Particle Production in the Central Rapidity Region of Ultrarelativistic Nuclear Collisions

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We discuss the central rapidity region of high-energy proton-nucleus collisions in the context of a flux-tube model. This model supposes the creation of a color flux tube by a random-walk color-charging process and its subsequent decay by $q\bar{q}$ and gluon-pair creation. The observed dependence of particle multiplicity on the number ν of projectile interactions is explained. Some further implications of the model for high-energy proton-nucleus and nucleus-nucleus collisions, including A-dependence in the latter case, are discussed.

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Quantum chromodynamics (QCD) predicts that strongly interacting matter will form a weakly interacting plasma of unconfined quarks and gluons at high temperatures or high baryon densities. It is hoped that this prediction will be tested in ultrarelativistic nucleus-nucleus collisions.[1] Much effort has been devoted to an understanding of the evolution of the quark-gluon plasma that may be formed in these collisions, and observational signatures of the plasma formation have been sought. Much less understood is the mechanism of energy deposition, which provides the initial condition for the plasma evolution.

In this paper we shall discuss a possible mechanism of energy deposition in the central rapidity region of high-energy nuclear collisions. We view the elementary nucleon-nucleon interaction as being dominated by an exchange of a single soft gluon.[2] This causes each of the two nucleons to acquire an octet color charge, so that the receding nucleons become linked by a color flux tube. The subsequent decay of the flux tube by pair creation leads to the copious production of secondary hadrons. In the case of nucleon-*nucleus* or *nucleus-nucleus* collisions there will be more gluons exchanged, which will change the details of the formation and decay of the flux tube(s). In the following we shall discuss several observational consequences of this model, focusing on proton-nucleus collisions.[3] We will see that our model can explain the ν dependence of the particle multiplicity in the central rapidity region, as observed by Elias *et al.*[4] and De Marzo *et al.*[5] We shall consider some implications of our model for nucleus-nucleus collisions at the end.

Consider a proton-nucleus collision with a small impact parameter, and suppose that the projectile nucleon suffers ν interactions while traversing the target nucleus. Our model assumes that the projectile exchanges one soft gluon with a target nucleon in each collision. The color charge will pile up both in the projectile and in the wounded target nucleus in a stochastic way. This process can be viewed as a random walk in the intrinsic color space. For an Abelian charge, the strength of the charge built up after the collision is given by

$$Q \propto \sqrt{\nu}. \tag{1}$$

This essential feature of the random-walk process does not change in the case of non-Abelian gauge theories, where we replace Q^2 by the quadratic Casimir operator.

In each interaction the projectile absorbs (or emits) a gluon which carries color $\mathbf{t}$ in the adjoint representation. After ν gluons have been exchanged, the total color vector of the projectile is $\mathbf{T}=\mathbf{t}_1+\mathbf{t}_2+\ldots+\mathbf{t}_\nu$. Hence the expectation value of the squared magnitude of total color is given by

$$\langle \mathbf{T}^2\rangle = \sum_{i=1}^{\nu} \langle \mathbf{t}_i^2\rangle + \sum_{i\neq j} \langle \mathbf{t}_i\cdot\mathbf{t}_j\rangle . \tag{2}$$

When the gluon exchange process takes place randomly, the color vectors of any pair of gluons are uncorrelated, $\langle \mathbf{t}_i\cdot\mathbf{t}_j\rangle=0$, which leads to

$$\langle \mathbf{T}^2\rangle = \nu\langle \mathbf{t}^2\rangle . \tag{3}$$

Thus we see that the expectation value of the quadratic Casimir operator indeed grows in proportion to ν.

Since the color charge at the ends of the flux tube is greater than that produced in pp collisions, the field strength in the tube is initially stronger. The particle-production process can be modeled as the quantum creation of $q\bar{q}$ pairs in the strong color field[6,7] (the Schwinger mechanism[8] in QED) and the subsequent combination of quarks into hadrons. The energy originally stored in the color field is gradually converted into the kinetic energy of the quarks. This process continues until the field energy is exhausted.

Schwinger's original formula for the pair-creation rate has been rederived[6] by WKB methods and used to calculate the $q\bar{q}$ pair production rate in the color flux tube. This method gives the explicit transverse-momentum dependence of the $q\bar{q}$ creation rate as

$$dP \propto -gE \ln\left[1-\exp\left(-\frac{2\pi(p_T^2+m^2)}{gE}\right)\right] dp_T^2, \quad (4)$$

where g is the effective QCD coupling constant and E is the strength of the color electric field. Here we have neglected the partial screening of the field by the produced pair. Inclusion of this effect is rather trivial[9] but does not lead to any qualitative change in the following discussion, since the initial field strength is much stronger than that of an elementary flux tube. In the massless limit the pair-creation rate becomes

$$P = aE^2, \quad (5)$$

where a is a dimensionless numerical constant. We note that Eq. (5) holds even in the presence of gluon-pair production.[10]

To relate the initial field strength to the particle multiplicity, we have to evaluate the space-time integral

$$N_{\text{pair}} = \int d^4x\, P. \quad (6)$$

Doing of this integral requires knowledge of the space-time dependence of the field strength $E(x)$. We first assume that the transverse cross section of the tube is fixed at $\mathscr{A}$ and that the field is uniform across the tube. We define the longitudinal coordinate z and time t in the center-of-mass frame, and define the light cone variables

$$\tau = \sqrt{t^2 - z^2}, \quad y = \frac{1}{2}\ln\left(\frac{t+z}{t-z}\right). \quad (7)$$

On the average, the longitudinal velocity v_z of a secondary hadron will be related to the position (t,z) where the particle is created by $v_z = z/t$. We take the position of hadron creation to be approximately equal to the position where $q\bar{q}$ pair creation occurs. Thus y defined by (7) can be identified with the rapidity of the final hadrons. Since the four-volume element is given by $d^4x = dy\, d\tau\, \tau \mathscr{A}$, Eqs. (5) and (6) lead to

$$\frac{dN_{\text{pair}}}{dy} = a\mathscr{A}\int_0^\infty d\tau\, \tau E^2. \quad (8)$$

The decay of the flux tube is a very complicated stochastic process. However, the average behavior of this process must meet the following rather simple condition: The process looks almost the same in any frame if it is not too close to the projectile or target rest frame. In other words, the space-time picture is invariant under Lorentz boosts in the longitudinal direction. (This point has been emphasized by Bjorken as a model-independent feature of the space-time development of high-energy collisions in the context of the parton model.[11]) In the present problem this implies that the average field strength E must be a function only of the proper time τ,

$$E(t) = E_0 f(\tau/\tau_0), \quad (9)$$

where E_0 is the initial field strength at $\tau = 0$, and $f(x)$ is a dimensionless function satisfying $f(0) = 1$. The constant τ_0 sets the time scale for attenuation of the color field, which by dimensional analysis must be inversely proportional to $\sqrt{E_0}$:

$$\tau_0 \propto 1/\sqrt{E_0}. \quad (10)$$

This is simply because the attenuation of the field due to pair creation is controlled according to Eq. (5) by the local strength of the field, and E_0 is the only parameter which has a dimension.

Substituting (9) into (8) and using (10), we find

$$dN_{\text{pair}}/dy = a\mathscr{A} E_0^2 \tau_0^2 \int_0^\infty dx\, x f^2(x) \propto \mathscr{A} E_0. \quad (11)$$

Since the initial field strength E_0 is related by Gauss's law to the color charge Q built up in the collision as

$$\mathscr{A} E_0 = Q, \quad (12)$$

we see that the particle density in the central rapidity region increases in proportion to Q,

$$dN_{\text{pair}}/dy \propto Q. \quad (13)$$

In the above discussion, we fixed the cross section of the flux tube. The tube will expand, however, because the field pressure $(=\frac{1}{2}E_0^2)$ is greater than the equilibrium pressure. The above derivation is right only if the field attenuation due to the pair creation is much faster than the expansion of the tube. Now let us consider the other extreme case, viz., that the pair creation is a very slow process and that the tube first expands and attains its equilibrium shape. The equilibrium cross section of the tube is given by

$$\mathscr{A}_{\text{eq}} = Q/E_{\text{eq}}, \quad (14)$$

where the equilibrium field E_{eq} is defined to balance the external bag pressure B via

$$\tfrac{1}{2}E_{\text{eq}}^2 = B. \quad (15)$$

In this case the numbers of pairs produced after the expansion can be estimated as

$$dN_{\text{pair}}/dy = a\mathscr{A}_{\text{eq}}\int_0^\infty d\tau\, \tau E^2(\tau) \propto \mathscr{A}_{\text{eq}} E_{\text{eq}} = Q. \quad (16)$$

Thus the Q dependence of the particle multiplicity does not depend much on the details of transverse evolution of the flux tube.

Using the random-walk relation between Q and the number of gluons exchanged $\sqrt{\nu}$, we find

$$dN_{\text{pair}}/dy \propto \sqrt{\nu}. \quad (17)$$

With the assumption that the number of $q\bar{q}$ pairs is proportional to the number of hadrons, this leads to

$$(dn/dy)_{pA}(dn/dy)_{pp}^{-1}=\sqrt{\nu}. \tag{18}$$

In Fig. 1 we compare our result (18) with recent streamer-chamber data[5] taken at the CERN Super Proton Synchrotron, where $\bar{\nu}$, the average number of collisions, was estimated from the number n_p of knocked-out fast protons.[12] The prediction of our model is in good agreement with the data. Our result also fits the data reasonably well[13] if we use the phenomenological relation

$$\bar{\nu}=A\sigma_{pp}^{\text{in}}/\sigma_{pA}^{\text{prod}}. \tag{19}$$

Along the same lines we can calculate the ν dependence of the average transverse momentum of the pairs. Recalling the formula (4) for the pair-creation rate which has explicit p_T dependence, the energy which is converted from the field into transverse kinetic energy, per unit four-volume, is given by

$$\frac{d\mathscr{E}_T}{dx^4}\propto -gE\int dp_T^2\,2p_T\ln\left[1-\exp\left(-\frac{2\pi p_T^2}{gE}\right)\right] \propto E^{5/2} \tag{20}$$

for massless quarks. The total transverse energy released by pair creation during the evolution of the flux tube is

$$\mathscr{E}_T^{\text{tot}}=\int d^4x\,\frac{d\mathscr{E}_T}{d^4x}\propto\int d\tau\,\tau\,dy\,\mathscr{A}\,E^{5/2}, \tag{21}$$

and hence,

$$d\mathscr{E}_T^{\text{tot}}/dy\propto E_0^{3/2}. \tag{22}$$

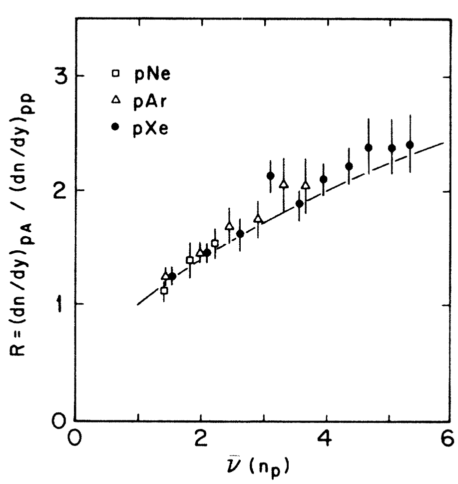

FIG. 1. The ratio $R=(dn/dy)_{pA}(dn/dy)_{pp}^{-1}$ in the central rapidity region of high-energy proton-nucleus collisions plotted as a function of the number of collisions $\bar{\nu}$ which the projectile suffers. The solid curve is the prediction of our model [see Eq. (18)]. The data are taken from Ref. 5.

If this transverse energy were to go directly into the secondary particles, the average transverse energy per particle would scale as

$$\langle E_T\rangle=\frac{d\mathscr{E}_T^{\text{tot}}/dy}{dN_{\text{pair}}/dy}\propto E_0^{1/2}\propto\nu^{1/4}. \tag{23}$$

This prediction of a slow increase of the average transverse energy distinguishes our model from other models[14–17] of multiparticle production in hadron-nucleus collisions. For example, the additive quark model,[17] which can equally well reproduce the ν dependence of dn/dy, gives $\langle E_T\rangle$ totally independent of ν. This is simply because, in such a model, the increase in the multiplicity is explained by an increase in the *number* of strings, and the nature of each string is assumed to be unchanged.

Unfortunately, we expect (23) to be affected strongly by the interactions inherent in the hadronization process. Currently available data on hadron-nucleus collisions[18] do not in fact show any clear evidence of an increase of the average p_T with increasing A. However, Eq. (23) has more direct implications for AA collisions, to which we proceed.

The use of Eq. (19) is equivalent, in effect, to the assumption that the number of proton interactions ν in the target nucleus with atomic number A scales as $A^{1/3}$. This observation, combined with the foregoing discussion, allows us to make predictions for particle production and energy deposition in ultrarelativistic nucleus-nucleus collisions. The central collision of two identical heavy nuclei may be considered as the creation of a large cylinder filled with color electric field. The coherence length, transverse to the cylinder, for color orientation will be the size of the proton. The number of interactions (color-exchange processes) which take place in each tube creation will be proportional to $A^{1/3}\times A^{1/3}$. Thus the average local color-charge density per unit transverse area built up after the nucleus-nucleus collision will grow as $\sqrt{(A^{2/3})}=A^{1/3}$. This implies that one can expect an energy density in the central rapidity region $\sim A^{2/3}$ times that in a pp collision. This initial condition also leads to faster quark-pair production characterized by $\tau_0\propto A^{-1/6}$, and the average transverse energy of pairs grows in proportion to $A^{1/6}$.

Under such circumstances, we may suppose that the matter produced by the decay of color flux takes the form of a plasma of unconfined quarks and gluons. Moreover, since the momentum distribution in the pair-creation formula is already very close to the Boltzmann distribution, thermalization would take place very rapidly. Hence we may interpret the A dependence of the average p_T of the produced particles as the A dependence of the initial temperature T_0 of the plasma, and τ_0 as the time when hydrodynamical expansion starts. We have the result

$$T_0 \propto A^{1/6}, \quad \tau_0 \propto A^{-1/6}. \tag{24}$$

Since, in the absence of dissipation, the scaling hydrodynamic expansion conserves the entropy per unit rapidity,[19] the final multiplicity of secondary hadrons produced in nucleus-nucleus collisions should scale as

$$(dn/dy)_{AA} \propto A. \tag{25}$$

This also seems to be consistent with available cosmic-ray data.[20]

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[1]For a recent review, see *Quark Matter '84, Proceedings of the Fourth International Conference on Ultrarelativistic Nucleus-Nucleus Collisions,* edited by K. Kajantie, Lecture Notes in Physics Vol. 221 (Springer-Verlag, New York, 1985).

[2]K. Johnson, Acta Phys. Pol. B **6**, 865 (1975).

[3]A model quite similar to ours has independently been proposed by several others for high-energy nucleus-nucleus collisions. See, for example, H. Ehtamo, J. Lindfors, and L. McLerran, Z. Phys. C **18**, 341 (1983); T. S. Biro, H. B. Nielsen, and J. Knoll, Nucl. Phys. **B245**, 449 (1984); A. Bialas and W. Czyż, Phys. Rev. D **31**, 198 (1985).

[4]J. E. Elias *et al.*, Phys. Rev. D **22**, 13 (1980).

[5]C. De Marzo *et al.*, Phys. Rev. D **29**, 2476 (1985).

[6]A. Casher, H. Neuberger, and S. Nussinov, Phys. Rev. D **20**, 179 (1979).

[7]N. K. Glendenning and T. Matsui, Phys. Rev. D **28**, 2890 (1983).

[8]J. Schwinger, Phys. Rev. **82**, 664 (1951); E. Brezin and C. Itzykson, Phys. Rev. D **2**, 1191 (1970).

[9]See Ref. 7 and the last paper in Ref. 3.

[10]A. Yildiz and P. H. Cox, Phys. Rev. D **21**, 1095 (1980); J. Ambjörn and R. J. Hughes, Ann. Phys. (NY) **145**, 340 (1983).

[11]J. Bjorken, in *Current Induced Reactions,* edited by J. G. Körner *et al.*, Lecture Notes in Physics Vol. 56 (Springer-Verlag, New York, 1976), p. 93.

[12]B. Anderson, I. Otterlund, and E. Stenlund, Phys. Lett. **73B**, 343 (1978).

[13]M. Faessler, in Proceedings of the NATO Advanced Study Institute on Heavy Ion Collisions, Cargèse, 1984 (to be published).

[14]K. Gottfried, Phys. Rev. Lett. **32**, 957 (1974).

[15]S. Brodsky, J. Gunion, and J. Kühn, Phys. Rev. Lett. **39**, 1120 (1977).

[16]A. Capella and J. Tran Than Van, Phys. Lett. **93B**, 146 (1980); K. Kinoshita, A. Minaka, and H. Sumiyoshi, Z. Phys. C **8**, 205 (1981).

[17]V. Anisovich, Phys. Lett. **57B**, 87 (1975); A. Białas, W. Czyż, and W. Furmanski, Acta Phys. Pol. B **8**, 585 (1977); N. Nikolaev, Phys. Lett. **70B**, 95 (1977).

[18]C. De Marzo *et al.*, Phys. Rev. D **29**, 363 (1985).

[19]J. Bjorken, Phys. Rev. D **27**, 140 (1983); M. Gyulassy and T. Matsui, Phys. Rev. D **29**, 419 (1984).

[20]O. Miyamura, report of JACEE collaboration, in Ref. 1.