\mathrm{tr}_1 A_6 \{\hat{D}_6, A_6\} + \tfrac{2}{3} \mathrm{tr}_1 A_6^3] \mathrm{tr} R_4^2 + 5 \int \omega_{3L}^{(d=4)} \mathrm{tr}_1 [\{D_4, A_6\} A_6] \mathrm{tr} \hat{R}_6^2$$

$$+ \int \mathrm{tr}_1 [\{D_4, A_6\} A_6] \omega_{3L}^{(d=4)} G + 2 \int \omega_{3Y_1}^{(d=4)} \hat{\omega}_{3L}^{(d=6)} [\mathrm{tr}_1 \{D_4, A_6\} \{\hat{D}_6, A_6\} + \tfrac{1}{3} d_4 \mathrm{tr}_1 A_6^3]$$

$$+ 2 \int [\hat{\omega}_{3Y_1}^{(d=6)} + \mathrm{tr}_1 A_6 \{\hat{D}_6, A_6\} + \tfrac{2}{3} \mathrm{tr}_1 A_6^3] \omega_{3L}^{(d=4)} [\mathrm{tr}_1 \{D_4, A_6\} \{\hat{D}_6, A_6\} + \tfrac{1}{3} d_4 \mathrm{tr}_1 A_6^3]$$

$$+ \int [\mathrm{tr}_1 A_6 \{\hat{D}_6, A_6\} + \tfrac{2}{3} \mathrm{tr}_1 A_6^3] \hat{\omega}_{3L}^{(d=6)} (\mathrm{tr}_1 F_4^2 - \mathrm{tr}_2 F_4'^2) + 30 \int B_6 \, \mathrm{tr} \hat{R}_6^2 \, \mathrm{tr}_2 F_4'^2$$

$$+ 2 \int \hat{\omega}_{3L}^{(d=6)} \omega_{3Y_2}^{(d=4)} [\mathrm{tr}_1 \{D_4, A_6\} \{\hat{D}_6, A_6\} + \tfrac{1}{3} d_4 \mathrm{tr}_1 A_6^3] \Big\}, \quad (6)$$

where

$$G = \mathrm{tr}_1 \hat{F}_6^2 + \mathrm{tr}_1 \{\hat{D}_6, A_6\}^2 + \mathrm{tr}_1 A_6^4 + 2 \mathrm{tr}_1 \hat{F}_6 A_6^2 + 2 \mathrm{tr}_1 \{\hat{D}_6, A_6\} A_6^2$$

and tr_1 and tr_2 are the traces of 248-dimensional representations of E_8 and E_8', respectively. We generically denote Chern-Simon forms by ω. For instance, $\hat{\omega}_{3Y_1}^{(d=6)}$ is the Chern-Simon three-form for the $\hat{\alpha}_6$ of E_8 where all differentials are in six-dimensional Calabi-Yau space. A hat denotes that the object is defined in terms of background fields alone. Finally, $\{D_4, A_6\} \equiv d_4 A_6 + \{A_4, A_6\}$. The rest of the notation is self-explanatory.

To exhibit the four-dimensional couplings which arise from the above structure, it is necessary to express A_6 and B_6 more explicitly as follows;

$$B_6 = \sum_{i=1}^{b_{11}} a^{(i)}(x) \Omega_{mn^*}^{(i)}(y) dz^m \wedge dz^{n^*} \quad (7)$$

and

$$A_6 = A_m^{(3,27)} dz^m + A_m^{(3^*,27^*)} dz^m + A_{m^*}^{(3,27)*} dz^{m^*} + A_{m^*}^{(3^*,27^*)*} dz^{m^*}, \quad (8)$$

where $a^{(i)}(x)$'s are model-dependent axion fields, due to the gauge invariance $B_6 \to B_6 + d\Lambda$ in the ten-dimensional supergravity Lagrangean. $A_m^{(3,27)}$ and $A_m^{(3^*,27^*)}$ also arise from the zero modes in Calabi-Yau space, and can be written as

$$A_m^{(3,27)} = \sum_{i=1}^{b_{12}} \sum_{\substack{a,X \\ n^*,l^*}} T^{Xa} C_X^{(i)}(x) \epsilon_a^{n^* l^*} \Omega_{mn^* l^*}^{(i)}(y) \quad (9)$$

and

$$A_m^{(3^*,27^*)} = \sum_{i=1}^{b_{11}} \sum_{\substack{X,a \\ n^*,l^*}} (T^*)_{aX} C'^{(i)X}(x) g^{an^*}(y) \Omega_{mn^*}^{(i)}(y). \quad (10)$$

$\Omega_{mn^* l^*}^{(i)}(y)$ and $\Omega_{mn^*}^{(i)}(y)$ are, respectively, harmonic (1,2)-forms and (1,1)-forms, b_{11} and $b_{12} = b_{21}$ are Betti numbers of the manifold and $C_X^{(i)}(x)$ [$C'^{(i)X}(x)$] are scalar fields in the **27** [**27***] representation of E_6. The "stable" matter spectrum consists of $|\chi|/2 = |b_{11} - b_{12}|$ massless E_6 multiplets. From now on, we treat the case $b_{12} > b_{11}$ and suppress, in most cases, the index which labels generations or axions. One can readily read off, from the discussions below, the formulas for the other case $b_{12} < b_{11}$, and also for the case in which some of the $C^{(i)}$ and some of the $C'^{(i)}$ remain simultaneously massless after compactification.

Let us now extract lower-dimension operators which arise from (4), (5), and (6). First of all, it is worth mentioning that a dimension-four operator

$$-\tfrac{1}{7200} c \int \hat{\omega}_{3L}^{(d=6)} \hat{\omega}_{3Y_1}^{(d=6)} \{\mathrm{tr}_1 F_4^2 - \mathrm{tr}_2 F_4'^2 - 5 \, \mathrm{tr} R_4^2\}$$

coming from S_3' vanishes identically thanks to the construction of Ref. 4. The counterterms do not spoil the anomaly-free property of the four-dimensional theory which is achieved by the choice of fermion representation. Actually, this is a version of Witten's observation[9] that anomaly cancellation in ten-dimensional sense ensures anomaly-free four-dimensional theories after compactification.[10]

The lowest-dimension operators left after the compactification of the counterterms are dimension-six operators. Let us exhibit these. After tedious but straightforward algebra, we obtain reasonably simple interactions:

$$S_1' + S_2 + S_3'|_{\rm dim6} = \frac{c}{80}\sum_{i=1}^{b_{11}} K^{(i)\prime}\int_{M_4} a^{(i)}(x)(-\mathrm{tr}_1 F_4^2 + \mathrm{tr}_2 F_4'^2 + 5\,\mathrm{tr} R_4^2) - \frac{c}{80}\sum_{i=1}^{b_{12}-b_{11}} K^{(i)}\int_{M_4} dB_4 C^{*(i)}\overleftrightarrow{\mathscr{D}} C^{(i)}$$
$$+ c\sum_{i=1}^{b_{12}-b_{11}} K^{(i)}\int_{M_4}\left(\tfrac{1}{1440}\omega_{3L}^{(d=4)} - \tfrac{1}{7200}\omega_{3Y_1}^{(d=4)} + \tfrac{1}{7200}\omega_{3Y_2}^{(d=4)}\right) C^{*(i)}\overleftrightarrow{\mathscr{D}} C^{(i)}$$
$$+ c\sum_{i=1}^{b_{12}-b_{11}} p^{(i)}\int_{M_4}\left(\tfrac{1}{288}\mathrm{tr} R_4^2 + \tfrac{1}{3600}\mathrm{tr}_2 F_4'^2\right) C^{*(i)}\cdot C^{(i)}, \qquad (11)$$

where $c = i^6/6!(2\pi)^6$,

$$K^{(i)} \equiv \int_k \epsilon_a^{\;n^* l^*}\Omega^{(i)}_{mn^*l^*}\epsilon^{an'l'}\Omega^{(i)}_{m'^*n'l'}\, dz^m \wedge dz^{m'^*}\,\mathrm{tr}\hat{R}_6^2, \qquad K^{(i)\prime} \equiv \int_k g^{an^*}\Omega^{(i)}_{mn^*}\Omega^{(i)}_{m^*a}\, dz^{m'} \wedge dz^{m^*}\,\mathrm{tr}\hat{R}_6^2, \qquad (12)$$

and

$$p^{(i)} \equiv \int_k \epsilon^{bnl}\Omega^{(i)}_{m^*nl}(\overleftrightarrow{\hat{\mathscr{D}}})_b^a \epsilon_a^{\;n'^*l'^*}\Omega^{(i)}_{m'n'^*l'^*}\, dz^{m'} \vee dz^{m^*}\,\hat{\omega}_{3L}^{(d=6)}.$$

Also $(\mathscr{D})_x^y$ and $(\hat{\mathscr{D}})_a^b$ are covariant derivatives acting on the **27** of E_6 and **3** of SU(3), respectively. Observe that $K^{(i)}$ and $K^{(i)\prime}$ are purely imaginary.

The first term is a typical coupling of "model dependent" axions to gauge and gravitational fields discussed previously in the literature.[9,11,12] The strength of the coupling differs in general for each axion. The other terms did not appear before. They have some features to be discussed below. A main purpose of the rest of this Letter is to see how (11) modifies Witten's truncated action[7,12,13] which fits into a standard $N=1$ supergravity form.

The truncated action contains symmetries related to the classical scale invariance[7] in addition to $N=1$ local supersymmetry and Peccei-Quinn-symmetry. To be more explicit, the ten-dimensional action is rescaled by λ^4 under the transformation $g_{MN} \to \lambda g_{MN}$, $\phi \to \lambda^{-1}\phi$ and some rescaling involving fermions. The truncated action inherits this symmetry. On the other hand, the counterterms do not contain g_{MN} or ϕ in the right proportion to scale like λ^4. Thus the counterterms do not scale like the tree-level action and violate this classical symmetry.

A similar argument also applies to a rescaling property[13,14] which is distinct from the above scale invariance: $\phi \to r^{1/2}\phi$, $\kappa_{10} \to r\kappa_{10}$ and some rescaling for fermions. This is also broken by the counterterms.

If the scale invariance is valid, it highly restricts the form of the Kähler potential:

$$G = -\ln(S+S^*) - 3\ln(T+T^*) - h(C^*\cdot C/(T+T^*)) + \ln|W|^2, \qquad (13)$$

where $h(C,S,T)$ is *a priori* an arbitrary function and generates D-type interactions between the matter fields and S and T given by

$$S = \phi^{-1}e^{3\sigma} - i2D \qquad (14)$$

and

$$T = \phi e^{\sigma} + i\sqrt{2}a, \qquad (15)$$

where the model-independent axion field D is related to $H_{\mu\nu\rho}$ through a constraint $\delta\mathscr{L} = D\, dH$, giving the equations of motion

$$H_{\mu\nu\rho} = g_4^{-1/2}(\phi e^{-3\sigma})^2\epsilon_{\mu\nu\rho\sigma}(\partial^\sigma D/3\sqrt{2}).$$

The symmetries mentioned above tell us that the argument of the function must be the combination[13] $x = C^*\cdot C/(T+T^*)$ alone and, with Witten's truncation procedure, $h(x) = -3\ln(1-2x)$.

Since the counterterms violate the above-mentioned symmetry, one naturally expects that the argument of the function h be more general. In fact, the second term and some pieces of the third term of (11) are combined to give the coupling of the model-independent axion to matter scalars $(cK/48) \times HC^*\overleftrightarrow{\mathscr{D}}C$. This changes the kinetic term for the D field from $-\frac{1}{2}\phi^2 e^{-6\sigma}(\partial_\mu D)^2$ into

$$\mathscr{L} = -\tfrac{1}{2}\phi^2 e^{-6\sigma}[\partial_\mu D - (\sqrt{2}/40)KC^*\overleftrightarrow{\mathscr{D}}_\mu C]^2. \qquad (16)$$

This form is suggestive of a more general Kähler potential:

$$G = -\ln(S+S^*) - 3\ln(T+T^*) - h(C^*\cdot C/(T+T^*), C^*\cdot C/(S+S^*)) + \ln|W|^2 \qquad (17)$$

with $h(x,y) = -3\ln(1-2x) - \ln[1 - i\sqrt{2}(ck/20)y]$. What we learned from (11) is the presence of axionlike interactions between S and the matter fields of the type given by (16) in the effective action.

One consequence of the violation of the scale invariance is that the "coupling function" f_{AB} of the gauge kinetic term $\int d^2\theta\, f_{AB} W_A W_B$ is no longer restricted to be of the form $f_{AB} \propto S\delta_{AB}$: In fact, because of the first term in (11), it will pick up contributions to its imaginary part of the form $\mathrm{Im} f_{AB} = \frac{1}{80} cK'a$ which after (15) lead to an effective kinetic coupling $f_{AB}^{(i)} = S + n_i(cK'/80\sqrt{2})T$, $i = E_6, E_8$, $n_6 = -1$, $n_8 = +1$, of different strength for both gauge fields. In addition, the fourth term in (11) proportional to $\mathrm{tr}_2 F\tilde{F}C^* \cdot C$ is CP nonconserving and amounts to a contribution to the imaginary part of f_{AB} unless the manifold is chosen to make the coefficients $p^{(i)}$ vanish, guaranteeing the CP invariance of the four-dimensional theory. A dimension-seven operator similar to this, namely, $\mathrm{tr}_2 F\tilde{F}(W + W^*)$, appears too.

To summarize, we have shown that the residue of the counterterms left after truncation can be understood in terms of a generalized Kähler potential, including D-type interactions between S and the matter superfields, and a gauge kinetic coupling involving S and T. The latter includes some unexpected seeds of CP nonconservation in the observable sector.

A theorem recently shown by Witten[15] on nonrenormalization of nonderivative F terms does not apply here since the counterterms typically contain exact forms, i.e., $\int B\, dX_7$. Corrections given in (11) are outside the domain of the conventional σ-model perturbation theory. They might contain some sources of destabilization relevant to low-energy phenomenology.

We acknowledge stimulating discussions with T. Taylor. We also thank M. Peskin for helpful comments.

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