Frisch, Rivier, and Wyler Respond: In his Comment,[1] Luban discusses the range of validity of the hard-sphere fluid equation of state [Eq. (1) of Ref. 1, or (5) of our Letter[2]],

$$p/(\delta kT) = 1 + \tfrac{1}{2}\rho v \qquad (1)$$

in $D=\infty$ dimensions, obtained in Ref. 2. Here $v = V_D(a)$ is the volume of a D-dimensional sphere of radius a, where a is the *diameter* of the original hard spheres. It is the natural scale of volume for the Mayer cluster expansion.[3]

Equation (1) is *not* an expansion. It is, as shown in Ref. 2, an exact reversion of the Mayer series for the density or the pressure of the system, which is unique within the radius of convergence $|zv| < 1/e$ of the Mayer series. Here z is the activity.

Consequently, the relevant series to analyze is the Mayer series itself. In Ref. 2 and elsewhere,[4] we argue that only tree diagrams contribute to the Mayer series coefficients b_n as $D \rightarrow \infty$. Here is a summary of the full analysis.

Recall first that, according to Groeneveld's theorem,[3] the Mayer series of a hard-sphere fluid has a finite radius of convergence, and that its coefficients are bounded [Eq. (9) of Ref. 2]. The Mayer coefficients b_n are sums of all possible connected graphs on n labeled points, multiplied by their respective weights [which are restricted (overlap) integrals over the positions of the n hard spheres in the cluster, divided by $n!$]. Such diagrams are called *labeled star trees,* and their enumeration is discussed by Ford and Uhlenbeck.[5] The weight of a diagram containing one closed loop in D dimensions is smaller than that of the tree diagram of the same order by a factor $\alpha^D/\sqrt{D}$, $\alpha \leqslant 4/3^{3/2} < 1$.[2,4]

For c independent loops, the restrictions in the overlaps may not all be independent, so that the weight of the diagram is lower than that of the tree diagram of the same order by a factor $(\alpha^D)^n$ only [rather than $(\alpha^D)^c$, $c = e - n + 1$, where e is the number of edges in the diagram].

At every order, the total number of labeled star trees is finite. If one only restricts the diagrams to be built out of stars from a (infinite) collection of types of stars containing no more than a given number of independent loops (i.e., for a finite number of homeomorphic types of stars), then the number of such graphs is down to

$$\sim x_0^{-n+1/2} n^{-5/2} n!, \qquad (2)$$

where x_0 is finite.[5] There are $(x_0 e)^{-n}$ more graphs of this type than tree diagrams of the same order n, but their weight is down by $(\alpha^D)^n$, and they vanish against the trees at large D. Note that the restriction above on the collection (which is still infinite) of types of stars, is only sufficient. The weaker, necessary condition is not known.[5] In the case of polygonal stars (mixed Husimi trees), $1/x_0 e = 1.53$.[5]

In the absence of any restriction, the only statement which can be made now is that, at every order n, there is a D above which b_n is dominated by tree graphs. But the critical value of D then depends on n.

We are, however, dealing with a *convergent* series, with bounded coefficients, for which tree diagrams realize the upper bound. It would be extraordinarily subtle of those diagrams not included in the restricted collection of stars (2) if they conspired to modify the fluid equation of state (1) above a finite density of hard spheres.

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Received 18 February 1986
PACS numbers: 05.20.$-$y, 64.30.+t, 64.70.$-$p

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