Stimulated Raman Scattering in the Presence of Filamentation in Underdense Plasmas

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A model of stimulated Raman scattering from underdense plasmas in which the laser intensity profile and plasma density have been corrupted by the filamentation instability is described. The model accounts in a unified way for inhomogeneity in the density, for Landau damping, and for local enhancements in light-wave intensities. In shallow filaments the concentration of the light gives rise to modest increases in growth. On the other hand, for deeper filaments the inhomogeneity and Landau damping dominate to suppress the instability. In addition, backscatter is enhanced relative to sidescatter.

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The importance of stimulated Raman scattering (SRS) from the extensive underdense coronas characteristic of contemporary laser fusion experiments has been recognized for some time.[1] While the observations confirm some of the characteristics of Raman emission there are features which do not readily conform to the predictions of the standard treatment of the instability in a linear density profile. If we are to understand the details of the various Raman observations it is important to model the plasma more realistically and in particular to take account of the possible effects of other instabilities taking place concurrently.[2] In this Letter we focus attention on ways in which SRS may be affected by filamentation. Filamentary structures are present in many target plasmas, these structures being localized on a spatial scale finer than that associated with nonuniformities in the incident beam.[3] There is some indirect evidence that filamentation can affect the Raman emission.[4] We would expect on the one hand that the enhanced intensity of radiation within the filament might lead to correspondingly enhanced Raman growth while, on the other, the density modulation resulting from filamentation is likely to restrict the development of the Raman instability through its effect on the plasma wave, since phase matching is only satisfied locally. Experiments in which filamentation has been observed provide some information about the width and, less certainly, the length of filaments but none about details such as depth or shape. In this work we adopt a model which is readily amenable to analysis and yet able to account fully for both the plasma inhomogeneity (without recourse to assumptions of weak inhomogeneity, to a local approximation, or to WKB analysis) and the kinetics (Landau damping). In the main, most approaches either adopt a fluid theory for weakly inhomogeneous plasma with an *ad hoc* Landau damping, or use kinetic theory with a local approximation to which a weak inhomogeneity may be added phenomenologically. We circumvent these limitations by assuming a sinusoidal density profile which allows the usual differential equations (fluid or Vlasov) describing SRS to be transformed to a difference-equation representation in wave-number space that is nonhomogeneous or homogeneous depending on the boundary conditions chosen. In this work we have chosen periodic boundary conditions giving a homogeneous equation. A finite-order difference equation results which may be routinely solved.

The filament is modeled by an electron plasma whose zero-order electron density is

$$n_0(y) = n_0(1+\epsilon\cos 2Ky) \tag{1}$$

and so is assumed to have a slab geometry with a wavelength $\lambda = \pi/K$ and prescribed depth 2ϵ relative to the mean background density n_0. The laser light, of frequency ω_0 and vacuum wavelength $\lambda_0 = 2\pi/k_0$, is incident along the filament in the x direction. The Vlasov equation with the ponderomotive force of the beating light waves included has been adapted to describe driven plasma waves in the presence of the profile given by (1). The light waves are adequately treated by fluid theory, incorporating (1).

The propagation of plasma waves parallel to a sinusoidal density gradient has been considered by others, the propagation characteristics being described by the Mathieu equation.[5] The propagation of light waves polarized perpendicular to the density gradient (s polarized) is similarly described. Plasma waves propagating obliquely to the density gradient satisfy a closely related equation. We assume that incident and scattered waves are both s polarized. The propagation of all three waves may then be described in wave-number space by the difference equation

$$(a-\kappa_n^2)E_n = q(\theta_n E_{n-1} + \theta_{n+1}E_{n+1}), \tag{2}$$

where $\mathbf{k}_n = \mathbf{k} + 2n\mathbf{K}$, $\mathbf{K} = K\hat{\mathbf{y}}$, $\boldsymbol{\kappa}_n = \mathbf{k}_n/K$, and $E_n = E(\mathbf{k}_n)$ is the electric field of the wave in question. For light waves $a = (\omega^2 - \omega_p^2)/K^2c^2$, $q = \epsilon\omega_p^2/2K^2c^2$, $\theta_n = 1$, and ω_p is the plasma frequency corresponding to the mean density n_0. For plasma waves $a = (\omega^2 - \omega_p^2)/3K^2V_T^2$, $q = \epsilon/6K^2\lambda_D^2$, and $\theta_n = \hat{\mathbf{K}}_n \cdot \hat{\mathbf{K}}_{n-1}$ is a geometric factor; V_T and λ_D represent the electron thermal velocity and the Debye length, respectively. Treating the plasma waves kinetically requires the replacement of $(\omega^2-\omega_p^2-3k^2V_T^2)/\omega_p^2$ by $-[1+1/\chi(\mathbf{k},\omega)]$ and χ is the usual electron susceptibility for a homogeneous Maxwellian plasma.

For a given $\mathbf{k}$, the periodic solutions of (2) yield an infinite but discrete set of eigenvalues $a=a_N$ ($N=0, 1, 2, \ldots$). The crucial quantity determining the nature of the solution is the coupling parameter q. When q is small, the wave can adjust its wave number to compensate for changes in density and hence propagates at every density present, i.e., between $\omega_\pm^2=\omega_p^2(1\pm\epsilon)$. When q is large this compensation is no longer possible and the wave is evanescent at higher densities. The wave then becomes trapped within the filament. Just what large or small q means must be seen in relation to the particular eigenstate $a_N(q)$. Higher N corresponding to higher frequency requires larger values of q to trap such waves within the filament. In addition, coupling into shorter-wavelength modes implied by (2) gives rise to enhanced Landau damping which can be the dominant effect for sufficiently deep filaments.

For light waves, $g_{\text{em}}=2\epsilon(n_0/n_c)(\lambda/\lambda_0)^2$ where n_c is the critical density for the laser light. The parameters used throughout this work are $n_0=0.1n_c$, $V_T=0.035c$, $\lambda=10\lambda_0$, and $v_0=0.01c$; v_0 is the quiver velocity of electrons in the laser electric field. Then $g_{\text{em}}=20\epsilon$ which can easily be greater than unity implying that the first few eigenstates are trapped within the filament, the remainder being free to propagate in what, for them, is an everywhere underdense plasma. To illustrate this, Fig. 1(a) shows the intensity variation across a filament when $\epsilon=0.15$ for the laser driver or backscattered light (both correspond to an $N=0$ state as given by the dashed curve) and sidescattered light (corresponding to an $N=6$ state as given by the solid curve). The figure shows the "filament," the concentrated intensity profile of the laser light consistent with the density channel (1), which is used as the driver in the SRS equations. Backscattered light similarly suffers filamentation while sidescattered barely "sees" the density variation.

Plasma waves, on the other hand, scale to much shorter lengths and therefore have a coupling parameter

$$q_{\text{es}}=(c^2/3V_T^2)q_{\text{em}}$$

$$\approx 344\epsilon(n_0/n_c)(\lambda/\lambda_0)^2[(1\ \text{keV})/T].$$

This can easily be very large implying strong localization of plasma-wave energy at specific points within the filament. Propagation anywhere requires $\omega>\omega_-$, the minimum plasma frequency, yet Landau damping implies an upper limit to the frequency, $\omega<\omega_+$. Within this range a finite number of eigenstates may be supported, the number being approximately $\surd(q/4)$. As the filament or cavity deepens successively more plasma waves may be excited.[6] Figure 1(b) displays the intensity profiles for three of the possible fourteen plasma-wave states supported by a cavity for which $\epsilon=0.15$. The lowest-frequency mode $\omega\simeq\omega_-$ ($N=0$) is that localized near the density minimum while the highest is $\omega\simeq\omega_+$ ($N=13$) localized near the density maximum. One intermediate state, $N=4$, is also shown localized midway up the filament wall.

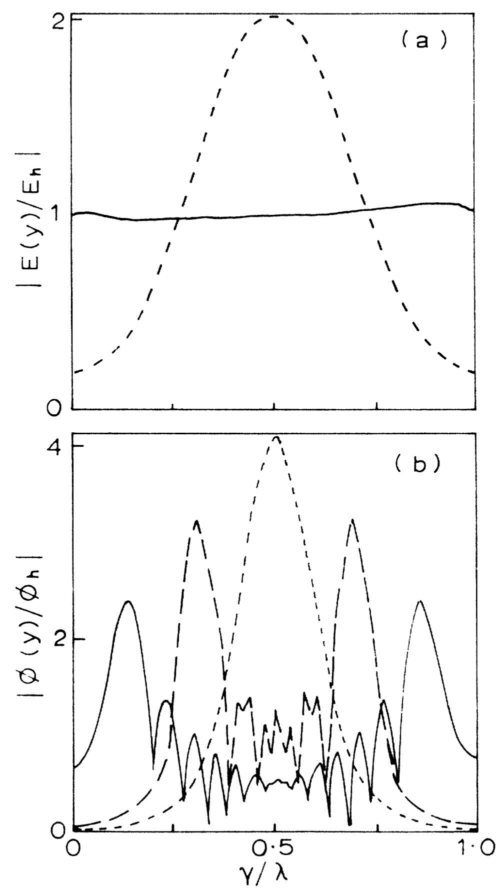

FIG. 1. Profiles of electric field and potential (normalized to homogeneous values) across a filament for light wave and plasma waves when $\epsilon=0.15$. (a) Incident and backscattered light, $N=0$ (dashed line); sidescattered light, $N=6$ (solid line). (b) Plasma-wave eigenstates $N=0$ (short-dashed line), $N=4$ (long-dashed line), or $N=13$ (solid line).

Of course, in an inhomogeneous plasma three-wave interactions may occur at any of the densities present (up to $n_c/4$). In the context of the present periodic system and the consequent discrete set of eigenstates we can have resonant three-wave interactions between a whole series of pairings of light-wave and plasma-wave eigenstates. In principle, then, we need to solve three coupled sets of equations of the form (2). In practice, we may simplify the problem by observing that since $q_{es} >> q_{em}$ we may assume, to lowest order, that the light-wave intensities are uniform across the filament. Figure 2 illustrates the effect of the inhomogeneity on the plasma wave alone. When $\epsilon = 0$, the homogeneous growth rate has a maximum value $\gamma_0 = k v_0 \omega_p / 2(\omega \omega_s)^{1/2}$ and bandwidth γ_0. Reduction in growth is rapid until the bandwidth associated with the inhomogeneity $\epsilon \omega_p \cong \gamma_0$; subsequently it is rather slower but such that the bandwidth for growth is determined by $\epsilon \omega_p$ rather than γ_0. This is akin to using bandwidth in the incident laser to reduce growth. A cold-plasma theory shows that oscillations exist only at density maximum, $\omega = \omega_+$, and density minimum, $\omega = \omega_-$, in other words only where density gradients are zero. These dominate as Fig. 2 shows clearly (ω_+ on the left, ω_- on the right) with growth rates deduced analytically to be $\gamma_\pm = \frac{1}{2}\sqrt{3}(\epsilon \omega_p^2 / \gamma_0 \omega_\pm)^{-1/3} \gamma_0$ when $\epsilon \omega_p > \gamma_0$. These expressions reproduce the numerical values to good accuracy. Finite temperature allows states at intermediate frequencies as clearly seen when $\epsilon = 0.15$ in Fig. 2. The frequency separation between peaks can also be deduced analytically to be $\delta\omega \cong 2KV_T\surd(6\epsilon)$. Each peak in Fig. 2 is associated with a given eigenstate, N, which is localized at a specific point within the filament or cavity [cf. Fig. 1(b)].

Given this strong localization of the plasma-wave energy it is straightforward to predict just how the nonuniform light-wave intensity profiles will modify Fig. 2. If we include in the first instance the effects of the filament on the scattered wave alone, Fig. 1(a) shows that backscattered light ($N = 0$) is concentrated around the filament bottom. The numerical results show that the resonance at ω_- is enhanced while that at ω_+ is diminished by just those factors which would be anticipated from Fig. 1(a). Growth at ω_- still shows a reduction with increasing ϵ, albeit now rather weak, while that at ω_+ is all but suppressed, there being little scattered light-wave energy near the density maximum where this resonance occurs.

Finally, including the nonuniform laser intensity profile produces Fig. 3; here all three interacting modes are correctly treated for the profile (1). Figure 3(a) shows the backscatter growth rates showing a fur-

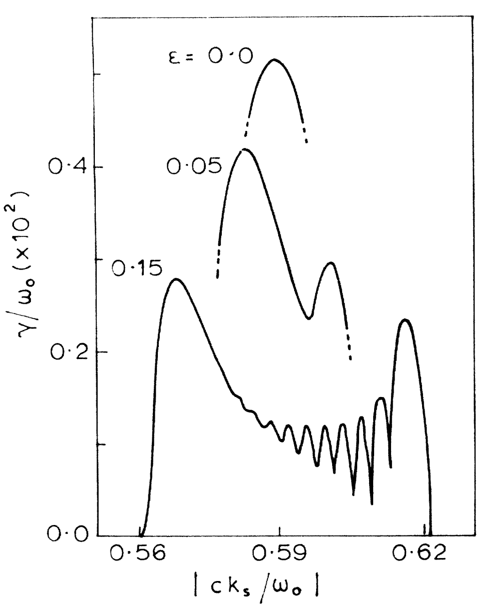

FIG. 2. The SRS growth rate for backscatter as a function of the scattered light wave number assuming light waves of uniform intensity across the filament.

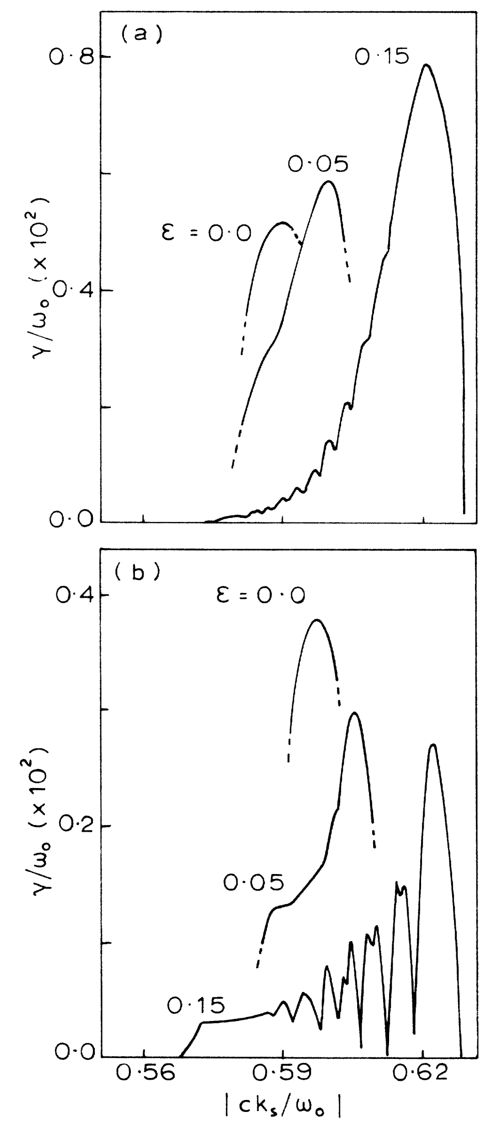

FIG. 3. The SRS growth rate for (a) backscatter and (b) sidescatter as a function of the scattered light wave number including the effects of filamentation on all waves.

ther expected strong bias towards the resonance at the filament bottom. For deeper filaments the net growth shows an increase over homogeneous values which continues until the minimum density reaches a value such that Landau damping becomes strong $k\lambda_D$ locally large) after which it quickly decreases with ϵ. Finally a stage is reached where Landau damping is so strong that around the density minimum SRS degenerates into stimulated Compton scattering and the maximally growing states are those at higher densities (i.e., localized up the filament wall) where Landau damping is relatively weaker and SRS still occurs. Ultimately, the regions of laser-light concentration and plamsa-wave propagation become mutually exclusive with only stimulated Compton scattering remaining.

Growth for sidescatter shows essentially the same features seen in backscatter with one exception. The difference is illustrated in Fig. 1(a) showing that sidescattered light ($N=6$) is barely affected by the filamented profile. Relative to the backscatter case, this means that the enhancement of the resonance at ω_- is correspondingly weaker as is clear from Fig. 3(b). One effect of filamentation, which might potentially signal its presence, is to make SRS backscatter relatively much stronger than sidescatter even when the plasma is sufficiently broad to allow significant lateral gain.[4]

Isolated filaments would give identical results to those discussed here for the assumed periodic array of filaments at least for backscatter where strong trapping prevents transmission of either plasma-wave or light-wave energy from one filament to the next. However, any density variation along the filament imposes a phase-mismatching length requirement as in the standard inhomogeneous theory. By contrast, sidescattered light which emanates from densities well away from the quarter-critical density is little affected, apart from needing enough filaments to be present to achieve lateral gain (thus imposing a condition on the lateral extent of the periodic plasma). Sidescattered light from densities approaching quarter critical is strongly trapped within a single filament. This is a problem we will return to elsewhere; for present purposes suffice it to say that complex interference effects occur and that absolute instability can arise over a wider range of densities.

In summary, this work has shown scattering to be strongly confined to the bottom of filaments. In shallow filaments modest increases in growth result, due mainly to the locally enhanced laser intensity there. The increase is less than might be anticipated since the inhomogeneity reduces growth, through bandwidth effects, by its action on the plasma-wave propagation. For deeper filaments, Landau damping at the lower densities present plays the dominant role strongly suppressing growth. Sidescatter exhibits no enhancement over its homogeneous value and is decreased relative to backscatter.

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