Generation, Detection, and Application of High-Intensity Photon-Number-Eigenstate Fields

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Near–photon-number-eigenstate fields lead to dramatic signal-to-noise improvement beyond the standard shot-noise limit in many optical systems, including interferometric gravitational-wave detectors. These fields may be generated from parametric processes with measurement feedback. Degradation from nonideal photodetection can be overcome by appropriate optical preamplification. It is estimated that an order-of-magnitude improvement can already be obtained with existing devices.

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For an optical beam in a photon-number eigenstate $|n\rangle$, no measurement error would result if the photon-number operator N is measured. In such an ideal situation, one can expect tremendous performance gain over an ordinary laser coherent-state beam which is shot-noise limited. In this paper, I will show how such an ideal situation may be approached, and how significant improvement over the coherent-state shot-noise limit may be obtained with current technology. It will be shown that dramatic improvement in precision interferometry can also be obtained, similar to squeezing.[1–4]

For photodetection with quantum efficiency $0 < \eta \leq 1$, the input field mode suffers an effective fractional loss $1-\eta$. Thus, the detected field mode is described[5] by a photon annihilation operator b,

$$b = \eta^{1/2} a + (1-\eta)^{1/2} c, \tag{1}$$

where a and c are the annihilation operators for the input mode and an independent vacuum-state mode, respectively. It follows that the mean square photon-number fluctuation for b is

$$\langle \Delta N_b^2 \rangle = \eta^2 \langle \Delta N_a^2 \rangle + \eta(1-\eta)\langle N_a \rangle, \tag{2}$$

where $N_b \equiv b^\dagger b$, etc. The direct detection signal-to-noise ratio is, for the a mode in a number state $|n\rangle$ with $n = S$,

$$\frac{S}{N} \equiv \frac{\langle N_b \rangle^2}{\langle \Delta N_b^2 \rangle} = \frac{\eta S}{1-\eta}, \tag{3}$$

which can be compared to ηS obtained from a coherent state $|\alpha\rangle$ with the same $\langle N_a \rangle = S$. It appears from (3) that a very high $\eta \sim 1$ is necessary for actualizing the benefit of $|n\rangle$.

Even if η could never be brought close to unity, one could still realize the full potential of $|n\rangle$ by optical preamplification via a "noiseless photon amplifier." Mathematically, this device effects the state transformation

$$|n\rangle \rightarrow |Gn\rangle, \tag{4}$$

where the gain $G > 1$ is an integer. That such a device is in principle possible can be seen from the realization consisting of N measurement followed by $|Gn\rangle$-state generation, which, of course, is useless for the present purpose. Recently, I showed[6] that possible realizations exist within a unitary development description on a larger system, say including a material system, without the need of measurement first. To illustrate how this device may be physically approximated, consider the system indicated in Fig. 1. The incident photon at frequency ω combines with a pump photon at frequency $(G'-1)\omega$ to excite an "atom" through resonant two-photon absorption via a virtual intermediate state, which then decays via resonance fluorescence with G output photons at frequency $\omega' = G'\omega/G \neq \omega$. A change in frequency to ω' away from ω does not affect the resulting S/N but may make the scheme more practical. To minimize multiple input-photon absorption and induced G'–ω-photon emission, we can either have moderate input power, or set G' away from an integer, or let G' be an odd integer since levels 2 and 1 have the same parity. To minimize multiple pump-photon absorption we can set $G' > 2$, or use moderate

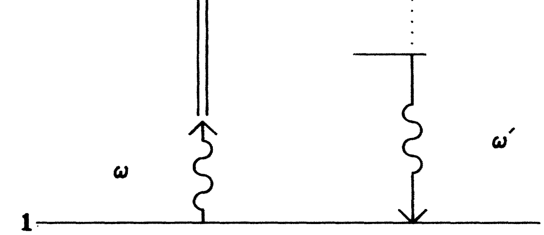

FIG. 1. A possible mechanism for the realization of a noiseless photon amplifier; level 1 is occupied initially and $G\omega' = G'\omega$.

pump power, or again use the parity-selection rule. With an obvious effective Hamiltonian describing the action of this device, the transformation $|n\rangle_\omega \rightarrow |Gn\rangle_{\omega'}$ is obtained in a first-order perturbation treatment of photoabsorption with sufficiently many absorbing atoms per input photon. To get a large G, the use of an avalanche mechanism may be contemplated. If a weak probe beam at ω' is applied with virtual intermediate states between levels 2 and 1 for induced G-ω'-photon emission, one can obtain space-time coherent outputs at the expense of introducing some photon-number uncertainty at the output via the probe. Further investigation is required for determining the ultimate practicality of this realization. But perhaps it is already clear that the noiseless photon amplifier is a viable device concept.

If the input state to the noiseless photon amplifier is $\sum_n \lambda_n |n\rangle$, the output state is $\sum_n \lambda_n |Gn\rangle$. Thus, if the a mode in an arbitrary state is first sent through this device before photodetection, it follows from (1) that $\langle N_b\rangle$ is given by $\eta G\langle N_a\rangle$ and the S/N becomes

$$\frac{S}{N}=\frac{\langle N_a\rangle^2}{\langle \Delta N_a^2\rangle}\left[1+\frac{1-\eta}{G\eta}\frac{\langle N_a\rangle}{\langle \Delta N_a^2\rangle}\right]^{-1}. \quad (5)$$

This goes to $\eta G\langle N_a\rangle/(1-\eta)$ in the limit $\langle \Delta N_a^2\rangle \rightarrow 0$. Equation (5) shows that the effect of nonunit η can be compensated by a sufficiently large G. It should be clear that such amplification in the photon number would *also suppress other randomness* introduced in the subsequent photodetection process, including dark current and thermal noise. Furthermore, the noiseless photon amplifier does not really have to be noiseless. It is *only required that* the added photon fluctuation be small compared to $\eta^2G^2\langle \Delta N_a^2\rangle$ similar to the noise suppression factor in (5). Thus, the S/N may not be significantly degraded by probe-beam-induced photon-number uncertainty or other nonideal disturbance and higher-order effects in the realization of Fig. 1. This device may lead to highly sensitive detection systems. In such case, the detection statistics for small $\langle \Delta N_a^2\rangle$ will be more than just sub-Poissonian ($\langle \Delta N_b^2\rangle < \langle N_b\rangle$); it will be strongly sub-Poissonian corresponding to that of a near number state.

Sub-Poissonian light was first observed in atomic fluorescence.[7] Recently, a variety of systems[8–11] have been suggested for generating sub-Poissonian light via measurement feedback. Consider the parametric interaction described by the Hamiltonian

$$H_I=\kappa a_s^\dagger a_i^\dagger a_p+\kappa^* a_s a_i a_p^\dagger, \quad (6)$$

where a_s, a_i, and a_p are the annihilation operators for the signal, idler and pump modes, respectively. From (6), the operator Manley-Rowe relation[12]

$$N_s(t)-N_s(0)=N_i(t)-N_i(0) \quad (7)$$

follows easily. In traveling-wave systems, t can be interpreted as a spatial variable. It is clear from (7) or in fact from (6) that each time the pump photon creates an idler photon, it must also create a signal photon. Let $N\equiv N_s-N_i$. From (7), $\langle \Delta N^2(t)\rangle \sim 0$ whenever $\langle \Delta N^2(0)\rangle \sim 0$, and $\langle \Delta N^2(0)\rangle \sim 0$ in parametric fluorescence, superfluorescence, and oscillation. Thus, if a photon is detected at the idler mode one can expect the presence of a corresponding photon at the signal mode. This has been verified experimentally in parametric fluorescence.[13] In fact, it can be shown that after one counts n photons at the idler mode the corresponding signal mode is indeed in state $|n\rangle$. So one may consider the generation of $|n\rangle$ at the signal mode by stopping the signal after n counts at the idler. However, parametric fluorescence[10,11] or atomic cascade emission[9] yields low-intensity output which can only be detected with high-gain low-η photodetectors, resulting in a near unity Fano factor $F\equiv\langle \Delta N_b^2\rangle/\langle N_b\rangle$. ($F=1$ is the coherent-state shot-noise limit.) While high-intensity output can be obtained in the scheme of Yamamoto and co-workers,[8] it seems that there is a strong limit on the F that can be so achieved.

These problems can be somewhat alleviated in a parametric amplifier or oscillator, and in certain corresponding four-wave mixer configurations. Inside a doubly resonant parametric oscillator one can get $\langle \Delta N^2(t)\rangle \sim 0$. However, the fields need to be coupled out of the cavity through a mirror with reflectance R. As a consequence, the output photon-number correlation is

$$\langle \Delta N^2\rangle = 2R(1-R)\langle N_s\rangle \quad (8)$$

for $\langle N_s\rangle$ generated inside the cavity. Equation (8) can be derived by relating the output and the cavity a_s's (and also a_i's), similar to the b and a of Eq. (1), with $\eta=1-R$. It imposes a major limitation on the achievable F to $2R$ outside the cavity. It may be possible to get $F\sim R$ for a singly resonant parametric oscillator by continuously tracking the idler count outside the cavity.

For parametric amplification,

$$\langle \Delta N^2(t)\rangle \sim \langle \Delta N^2(0)\rangle \sim \langle \Delta N_s^2(0)\rangle.$$

In contrast to fluorescence and oscillation, here the signal photon-number fluctuation for a given idler count is no longer simply given by $\langle \Delta N^2(t)\rangle$. It depends on the specific number counted with $\langle \Delta N^2(t)\rangle$ being the average over all idler counts. Neglecting pump quantization and depletion, one can compute explicitly the joint counting probability $P(n_s,n_i)$ and also $P_M(n_s,n_i)$ for counting over M independent signal and idler modes with initial signal coherent states $|\alpha_i\rangle$, vacuum idler states, and the same real $g\equiv\kappa\langle a_p\rangle$. Let $\mu\equiv\cosh(gt)$, $\nu\equiv\sinh(gt)$, and $I\equiv\sum_{i=1}^M|\alpha_i|^2/|\mu|^2$. The mean of N_s conditioned on a count n_i can be derived[14] as

$$S_{n_i} \equiv E[n_s | n_i] = n_i + \{1 + L^M_{n_i-1}(-I)/L^{M-1}_{n_i}(I)\} I, \quad (9)$$

where L^α_n is the Laguerre polynomial. The conditional variance is[14]

$$\mathrm{Var}[n_s|n_i] = (M + S_{n_i})I - (M-1)(S_{n_i} - n_i) - (S_{n_i} - n_i)^2. \quad (10)$$

For large n_i, $S_{n_i} \sim n_i + (n_i I)^{1/2} \sim n_i$. The conditional variance is $\sim \frac{1}{2}(n_i I)^{1/2}$ from a careful asymptotic analysis,[14] so that the F of the output signal is

$$F \sim \tfrac{1}{2}(I/n_i)^{1/2}. \quad (11)$$

Thus, a very small F may be obtained for any given I, albeit with a correspondingly small probability $P_M(n_i)$ which is the M-fold Laguerre counting probability.[15] Let $A \sim |\mu|^2 \sim |\nu|^2$ be the power gain per mode. For $n_i \sim A^2 I \lesssim \langle N_i \rangle$, the mean number of idler photons generated, (11) becomes $F \sim 1/2A$. The plausibility of this result, which is what is needed in the following, can be seen from the estimate $F \sim \langle \Delta N^2(t) \rangle / \langle N(t) \rangle \sim 1/A$ in which the unconditional variance $\langle \Delta N^2(t) \rangle \sim AI$ is used in place of (10). It can be shown that (8) and (11) are not sensitive to thermal background and loss in the generation process.

Thus, strongly sub-Poissonian light can be generated at the signal mode when an optical shutter is used to turn off the signal output after photodetection at the idler, as indicated schematically in Fig. 2. If one wants to generate S signal photons over a time interval T by this method, one can set $\langle N_i \rangle$ to be S plus, say, ten standard deviations of $P_M(n_i)$. It is no mystery that the signal photon statistics can be controlled this way, as the "statistical ensemble" of signal photons is being directly manipulated. For average idler power P_i and optical-shutter response time τ, additional signal photon fluctuation is induced through the idler photon-number variance δ over the interval τ. From $P_M(n_i)$, one finds $\delta \sim 2AP_i\tau$ for amplification and $\delta \sim AP_i\tau$ for oscillation and superfluorescence. If the idler counts are continuously fed back to vary the pump intensity

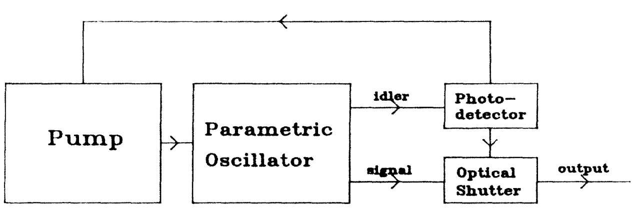

FIG. 2. Output number-eigenstate generation via idler-measurement feedback on the signal shutter; additional feedback on the pump permits better control of the generation.

according to an appropriate control law, it should be possible to reduce this tail fluctuation greatly and to bring the required idler power P_i very close to the desired signal count.

A great advantage of near-number-state beams compared to squeezed light is their robustness with respect to disturbance. There is no phase-sensitivity requirement in the system. Another *key point* is that one cannot expect but *does not need to resolve individual photons* for useful application. If one can resolve $\sim (S/r)^{1/2}$ photons, one can improve the signal-to-noise ratio by a factor of r with S signal photons. For any given improvement level or tolerance limit, fluctuations in the generation and detection processes *would not matter so long as they are below that limit. While one added noise photon destroys squeezing completely, it would do nothing to a near-number-state beam* unless one wants to resolve individual photons. Thus, the state generation would not be affected by spontaneous emission even in media with near-resonant transitions, and the output signal and idler photon numbers need only be correlated to the tolerance level. A host of other problems that affect squeezing generation in a parametric process also dissolve here.

As a consequence, it appears possible to perform a demonstration experiment with noise significantly below the shot-noise limit by just using existing devices. The counts n_s and n_i can be obtained by measurement of the photocurrents of p-i-n silicon diode detectors. Near 1 μm, one can get such photodiodes with $\eta \sim 0.95$, milliwatt saturation, and essentially no dead time. The achievable F can be estimated from the above consideration, with neglect of some finer details on the operation of a parametric amplifier or oscillator. With only feedback on the shutter, the output signal F at idler count $n_i \sim P_i T \sim S_{n_i}$ is

$$F \sim \frac{\langle \Delta N_S^2(0) \rangle}{2P_i T} + 2\eta(1-\eta) + \frac{2\eta(P_d + P_{\mathrm{th}})}{P_i} + 2R(1-R) + \frac{2A\tau}{T}, \quad (12)$$

where T is the observation interval, P_d and P_{th} the dark and thermal noise power. Equation (12) incorporates the important fluctuations in both the generation and the detection processes, including nonideal idler detection. If the last term in (12) can be effectively suppressed, it is easy to get $F \sim 0.1$ in a pulse experiment even for parametric superfluorescence, the limit on F being set by η. In the pulse case, the term $2R(1-R)$ would also drop out for parametric oscillators by use of cavity dumping. With the $2A\tau/T$ term in full effect for $\tau \sim 1$ nsec, an $F \sim 0.5$ may be obtained from a cw or quasi cw parametric oscillator with $R \sim 0.2$. A better choice would be a parametric amplifier $(R=0)$ for which one may get[16] a gain $A \gtrsim 10^4$

for $T \sim 1$ μsec with a 5-cm $LiNbO_3$ crystal. Of the many possibilities from (12) I merely quote one set of numbers: $A \sim 10^2$, $I \sim 10^5$, $T \sim 10^{-5}$ sec so that $P_i T \sim 10^9$ with $F \sim 0.1$ limited by η. Note that the second and third terms in (12) can be suppressed with a noiseless photon amplifier. Thus, with good feedback control it is possible in principle to generate near number states this way.

Loss severely limits the advantage of any nonclassical state.[17,18] However, one may expect that near-number-state beams and low-loss fibers contribute a powerful combination that may find application in communication and data processing systems. In fact, the information capacity of a lossless channel is maximized by $|n\rangle$ and N measurement among all possible quantum states and quantum measurements, under an average power constraint on the states.[17] Furthermore, correlated number-state beams also lead to significant improvement in precision interferometry. For almost all interferometer configurations, the input modes a_1, a_2 and output modes b_1, b_2 are related as[3,4]

$$b_1 = a_1 \cos\phi + i a_2 \sin\phi, \quad b_2 = i a_1 \sin\phi + a_2 \cos\phi, \tag{13}$$

where ϕ is a small phase to be estimated. The interference may be observed through

$$N_{b_1} - N_{b_2} = (N_{a_1} - N_{a_2})\cos 2\phi + i(a_1^\dagger a_2 - a_2^\dagger a_1)\sin 2\phi, \tag{14}$$

with the interferometer performance measured by the mean square error in estimating ϕ from this difference count. This mean square error is $\sim (S/N)^{-1}$ for small ϕ; thus the rms photon-counting error ξ is $\sim (S/N)^{1/2}$. For the a_1 mode in a number state $|S\rangle$ and the a_2 mode in vacuum, ξ is $S^{-1/2}$ from (14), identical to that obtained from a coherent state $|S^{1/2}\rangle$. If a correlated photon-number state for the input modes

$$|\psi\rangle = \tfrac{1}{2}\sqrt{2}(|n\rangle_1 |n-1\rangle_2 + i|n-1\rangle_1 |n\rangle_2) \tag{15}$$

is used with $n = S/2$, the resulting ξ becomes $\sim S^{-1}$, the same as that obtained from an optimized single-frequency two-photon coherent-state system.[4] Note that S^{-1} is also the discretization error in a number-state system. Similar discretization error occurs in coherent detection with finite local oscillator power S. This error S^{-1} is directly applicable to laser gyroscopes. In particular, one may consider the possibility of detecting gravitational radiation by measuring the gravitational-wave induced phase shift of an optical beam in a fiber gyroscope. For application to the laser-interferometer gravitational-wave detector[3,4] in which the gravitational-wave–induced mirror motion is monitored, the radiation-pressure error on the mirrors has to be included. In such a detector, (15) still provides the same performance as the optimized single-frequency two-photon coherent-state system, namely,[4] achievement of the standard quantum limit with input power $\sim S_{\min}^{1/2}$ where $S_{\min}$ is that required for a coherent-state system. One may consider generating (15) from two independent number-state beams via spatial interference. And there are many other correlated number-state beams that would also lead to improved performance.

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[1] H. P. Yuen, Phys. Rev. A **13**, 2226 (1976).

[2] D. F. Walls, Nature (London) **306**, 141 (1983).

[3] C. M. Caves, Phys. Rev. D **23**, 1693 (1981).

[4] R. S. Bondurant and J. H. Shapiro, Phys. Rev. D **30**, 2548 (1984).

[5] H. P. Yuen and J. H. Shapiro, in *Coherence and Quantum Optics IV*, edited by L. Mandel and E. Wolf (Plenum, New York, 1978), p. 719.

[6] H. P. Yuen, Phys. Lett. **113A**, 405 (1986).

[7] R. Short and L. Mandel, Phys. Rev. Lett. **51**, 383 (1983).

[8] Y. Yamamoto, talk presented at the U. S.–Japan Seminar on Coherence, Incoherence and Chaos, Nara, Japan, August 1984 (unpublished); Y. Yamamoto, N. Imoto, and S. Machida, to be published; N. Imoto, H. A. Haus, and Y. Yamamoto, Phys. Rev. A **32**, 2287 (1985).

[9] B. E. A. Saleh and M. C. Teich, Opt. Commun. **52**, 429 (1985).

[10] E. Jakeman and J. G. Walker, Opt. Commun. **55**, 219 (1985).

[11] C. K. Hong and L. Mandel, Phys. Rev. Lett. **56**, 58 (1986).

[12] W. H. Louisell, A. Yariv, and A. E. Siegman, Phys. Rev. **124**, 1646 (1961).

[13] S. Friberg, C. K. Hong, and L. Mandel, Phys. Rev. Lett. **54**, 2011 (1985).

[14] H. P. Yuen, to be published.

[15] R. M. Gagliardi and S. Karp, *Optical Communications* (Wiley, New York, 1976), p. 70.

[16] R. L. Byer, private communication.

[17] H. P. Yuen, in Proceedings of the 1975 Conference on Information Sciences and Systems, Baltimore, Md. (unpublished), p. 171.

[18] H. P. Yuen and J. H. Shapiro, IEEE Trans. Inf. Theory **24**, 657 (1978).