Applicability of Asymptotic QCD for Exclusive Processes

O. C. Jacob and L. S. Kisslinger
Physics Department, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213
(Received 9 September 1985)

By directly applying light-front boosts to model wave functions, we calculate the soft parts as well as the hard parts of exclusive processes. In the model used by Isgur and Llewellyn Smith, the asymptotic terms begin to dominate for $Q_c^2 \approx 3.5$ $(\mathrm{GeV}/c)^2$ for the pion charge form factor, a much smaller value than previously reported.

PACS numbers: 13.40.Fn, 12.38.Bx, 25.30.Bf

With strong evidence that hadrons are composed of quarks and glue as expected in quantum chromodynamics (QCD), medium-energy nuclear/particle physics has begun to enter a new era. Models of the quark structure of hadrons are being developed in which the static and low-energy properties of baryons and mesons are fitted rather well, and theorists are studying various models of the modification of baryons in nuclei, such as the effects of six-quark and other multiquark structures. In the carrying out of this program, perhaps the most fundamental problem is the nature of the confinement phenomenon, the mechanism by which colored quarks and gluons are confined. Undoubtedly deconfinement occurs at some level in nuclei, and the determination of the quark and gluon structure of nuclei can play a major role in our understanding of how QCD is realized in nature.

However, the nonperturbative nature of QCD when applied to low- to medium-energy nuclear/particle physics makes it difficult to connect models of hadronic and nuclear properties with QCD itself. As Brodsky and Lepage[1,2] and others have stressed, if one can carry out experiments at such high momentum transfer that perturbative QCD can be reliably used, then QCD can be applied directly for hadronic and nuclear processes. A detailed methodology of light-front perturbation theory has been developed, with application to a wide variety of exclusive hadronic and nuclear processes.[1,2] However, the key question is this: At what momentum transfer for a specific process can perturbative QCD be applied? It has been argued that since the Λ parameter sets the scale, with the effective strong-coupling constant for momentum $Q^2 >> \Lambda^2$, $\alpha_s(Q^2) \propto 1/\log(Q^2/\Lambda^2)$, and $\Lambda \lesssim 200$ MeV, then one might be able to use perturbative QCD methods at Q values of 1 to a few gigaelectronvolts and larger.[2]

In an attempt to answer this question, Isgur and Llewellyn Smith calculated[3] the pion electric and nucleon magnetic form factors, both for the asymptotic (hard) parts and for the "soft parts," which are expected to dominate at low momentum transfers. A necessary condition for perturbative QCD to apply is that the hard part dominates, and thus a minimum value of momentum transfer, Q_c, at which one might attempt to use the asymptotic light-cone formalism is that value for which the hard contributions become larger than the soft ones. It was the conclusion of Ref. 3 that this occurred at momentum transfers much larger than a few gigaelectronvolts, and that with present and contemplated accelerators it might not be possible to test the predictions of perturbative QCD for exclusive processes. An essential aspect of this work is that the question of normalization of the asymptotic parts of exclusive processes must be seriously considered.

However, there are a number of difficulties which must be considered in evaluating the conclusions of Ref. 3. First, almost all of the results are model dependent at the momentum transfers being considered. Second, there are significant technical difficulties in carrying out the boosts of the wave functions, which are necessary in the calculation. This latter is an essential ingredient in defining the soft parts. It is the purpose of the present Letter to show that the methods of constructing boosted light-front wave functions which have been recently developed[4] enable one to use light-cone perturbation methods[5] in a more general way than the asymptotic methods of Ref. 1. We restrict our present discussion to the pion form factor, where a model-independent calculation of the hard part has been derived.[6] Starting with the same rest-system wave function for the pion as used in Ref. 3, we conclude that the asymptotic terms will dominate at $Q_c \approx 2$ GeV, a value considerably smaller than that found in Ref. 3. The method can be applied for baryons and nuclei, and the implications for hadronic and nuclear physics are briefly discussed.

The definition of soft versus hard parts must be considered carefully. If one were able to derive the correct QCD wave functions, then the impulse approximation—the one-body term for the transition operator—could be used for all Q. The wave function would contain all the effects of hard as well as soft gluons. Indeed, the methods of Ref. 1 are used to obtain the hard-gluon contributions starting from wave functions which model only soft-gluon effects. The assumption of Ref. 3 is that the model wave functions do not contain hard-gluon effects, which is demonstrated to be correct in the present work. That is why the results of Ref. 3 are so surprising.

The asymptotic part of the pion electric form factor for momentum transfer Q is given in light-cone perturbation theory by the form

$$F_\pi(Q^2) = \int_0^1 dx \int_0^1 dy\, \Phi_Q^*(x) T_H(x,y,Q) \Phi_Q(y), \tag{1}$$

where x and y are relative longitudinal quark momentum fractions (i.e., $x = x_1 - x_2$), and T_H is the light-cone scattering amplitude calculated with single hard-gluon exchanges. For high Q the quark amplitudes, $\Phi_Q(x)$, are of factorized form[1,7]

$$\Phi_Q(x) = \Phi(p_\perp)[x(1-x)]^{\eta(Q)}, \tag{2}$$

with $\eta(Q) = 1$ as $Q \to \infty$, and an integral over $p_\perp$ is implied.[1] With the form (2) and Eq. (1), one does indeed approximately recover the model-independent asymptotic result,[6] with the normalization given by the pion decay constant. An assumption of Ref. 3 is that the $\Phi_Q(x)$ contain the soft-gluon effects, while the hard-gluon processes are calculated directly in T_H.

In Ref. 3 harmonic-oscillator-type wave functions are used, so that for the pion in the rest frame with relative $q\bar{q}$ momentum $\mathbf{p} = \mathbf{p}_1 - \mathbf{p}_2$

$$\phi_\pi(\mathbf{p}) = e^{-p^2/2b^2}/[(\pi b)^{1/2}]^{3/2}. \tag{3}$$

It is shown that with factorization assumptions such as Eq. (2) to obtain the longitudinal momentum dependence on the light cone, with Eq. (1) one can approximately reproduce the correct asymptotic result.

The soft part is essentially the impulse approximation, since the model wave function of Eq. (3) most certainly does not contain hard-gluon effects [as can be seen, e.g., from Eq. (2)]. In general, the soft part is model dependent. Moreover, with typical confining quark models it is difficult to carry out the boosts. In Ref. 3, factorized forms such as Eq. (2) are used also for the calculation of the soft part. A typical result is shown in Fig. 1, the dashed curve labeled I-LS. One can see why it was concluded that asymptotic QCD perturbation theory cannot be used for Q in the few-GeV/c range.

On the other hand, under certain circumstances one can boost composite model wave functions from the rest frame, where static properties can be fixed to the light cone in a light-front representation. For point constituents, if one starts with a model in which the total linear and angular momenta are defined, then it is possible to carry out the boost in one direction with

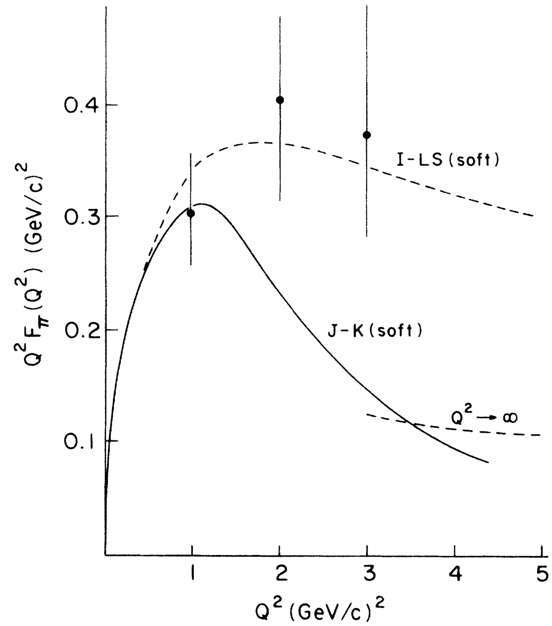

FIG. 1. Pion charge factor as calculated in Ref. 3 [I-LS] and as calculated in the present work [J-K], both starting with the pion wave function of Eq. (3). The $Q^2 \to \infty$ curve is from Ref. 6.

a light-front boost generator independent of the interaction Lagrangean.[5] Harmonic-oscillator wave functions are particularly easy to use, since projection of total linear momentum is very straightforward. In Ref. 4, explicit prescriptions for boosting a state from the instant form $|\pi\rangle_T$ to the light-front state $|\pi\rangle_F$ are given,

$$|\pi\rangle_F = U|\pi\rangle_T, \tag{4}$$

so that one can obtain normalized light-front wave functions from the wave functions in the instant form

$$\langle p_1, p_2|\pi\rangle_T = \Phi_\pi(p,P), \tag{5}$$

with four-vectors $P = p_1 + p_2$, $p = p_1 - p_2$. Note that for the pion the transformation of Eq. (4) is particularly simple, since the Wigner rotations of the $q\bar{q}$ system are reduced to unity.[4] One can work either in the Breit frame, where one boost direction is specified for initial and final hadronic states, or in the laboratory system, where we obtain

$$F_\pi^{\text{soft}}(Q^2) = \int dp\, \phi_F^*(p, P+Q)\phi_F(p,P). \tag{6}$$

For the soft part we take $\mathbf{Q} = (\mathbf{q}_\perp, 0)$ so that Eq. (6) reduces to the conventional Drell-Yan form.[8] For the calculation of the hard part we use the Breit frame, where the front-form wave functions are given by

$$\Phi_F(p_\perp, p_l(x,Q), P) e^{P^2/2b^2} = N \exp\left(-\frac{p_\perp^2}{2b^2}\right) \times \exp\left\{-\frac{x^2}{8}\left[\left(\frac{Q^2}{4b^2} + \frac{4p_\perp^2}{b^2(1-x^2)} + \frac{4m^2}{b^2(1-x^2)}\right)^{1/2} - \frac{Q}{2b}\right]^2\right\} \tag{7}$$

if one starts with $\Phi_\pi(\mathbf{p})$ as given by Eq. (3) in the rest system. Note the unusual, nonfactorized form of Φ_F, and the P dependence, which follows from the nature of the light-front variables and the boost to the light cone.[4] Note also that $|\pi\rangle_F$ is a well-defined $J^P=0^-$ pion state, that $F_\pi(Q^2)$ of Eq. (6) is equivalent to the standard definition, and that the usual pion decay constant, f_π, is obtained with use of this state.

The values used for the parameters m and b of the wave function are those of Ref. 3, $m=0.33$ GeV, $b=0.22$ GeV, and N of Eq. (7) is the normalization. The results are shown in Fig. 1. Our results for the soft part agree with those of Ref. 3 for small Q, but the soft part drops below the asymptotic contribution at $Q_c^2=3.5$ $(\mathrm{GeV}/c)^2$. It is not unreasonable to expect that QCD perturbation theory might be aplicable for such Q for the pion. We emphasize that these results are model dependent, and that we start with the model of Ref. 3 in the rest frame.

Our "hard" contribution, obtained with use of Eq. (1) with the quark distribution amplitudes $\Phi_Q(x)$ derived from Φ_F of Eq. (7) with the $k_\perp$ cutoff at 1 GeV, is approximately $F_\pi(Q^2)\approx 16\pi^2 f_\pi^2 \alpha_s(Q^2)/Q^2$, the Farrar-Jackson result shown in the figure. We expect a very slow evolution with Q^2. These results are similar to the hard part of Ref. 3, although it is not evident that they contain the same physics. In Ref. 3 the "soft" wave functions are modified by the *Ansatz* of Eq. (2) from considerations of the evolution equation, while we use the soft wave function boosted onto the light front. An essential point of the present Letter is that by using soft wave functions such as those of Ref. 3 one can successfully separate the soft and hard parts, as suggested in that work. However, the *Ansatz* of Eq. (2) actually includes some hard-gluon effects. Our results shown in Fig. 1 demonstrate that the soft parts are indeed soft, and that hard gluonic terms must be considered by 1 GeV/c.

For the proton there is no model-independent asymptotic result. Calculations in the spirit of Ref. 3, but with improved models, are being carried out.[9,10] We are also studying models appropriate for the method used in the present paper for the proton form factor. From the nature of the soft processes, we expect that the hard scattering term will become larger than the soft terms at higher momentum transfers than found for the pion.

For application to complex nuclei the situation is somewhat different, since the asymptotic terms require color to move freely between all of the quarks. Therefore, only the largest multiquark clusters can contribute for each nucleus. In the hybrid quark-hadron model it has been estimated that the six-quark cluster probability of the deuteron is about 0.03,[11] and that the nine-quark cluster content of ^{3}He is about 0.006.[12] These are normalization factors that reduce the asymptotic terms. From this and the fact that momentum sharing in multiquark clusters implies a large Q to reach the asymptotic stage for all internal lines, we expect that QCD asymptotic methods will require higher Q with increasing mass number. Detailed calculations which are required can be carried out via the methods given in this Letter. In any case, the study of the quark structure of nuclei is of great interest and an essential part of the physics at a few GeV/c, even if the asymptotic methods fail in this region.

The authors would like to thank Dr. S. Brodsky, Dr. F. Gross, Dr. N. Isgur, and Dr. B. Keister for helpful discussions. The work was supported in part through National Science Foundation Grant No. PHY 84-06555.

[1]G. P. Lepage and S. J. Brodsky, Phys. Rev. D **22**, 2157 (1980).

[2]S. J. Brodsky, in *Short-Distance Phenomena in Nuclear Physics,* edited by D. H. Boal and R. M. Woloshyn, NATO Advanced Study Institute Series B Vol. 104 (Plenum, New York, 1983), p. 141.

[3]N. Isgur and C. H. Llewellyn Smith, Phys. Rev. Lett. **52**, 1080 (1985).

[4]B. L. G. Bakker, L. A. Kondratyuk, and M. V. Terent'ev, Nucl. Phys. **B158**, 497 (1979); L. A. Kondratyuk, and M. V. Terent'ev, Yad. Fiz. **31**, 1087 (1980) [Sov. J. Nucl. Phys. **31**, 561 (1980)]; L. I. Frankfurt and M. I. Strikman, Phys. Rep. **76**, 215 (1981).

[5]P. A. M. Dirac, Rev. Mod. Phys. **21**, 392 (1949).

[6]G. R. Farrar and D. R. Jackson, Phys. Rev. Lett. **43**, 246 (1979).

[7]A. Duncan and A. H. Mueller, Phys. Rev. D **21**, 1636 (1980).

[8]S. D. Drell and T.-M. Yan, Phys. Rev. Lett. **24**, 181 (1970).

[9]C. E. Carlson, in *Nucleon and Nuclear Structure and Exclusive Electromagnetic Reaction Studies,* edited by George H. Rawitscher (BATES Users Theory Group, Middleton, Mass., 1984), p. 231; C. E. Carlson and F. Gross, private communication.

[10]V. L. Chernyak and I. R. Zhitnitsky, Nucl. Phys. **B246**, 52 (1984).

[11]L. S. Kisslinger, Phys. Lett. **112B**, 307 (1982); E. M. Henley, L. S. Kisslinger, and G. A. Miller, Phys. Rev. C **28**, 1277 (1983).

[12]P. Hoodbhoy and L. S. Kisslinger, Phys. Lett. **141B**, 163 (1984).