Onset of Global Phase Coherence in Josephson-Junction Arrays: A Dissipative Phase Transition

Sudip Chakravarty, Gert-Ludwig Ingold, Steven Kivelson, and Alan Luther[a]

Department of Physics, State University of New York at Stony Brook, Stony Brook, New York 11794

(Received 11 February 1986)

A theoretical analysis is presented of the problem of the onset of global phase coherence in granular superconductors and Josephson-junction arrays. It is shown that the ratio $(h/e^2)/R$, where R is the resistance, plays an important role in the thermodynamics. This is due to quantum fluctuations of the order parameter and occurs in superconductivity only because the phase of the order parameter is both a statistical as well as a dynamical variable. The theory provides a natural qualitative explanation of the recent experiments of Orr, Jaeger, Goldman, and Kuper.

PACS numbers: 74.40.+k, 74.20.De, 74.50.+r

On the basis of experiments on thin films of granular superconductors, Orr, Jaeger, Goldman, and Kuper[1] concluded that the onset of global phase coherence is governed by the normal-state sheet resistance R; a film was found to become superconducting only if R was less than a critical resistance proportional to $R_0 = h/e^2$. Moreover, they suggested that this behavior is not due to disorder as in percolation or Anderson localization, but rather is due to the dissipation in the resistive elements.

This experiment and others[2] which are consistent with the present one raise a number of fascinating issues. Classically, the dynamics and the thermodynamics of a system are separate, so that dissipation could in no way affect thermodynamics! Moreover, it is an established, but nevertheless remarkable fact that phenomena involving superconducting and many other condensed states can be treated in terms of an order-parameter theory in which the expectation value of an appropriate quantum field operator is treated as a macroscopic dynamical variable. Even when the underlying condensed state is highly quantum mechanical, as is the case in superconductivity, the order parameter can usually be treated classically since quantum fluctuations of a macroscopic variable are usually small. The validity of this approach has been spectacularly confirmed by experiments involving the Josephson effect. Only recently has it been demonstrated that quantum fluctuations of the order parameter can be observed.[3]

It is the purpose of this paper to show that under conditions in which quantum fluctuations of the order parameter are important the dissipation enters the thermodynamics as a critical parameter. This can occur in superconductivity only because the phase of the order parameter is both a statistical as well as a quantum dynamical variable. Our theory thus provides a natural qualitative explanation of the experiments of Orr *et al.*

To be concrete, we consider a collection of superconducting grains with Josephson coupling between neighboring grains. We imagine that the largest Josephson coupling V is weak, i.e., $V \ll k_B T_C$, where T_C is the bulk superconducting transition temperature, and we focus on temperatures $T \lesssim V/k_B$. Since $T \ll T_C$ and the grains themselves are macroscopic, we can ignore the fluctuations of the magnitude of the order parameter, $\Delta(T)$; but the phase θ_j on each grain j remains a dynamical variable. A rather general Hamiltonian for such a system is ($\Delta\theta_{ij} = \theta_i - \theta_j$)

$$H = \frac{1}{2}\sum_i M_i \dot{\theta}_i^2 + \sum_{\langle ij\rangle} V_{ij}(1-\cos\Delta\theta_{ij}) + \sum_{\langle ij\rangle}\sum_\alpha \frac{1}{2} m_\alpha (\dot{x}_\alpha^{ij\,2} + \omega_\alpha^2 x_\alpha^{ij\,2}) + \sum_{\langle ij\rangle} \Delta\theta_{ij} \sum_\alpha f_\alpha^{ij} x_\alpha^{ij}, \tag{1}$$

where $M_i = C_i(\hbar/2e)^2$, C_i being the capacitance of the ith grain. The Josephson coupling energy V_{ij} between the grains i and j is given by $I_C^{ij}(\phi_0/2\pi)$, where I_C^{ij} is the critical current and $\phi_0 = h/2e$. The sum $\langle ij\rangle$ is over nearest-neighbor pairs. The first two terms constitute the widely studied[4] standard phase Hamiltonian. Here, we have added to the model the effect of dissipation in the junctions by coupling to a heat bath.[5] This Hamiltonian is constructed to be consistent with the usual resistively-shunted-junction equation[6] with Ohmic damping. C_i, V_{ij}, and f_α^{ij} are in general random variables.

After integrating out the oscillator degrees of freedom, the effective Euclidean action functional from which we compute the thermodynamics is given by

$$S_{\text{eff}} = \frac{1}{2}\int_0^{\beta\hbar} d\tau \left[\sum_i M_i \dot{\theta}_i^2 + 2\sum_{\langle ij\rangle} V_{ij}(1-\cos\Delta\theta_{ij})\right] + \frac{\beta\hbar}{4\pi}\sum_n |\omega_n| \sum_{\langle ij\rangle} \alpha_{ij} \Delta\theta_{ij}(n)\Delta\theta_{ij}(-n), \tag{2}$$

where $\alpha_{ij} = (\pi/2)(\hbar/e^2)/R_{ij}$ with R_{ij} the shunting resistance between the grains i and j. The last term in S_{eff} represents the dissipative effect of the environment. For later convenience we have expressed it in the form of a series, where $\omega_n = 2\pi n/\beta$ are the Matsubara frequencies and $\Delta\theta_{ij}(n)$ is the Fourier transform of $\Delta\theta_{ij}(\tau)$. We have assumed Ohmic dissipation characterized by a spectral density of the bath[5]

$$J_{ij} = \frac{\pi}{2}\sum_{\alpha}\frac{(f_{\alpha}^{ij})^2}{m_{\alpha}\omega_{\alpha}}\delta(\omega-\omega_{\alpha}) = \left(\frac{\hbar}{2\pi}\right)\alpha_{ij}|\omega|\theta(\omega_c-\omega), \quad (3)$$

where ω_c is a high-frequency cutoff of the order of a typical microscopic frequency.

The action functional given in Eq. (2) is very general. We start with the simplest case of a perfectly ordered d-dimensional simple-cubic array where $C_i = C$, $V_{ij} = V$, and $R_{ij} = R$ for all i and j. There are two reasons for considering this case: (a) Such arrays can and have been fabricated lithographically, and (b) the dissipative aspect of the phase transition that we are interested in depends very little on the disorder as we shall discuss.

The model can still not be solved exactly even if the dissipative term were absent. We shall pursue a variational approach to argue that an important aspect of the dissipative transition is associated entirely with the infrared property of the spectral density of the Ohmic heat bath $[J_{ij}(\omega) \rightarrow \omega,\ \omega \rightarrow 0]$ which leads to results similar to those obtained earlier[7] in the context of macroscopic quantum effects in Josephson systems. Moreover, we believe that the variational approach should give semiquantitative results for the phase diagram.

A variational estimate F' of the true free energy F is constructed by use of the Gibbs-Helmholtz inequality[8]

$$F' = F_{\rm tr} + \langle H - H_{\rm tr}\rangle_{\rm tr} \geq F, \quad (4)$$

where H is the true Hamiltonian, and $F_{\rm tr}$ and $\langle \cdots \rangle_{\rm tr}$ are the free energy and thermal average computed with the trial Hamiltonian $H_{\rm tr}$. We choose the trial Hamiltonian or equivalently the effective trial action[8] to be the following:

$$S_{\rm tr} = \tfrac{1}{2}\int_0^{\beta\hbar} d\tau \left[M\sum_i \dot{\theta}_i^2 + D\sum_{\langle ij\rangle}\Delta\theta_{ij}(\tau)^2\right] + \frac{\beta\hbar\alpha}{4\pi}\sum_n |\omega_n| \sum_{\langle ij\rangle}\Delta\theta_{ij}(n)\Delta\theta_{ij}(-n), \quad (5)$$

where the variational parameter D is the effective spin-wave stiffness constant. The underlying physical picture is that if the system has a global phase coherence, then the spin-wave stiffness will be finite, but if it does not, D will be zero. Global phase coherence in the spin language is also the same as the global phase coherence in the sense of superconductivity.

At this point we mention a technicality[9] concerning our trial action; to be more precise, the potential should be periodic under $\theta_j \rightarrow \theta_j + 2\pi$ which the quadratic potential in Eq. (5) is not. Thus, one should really consider a periodic ("scalloped") potential which is quadratic about each of its equivalent minima. For large D, where the phase fluctuations are small, this is a trivial change since the different wells make, to an excellent approximation, independent contributions to F' which can be evaluated directly from Eq. (5). When phase fluctuations are of the order of π (i.e., for small D) the periodicity of the potential must be treated carefully. This has not been necessary for our results. Specifically, to determine the location of the spinodal and the continuous transition points, it is only necessary to evaluate the derivative of the free energy with respect to D. Thus we never need to compute F' as $D \rightarrow 0$ (cf. below).

With this in mind, minimization of F' with respect to D leads to the self-consistent equation for a d-dimensional hypercubic lattice

$$D/V = \exp(-\tfrac{1}{2}\langle\Delta\theta_{ij}^2\rangle_{\rm tr}), \quad (6)$$

where

$$\langle\Delta\theta_{ij}^2\rangle_{\rm tr} = \frac{2}{\beta N d}\sum_k\sum_{n=0}^{\infty}\frac{z - 2\sum_{i=1}^{d}\cos k_i a}{\omega_n^2/E_0 + (D + \alpha\omega_n/2\pi)\left[z - 2\sum_{i=1}^{d}\cos k_i a\right]}, \quad (7)$$

with $E_0 = 4e^2/C$ and $\alpha = (\pi/2)(\hbar/e^2)/R$. z is the coordination number and Nd is the total number of bonds in the lattice. The k sum runs over the first Brillouin zone.

The phase diagram is obtained from Eq. (6). We characterize the transition to be first or second order depending on whether the nontrivial $(D \neq 0)$ solution of Eq. (6) appears discontinuously or continuously. Actually what we are calling a first-order transition is, strictly speaking, a spinodal point. The true transition takes place only when $F'(D) = F'(D=0)$.

The zero-temperature phase diagram is shown in Fig. 1 for $d = 1, 2,$ and 3. The vertical boundary at $\alpha = 1/d$ is a line of second-order transition. We speculate that this is an exact result (see below). It is clear from the figure that for low dissipation, when V/E_0 is sufficiently small, quantum fluctuations destroy the long-range phase coherence even at $T = 0$. The interesting point to note here is that for all values below a threshold the second-order transition takes place at $\alpha = 1/d$ regardless of the value of V/E_0. Since the only place at which capacitance enters the problem is E_0, the capacitance is an irrelevant variable in this regime and the transition is entirely dictated by α. It is perhaps in this sense that one has to interpret the statement of Orr, Jaeger, Goldman, and Kuper that α is the only relevant variable. When α is greater than $1/d$, the dissipation suppresses the quantum fluctuations and restores global superconductivity. At finite temperature, the transition is always first order in the sense described above. The transition temperature $T^*(\alpha, V/E_0)$ is an increasing function of both of its arguments.

The above self-consistent harmonic approximation is, of course, not reliable in the critical region. In order to understand the critical region we are pursuing renormalization-group calculations.[10] Partial results allow us to conclude the following: At any nonzero temperature the phase transition is what one would get from an effective classical XY model of the same dimension as the array provided that one goes very close to the transition temperature. In particular, the parameters of the effective classical Hamiltonian are affected in a novel way by the presence of dissipation. For instance, in the lower left-hand corner of the phase diagram in the figure the effective coupling between grains renormalizes to zero. There is also an interesting crossover from zero-temperature quantum critical behavior controlled by dissipation to the finite-temperature classical behavior. As a result, for large α the critical region becomes extremely small.

We now describe in more detail our calculations and results in the novel regime of weak coupling and low temperatures, $E_0 >> 2\pi/\beta$, $2\pi D/\alpha$, where quantum effects are most important. In this regime E_0 serves only to define a high-energy cutoff for the sum in Eq. (7) at $n_0 = \beta E_0/2\pi$; otherwise ω_n^2/E_0 can be ignored. The k sum is trivially performed by assumption of a spherical Brillouin zone. (It can be shown that a better approximation to the Brillouin-zone sum does not change the conclusion that the transition is at $\alpha = 1/d$.) The self-consistent equation determining D now becomes

$$\frac{D}{V} = \left(\frac{2\pi}{\beta E_0}\right)^{1/d\alpha} \exp\left[\frac{1}{d\alpha}\psi\left(\frac{\beta D}{\alpha}\right)\right], \tag{8}$$

where $\psi(x)$ is the digamma function. In the limit $\beta \to \infty$ this equation leads to the solution[11]

$$D = E_0\left(\frac{2\pi}{\alpha}\right)^{1/(d\alpha-1)}\left(\frac{V}{E_0}\right)^{d\alpha/(d\alpha-1)} \quad \alpha > 1/d,$$
$$D \equiv 0, \quad \alpha \leq 1/d. \tag{9}$$

Thus the transition takes place at $\alpha = 1/d$ in d dimensions. As noted earlier the capacitance only enters through E_0 which merely sets the energy scale, but the critical parameter α depends only on the resistance. It is important to realize that this result is critically dependent on the fact that the heat bath we have considered is Ohmic [cf. Eq. (3)]. At low temperatures it is not difficult to show that the transition line between the normal and the superconducting phase in the α-T plane is given by

$$\frac{kT^*}{E_0} = \left\{\frac{\alpha x_0}{V/E_0}\left(\frac{1}{2\pi}\right)^{1/d\alpha} \exp\left[-\frac{1}{d\alpha}\psi(x_0)\right]\right\}^{d\alpha/(1-d\alpha)}, \tag{10}$$

where T^* denotes the transition temperature and x_0 is the solution of the equation $x_0 = d\alpha/\psi'(x_0)$. The first-order lines in Fig. 1 were obtained numerically. With regard to the first-order line we would like to caution the reader that its position is sensitive to the details of the model, in particular to the precise form of the spin-wave spectrum at all k. The results in Fig. 1 assume a Debye spectrum. In short, the exact location of the first-order line, but not its very existence, is quite model dependent.

Finally, we would like to discuss the effect of disorder. Since there exists a correlation length that diverges at the second-order line, weak disorder can only change the result from $\alpha = 1/d$ to $\alpha = A/d$, where A is a number of the order of unity. However, we expect strong disorder to be a common feature of the composites such as those studied by Orr *et al.* because the Josephson coupling between the grains i and j is proportional to a tunneling matrix element, as is $1/R_{ij}$. Thus, we expect V_{ij} and R_{ij} to be log-normally distributed and strongly correlated with each other. In fact, most naively, we expect $R_{ij}V_{ij} = \text{const} \times \Delta$.

We now argue that even though the distributions of

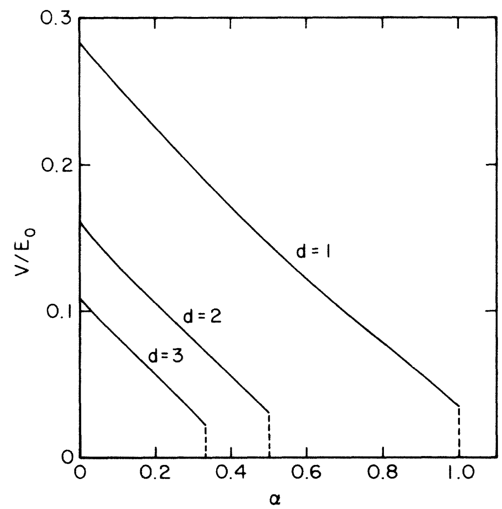

FIG. 1. The phase plane for $T = 0$ for $d = 1, 2$, and 3: The superconducting phase lies above the curves, and the normal state below. D changes continuously across the dashed lines and discontinuously across the solid lines.

resistances $P(R)dR$ and of Josephson couplings $Q(V)dV \sim P(\Delta/R)(V/R)dR$ are extremely broad, the problem is equivalent to a network with a larger lattice constant and a narrow distribution of R and V. Our argument is based on a percolation treatment akin to that of Ambegaokar, Halperin, and Langer[12] and Deutscher *et al.*[13] We imagine removing all resistors with $R_{ij} >> R_c$, and replacing those with $R_{ij} << R_c$ with short circuits. We are left with a new network with $P(R_c)\Delta R$ resistors, narrowly distributed over a range ΔR about R_c. If R_c is chosen so that the network just percolates (for $\Delta R \rightarrow 0$) and ΔR is chosen appropriately, the resulting network has approximately the same normal-state resistance as the original network. In much the same way, as we approach the superconducting transition, all those junctions with $V_{ij} << V_c$ and $R_{ij} >> R_c$ ($V_c R_c \sim \Delta$) can be ignored since the phase fluctuations across the junctions are enormous. Similarly, all junctions with $V_{ij} >> V_c$ and $R_{ij} << R_c$ can be treated as superconducting wires since the phases of the two grains connected by that junction are essentially locked, and the two grains can be treated as a single grain. Thus, for the purposes of computing both the normal-state resistance and the superconducting transition the highly disordered system is equivalent to a weakly disordered system.

In conclusion, we note that we have shown that if the quantum fluctuations of the phase of the superconducting order parameter are important, the dissipation can play a crucial role in determining the onset of global superconductivity, and have thus provided a natural qualitative explanation of the recent experiments. Orr *et al.* have made the interesting speculation that R is the only relevant variable determining the onset of global phase coherence and that therefore in two dimensions, where $R \rightarrow \infty$ and $T \rightarrow 0$, there is a universal disappearance of superconductivity. It is clear from our work that for strongly coupled Josephson junctions (cf. Fig. 1) global phase coherence persists when $\alpha \rightarrow 0$ ($R \rightarrow \infty$). However, in the quantum regime where V/E_0 is small and T is near zero, R is indeed the sole critical parameter. For a better understanding of the fascinating issues that these experiments raise, it would be useful to perform these experiments with use of ordered arrays.

We would like to thank B. Orr and A. Goldman for discussions and for sending us a preprint of their work before its publication. This work was partially supported by National Science Foundation Grants No. DMR-83-18051 (S.K.) and No. DMR-83-01510 (S.C.) and by a grant from the Department of Physics, State University of New York at Stony Brook (A.L.). Two of us (S.K. and S.C.) also acknowledge support through fellowships from the Alfred P. Sloan Foundation. One of us (G.I.) would like to acknowledge support from the Studienstiftung des deutschen Volkes.

[a]Permanent address: NORDITA, Blegdamsvej 17, DK-2100 Copenhagen, Denmark.

[1]B. G. Orr, H. M. Jaeger, A. M. Goldman, and C. G. Kuper, Phys. Rev. Lett. **56**, 378 (1986).

[2]See references to earlier experiments given in Ref. 1.

[3]"New Techniques and Ideas in Quantum Measurement Theory" (New York Academy of Sciences, New York, to be published).

[4]K. B. Efetov, Zh. Eksp. Teor. Fiz. **78**, 2017 (1980) [Sov. Phys. JETP **51**, 1015 (1980)]; D. M. Wood and D. Stroud, Phys. Rev. B **25**, 1600 (1982); R. M. Bradley and S. Doniach, Phys. Rev. B **30**, 1138 (1984); L. Jacobs, J. V. José, and M. A. Novotny, Phys. Rev. Lett. **53**, 2177 (1984), and references therein.

[5]A. O. Caldeira and A. J. Leggett, Ann. Phys. (N.Y.) **149**, 374 (1983), and **153**, 445 (E) (1984); see also V. Ambegaokar, U. Eckern, and G. Schön, Phys. Rev. Lett. **48**, 1745 (1982).

[6]A. Barone and G. Paterno, *Physics and Applications of the Josephson Effect* (Wiley, New York, 1982).

[7]A. Schmid, Phys. Rev. Lett. **51**, 1506 (1983); F. Guinea, V. Hakim, and A. Muramatsu, Phys. Rev. Lett. **54**, 263 (1985).

[8]R. P. Feynman, *Statistical Mechanics* (Benjamin, New York, 1972).

[9]For the same reason, our results are substantially unchanged if we identify θ_j and $\theta_j + 2\pi$. However, we do not feel that this identification is appropriate in the presence of the heat bath since it records the history of the phase.

[10]S. Chakravarty, G.-L. Ingold, and S. Kivelson, to be published.

[11]Note that the solution given in Eq. (9) is very similar to that obtained for a particle in a cosine potential and coupled to an Ohmic heat bath as discussed by M. P. A. Fisher and W. Zwerger, Phys. Rev. B **32**, 6190 (1985). This is a surprising result which is due to a very small behavior of $\langle \Delta\theta_{ij}^2 \rangle$ in the limit of large E_0; the k dependence simply drops out, giving only an overall factor which is related to the volume of the Brillouin zone. Moreover, as noted earlier, the infrared behavior of the heat bath plays a special role and in a very subtle way its effect is the same as that discussed in Ref. 7.

[12]V. Ambegaokar, B. I. Halperin, and J. S. Langer, Phys. Rev. B **4**, 2612 (1971).

[13]G. Deutscher, O. Entin-Wohlman, S. Fishman, and Y. Shapira, Phys. Rev. B **21**, 5041 (1980).