Singh and Pathria Respond: Shapiro[1] has raised some valid questions on the contents of our recent Letter[2] and in the process has provided a valuable lead for the direction in which answers to his questions may be sought. First of all, we agree with his observation that his Eq. (1) for $\zeta(d,d')$, with $2 < d < 4$ and $d' < 2$, is valid only for the spherical model ($n = \infty$) whereas for the general O(n) model, with $n \geqslant 2$, the correct formula is the one given by his Eq. (2). The predictive power of the latter is, however, contingent upon the knowledge one has of the exponents p and q. Using the well-known result, $\zeta(d,0) = d$, Shapiro has shown that $p/q = \beta$. Supplementing this with another known result,[3] viz. $\zeta(d,1) = 2(d-1)$, we can write down explicit expressions for both p and q, i.e.,

$$p = \beta/(d-2)\nu, \quad q = 1/(d-2)\nu. \tag{3}$$

Substituting (3) into his Eq. (2), and using standard scaling relations, we obtain the remarkable result

$$\zeta(d,d') = 2 + (d-2)\dot{\gamma}, \tag{4}$$

valid for all O(n) models with $n \geqslant 2$. Note that, for the spherical model, (4) follows from Shapiro's[1] Eq. (1) rather trivially.

Following the same procedure, we obtain for the (singular part of the) specific heat

$$\zeta_c(d,d') = [(\alpha - 2) + \dot{\alpha}/q]/\nu = -d + (d-2)\dot{\alpha} \tag{5}$$

and for the correlation length

$$\zeta_\xi(d,d') = (\nu + \dot{\nu}/q)/\nu = 1 + (d-2)\dot{\nu}. \tag{6}$$

Now, using scaling arguments for the d'-dimensional bulk system near $T = 0$, one can show that $\dot{\alpha} = 1 - \dot{\gamma}$ and $\dot{\nu} = \frac{1}{2}\dot{\gamma}$, with the result that

$$\zeta_c(d,d') = -\zeta(d,d'), \quad \zeta_\xi(d,d') = \tfrac{1}{2}\zeta(d,d'). \tag{7}$$

Thus, not only do we obtain "approach exponents" valid for all O(n) models with $n \geqslant 2$ but also find that, in the last analysis, these exponents are determined only by (i) the critical exponents pertaining to the d'-dimensional bulk system and (ii) the total dimensionality d of the given system. As anticipated by Shapiro, the critical exponents pertaining to d dimensions do not appear in the final formulas.[4] If we now substitute $\dot{\gamma} = 2/(2-d')$ for $d' < 2$, which should be valid for all O(n) models with $n \geqslant 2$, we obtain *quite generally*

$$\zeta(d,d') = 2(d-d')/(2-d'), \tag{8}$$

which is exactly the same as obtained earlier for the spherical model.[2]

Next, we address ourselves to the question as to how $\tilde{t}$ and $\tilde{C}_2$ may be determined as functions of T for $T < T_c$. Originally we made use of the correlation length $\xi(T)$ of the bulk system and employed the resulting form of $\tilde{t}(T)$ all the way down to $T = 0$, although ξ itself diverged for $T \leqslant T_c$. In spite of the fact that it worked successfully, the basis of that determination was rather unsatisfactory. Since then we have been able to show[5] that the desired forms of $\tilde{t}$ and $\tilde{C}_2$ can be obtained by making use of the bulk correlation function $G(\mathbf{R},T;\infty)$ for $T < T_c$. Using some results of Ref. 5, with derivations generalized to O(n) models, and some of Fisher, Barber, and Jasnow,[6] we find that

$$|\tilde{t}| \sim [\Upsilon(T)/T]^{1/(d-2)\nu},$$
$$\tilde{C}_2 \sim \mathscr{M}_0(T)[T/\Upsilon(T)]^{\beta/(d-2)\nu}, \tag{9}$$

where $\Upsilon(T)$ is the helicity modulus and $\mathscr{M}_0(T)$ the spontaneous magnetization of the corresponding bulk system. As $T \to 0$, we do obtain $|\tilde{t}| \sim T^{-q}$, $\tilde{C}_2 \sim T^p$, with p and q the same as given in Eq. (3). The remarkable feature of formulas (9), however, is that not only do they provide a prescription for determining $\tilde{t}$ and $\tilde{C}_2$ for *all* $T < T_c$ but, in the spirit of the Privman-Fisher hypothesis, they are based *solely* on the properties of the bulk system. For the spherical model, one readily obtains $|\tilde{t}| \sim (T_c - T)/T$, $\tilde{C}_2 \sim T^{1/2}$, in perfect agreement with Ref. 2.

Finally, we observe that, in view of formulas (9), the temperature (and size) dependence of the scaled variables x_1 and x_2 appearing in the Privman-Fisher hypothesis is now given by

$$x_1 \sim [L^{d-2}\Upsilon(T)/T]^{1/(d-2)\nu},$$
$$x_2 \sim [L^d \mathscr{M}_0(T)H/T][T/L^{d-2}\Upsilon(T)]^{\beta/(d-2)\nu}, \tag{10}$$

so that, for $0 \leqslant T < T_c$, $L^{d-2}\Upsilon(T)/T$ and $L^d\mathscr{M}_0(T)H/T$ may be regarded as the basic scaled variables of the problem; this indeed agrees with the findings of Privman and Fisher[3] for the special cases $d' = 0$ and 1.

Surjit Singh and R. K. Pathria
Department of Physics
University of Waterloo
Waterloo, Ontario N2L 3G1, Canada

Received 17 March 1986
PACS numbers: 05.70.Jk, 64.60.−i, 74.40.−s

[1] J. Shapiro, preceding Comment [Phys. Rev. Lett. **56**, 2225 (1986)].
[2] S. Singh and R. K. Pathria, Phys. Rev. Lett. **55**, 347 (1985).
[3] V. Privman and M. E. Fisher, Phys. Rev. B **32**, 447 (1985).
[4] A separate calculation shows that this is true for $d \geqslant 4$ as well. Details of this calculation will be published shortly.
[5] S. Singh and R. K. Pathria, Phys. Rev. B **33**, 672 (1986).
[6] M. E. Fisher, M. N. Barber, and D. Jasnow, Phys. Rev. A **8**, 1111 (1973).