Cosmic Production of Quarkonium?

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It has been suggested that pair annihilation of heavy Majorana fermions in the galactic halo into quarkonium plus a monochromatic photon could occur at an observable rate. Here we show that a calculation of Srednicki, Theisen, and Silk seriously overestimates the rate for this process, by an order of magnitude or more, as a result of neglecting the bound-state structure of quarkonium. It may nevertheless still be possible to resolve the correspondingly smaller gamma-ray line flux over the diffuse cosmic background by use of the currently planned space-borne superconducting-magnet spectrometer facility with an energy resolution of 1% or better.

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Much effort has recently been directed to searching for cosmic-ray signatures[1] which could confirm the widely entertained hypothesis that heavy (mass >1 GeV) weakly interacting particles could remain as relics of the "big bang" and constitute the dark matter in our galactic halo. In a recent Letter, Srednicki, Theisen, and Silk[2] suggested that the annihilation of relic nonrelativistic Majorana fermions (here denoted χ) in the halo into quarkonium and a photon could occur at an observable rate, yielding monochromatic gamma-ray lines which could be distinguished from the diffuse cosmic gamma-ray background. In this paper, I will argue that their calculation of the rate for this process neglects in a rather crucial way the bound-state dynamics of quarkonium and leads to an overestimate of the rate by an order of magnitude or more, depending on the Majorana-fermion mass M_χ.

It is convenient to normalize the rate for the reaction $\chi\chi\to V\gamma$, where V is a quarkonium state, to the annihilation cross section into the corresponding heavy quark-antiquark pair, $\chi\chi\to Q\bar{Q}$. In the local limit the effective interaction is[3]

$$\mathscr{L}_{\text{eff}}=\bar{\chi}\gamma^\mu\gamma_5\chi\bar{Q}\gamma_\mu(a+b\gamma_5)Q. \qquad (1)$$

In the halo, $v/c\simeq 10^{-3}$ and it is appropriate to consider the annihilation cross section in the limit $v_{\text{rel}}\to 0$, where v_{rel} is the relative velocity in the initial state. The result is then

$$\sigma(\chi\chi\to Q\bar{Q})v_{\text{rel}}=\frac{6}{\pi}b^2m_Q^2\left(1-\frac{m_Q^2}{M_\chi^2}\right)^{1/2}, \qquad (2)$$

which is larger than the result quoted in Ref. 2 by a factor of 12, which can be accounted for as follows: A factor of 3 comes from the sum over final-state quark colors, and a factor of 4 arises because the Feynman rule for the amplitude derived from the Lagrangean in Eq. (1) includes a factor of 2 due to the Majorana nature of χ, which would in fact not be there in the more familiar Dirac case.

The amplitude for the process $\chi\chi\to V\gamma$ is given by the sum of the two Feynman diagrams shown in Fig. 1. Srednicki, Theisen, and Silk chose to evaluate the corresponding loop integral by assuming a pointlike interaction $fV^\mu(x)\bar{Q}(x)\gamma_\mu Q(x)$ between quarkonium and its constituent quarks, with f a momentum-independent constant. This seems completely unreasonable: Surely some momentum-dependent form factor should appear in the matrix element to account for confinement effects. As a result of their assumption, quarks of all virtualities contribute to the integral, and indeed their calculation yields a piece accounting for the expected triangle anomaly, which is simply thrown away on the grounds that the resulting amplitude is unphysically large. In fact, viewing quarkonium as a nonrelativistic bound state would lead us to expect that it should be dominated by only slightly off-shell quarks and antiquarks, with relative momenta small on the scale of the quarkonium mass. A more realistic and consistent calculation of the rate for $\chi\chi\to V\gamma$ will take the bound-state nature of quarkonium into account from the outset: The calculational method dates back to work by Van Royen and Weisskopf,[4] with a convenient systematic formalism described, e.g., by Kuhn, Kaplan, and Safiani.[5] This approach has been applied to a wide variety of

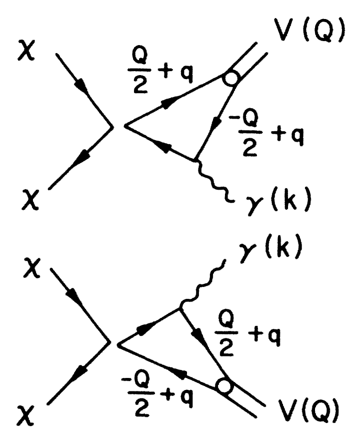

FIG. 1. Diagrams contributing to the process $\chi\chi\to V\gamma$, where V is a quarkonium state.

processes involving quarkonia, such as orthoquarkonium decay to a Higgs boson and a photon[6] ($V \to H\gamma$) and to rare decays of the Z^0,[7] to mention but a few.

Let Q be the four-momentum of the final-state quarkonium, and q the relative momentum between the quark and antiquark in the diagrams of Fig. 1. Then the amplitude for $\chi\chi \to V\gamma$ is written in the form[4,5]

$$A(\chi\chi \to V\gamma) = \int d^4q\,(2\pi)^{-4}\,\mathrm{Tr}M(q)\chi(Q,q). \quad (3)$$

Here, $\chi(Q,q)$ is the Bethe-Salpeter wave function appropriate for the quarkonium state, and $M(q)$ represents the rest of the matrix element obtained from Fig. 1. Specifically,

$$M(q) = 2(4\pi\alpha)^{1/2}\hat{e}_Q\bar{v}(p_2)\gamma^\mu\gamma_5 u(p_1)$$
$$\times\left\{\gamma_\mu(a+b\gamma_5)\frac{\gamma\cdot(Q/2+q+k)+m_Q}{(Q/2+q+k)^2-m_Q^2}\gamma\cdot\epsilon_\gamma(k)+\gamma\cdot\epsilon_\gamma(k)\frac{\gamma\cdot(-Q/2+q-k)+m_Q}{(Q/2-q+k)^2-m_Q^2}\gamma_\mu(a+b\gamma_5)\right\}, \quad (4)$$

where p_1 and p_2 are the initial Majorana fermion four-momenta, ϵ_γ and k are respectively the polarization and momentum four-vectors of the outgoing photon, and $\hat{e}_Q$ is the quark electric charge in units of the proton charge.

We now adopt a nonrelativistic bound-state picture for quarkonium, and reduce the Bethe-Salpeter wave function to its nonrelativistic form: $\chi(Q,q)$ is constructed in terms of quark and antiquark spinors in a given total-spin configuration multiplied by the nonrelativistic momentum-space wave function $\psi_{LM}(\mathbf{q})$ in a given orbital angular momentum state, forming a state of given total angular momentum. Here we will restrict our attention to $^3S_1$ quarkonium states and $\chi(Q,q)$ can be written in the form

$$\chi(Q,q;J=1,J_z) = 2\pi\delta(q^0-\mathbf{q}^2/2m_Q)\psi(\mathbf{q})(3/m_Q)^{1/2}\sum_{s,\bar{s}}u(Q/2+q;s)\bar{v}(Q/2-q;\bar{s})\langle\tfrac{1}{2}s;\tfrac{1}{2}\bar{s}|1\,J_z\rangle. \quad (5)$$

We adopt the normalization $\bar{u}u=2m_Q$. The factor of $\sqrt{3}=(\mathrm{Tr}1)/\sqrt{3}$ is the appropriate color factor for a properly normalized color-singlet quarkonium state. In terms of zero-three-momentum spinors, we can write

$$\sum_{s,\bar{s}}u(Q/2+q;s)\bar{v}(Q/2-q;\bar{s})\langle\tfrac{1}{2}s;\tfrac{1}{2}\bar{s}|1\,J_z\rangle$$
$$=\sum_{s,\bar{s}}\frac{\gamma\cdot(Q/2+q)+m_Q}{[2m_Q(E_Q+m_Q)]^{1/2}}u(0;s)\bar{v}(0;\bar{s})\frac{\gamma\cdot(-Q/2+q)+m_Q}{[2m_Q(E_{\bar{Q}}+m_Q)]^{1/2}}\langle\tfrac{1}{2}s;\tfrac{1}{2}\bar{s}|1\,J_z\rangle$$
$$=\frac{1}{2m_Q}[\gamma\cdot(Q/2+q)+m_Q]\frac{\gamma\cdot Q+M_V}{2\sqrt{2}M_V}\gamma\cdot\epsilon_V(Q;J_z)[\gamma\cdot(-Q/2+q)+m_Q][1+O(\mathbf{q}^2/m_Q^2)], \quad (6)$$

where ϵ_V is the quarkonium polarization four-vector.

If we now go to the rest frame of a nonrelativistic quarkonium state, the relative quark-antiquark momentum is much less than the quarkonium mass, and the bound-state wave function is sharply damped for all but small values of relative momentum. We may then evaluate the amplitude A in Eq. (3) keeping only the leading behavior by replacing $M(q)$ by $M(q=0)$, and setting $q=0$ in Eq. (6). We can then put everything together and arrive at

$$A(\chi\chi\to V\gamma) = (3/4M_V)^{1/2}\,\mathrm{Tr}M(q=0)(\gamma\cdot Q+M_V)\gamma\cdot\epsilon_V(Q;J_z)\int d^3q\,(2\pi)^{-3}\psi(\mathbf{q}), \quad (7)$$

where here and from now on we simply take $M_V=2m_Q$ for $n=1$ quarkonia, consistent with the above assumptions. It is now a simple matter of algebra to reduce this to the form

$$A(\chi\chi\to V\gamma) = (3\alpha M_V)^{1/2}\frac{16ib\hat{e}_Q R(0)}{4M_\chi^2-M_V^2}\epsilon_{\mu\nu\lambda\sigma}\bar{v}(p_2)\gamma^\mu\gamma_5 u(p_1)k^\nu\epsilon_\gamma^\lambda(k)\epsilon_V^\sigma(Q), \quad (8)$$

where

$$R(0) = \sqrt{4\pi}\int d^3q\,(2\pi)^{-3}\psi(\mathbf{q}) \quad (9)$$

is the radial wave function at the origin in configuration space, in terms of which the quarkonium e^+e^- decay width is written

$$\Gamma(V\to e^+e^-) = 4\alpha^2\hat{e}_Q^2|R(0)|^2/M_V^2. \quad (10)$$

In the limit $v_{\rm rel} \to 0$ we then obtain the result

$$\sigma(\chi\chi \to V\gamma)v_{\rm rel} = \frac{6\alpha}{\pi}\hat{e}_Q^2 b^2 |R(0)|^2 \frac{M_V}{M_\chi^2}\left[1-\frac{M_V^2}{4M_\chi^2}\right], \tag{11}$$

or, in terms of $\Gamma(V \to e^+e^-)$,

$$\sigma(\chi\chi \to V\gamma)v_{\rm rel} = \frac{3b^2}{2\pi\alpha} M_V \Gamma(V \to e^+e^-)\frac{M_V^2}{M_\chi^2}\left[1-\frac{M_V^2}{4M_\chi^2}\right]. \tag{12}$$

Finally, the branching ratio is given in a very simple form:

$$\frac{\sigma(\chi\chi \to V\gamma)}{\sigma(\chi\chi \to Q\bar{Q})} = \frac{\Gamma(V \to e^+e^-)}{\alpha M_V}\frac{M_V^2}{M_\chi^2}\left[1-\frac{M_V^2}{4M_\chi^2}\right]^{1/2}. \tag{13}$$

This is plotted for both $J/\psi(3100)$ and $\Upsilon(9460)$ in Fig. 2 with use of the values 4.8 and 1.2 keV for the respective electronic decay widths of J/ψ and Υ.

The result Eq. (13) is expected to be quite reliable as to the order of magnitude (e.g., to the same extent as the oft-quoted estimate of the branching ratio for $V \to H\gamma$ presented in Ref. 6). It will of course be subject to corrections of two types: from bound-state effects (long distance), and from gluon exchange (short distance) calculable in perturbative QCD. The bound-state corrections include going beyond the approximation of the wave function at the origin, as well as considering initial-state $Q\bar{Q}$ interactions when M_χ is only slightly larger than m_Q. Both of these effects tend to suppress the rate given by Eq. (13) by perhaps a factor of 2 or so.[8] In addition, one can include QCD corrections to the various pieces, Eqs. (2), (10), and (11): The first two are known, while the third has not yet been calculated. Nevertheless, one can get an idea of the overall effect of first-order QCD corrections here by looking at the calculation of Vysotsky[9] for the closely related process $V \to H\gamma$. There it was found that short-distance corrections in fact led to a suppression of the tree-level result as well. These qualitative considerations lead us to conclude that one should probably view Eq. (13) as an upper bound to the branching ratio. Notwithstanding this caveat, I proceed with a short discussion of the implications of Eq. (13) as it stands.

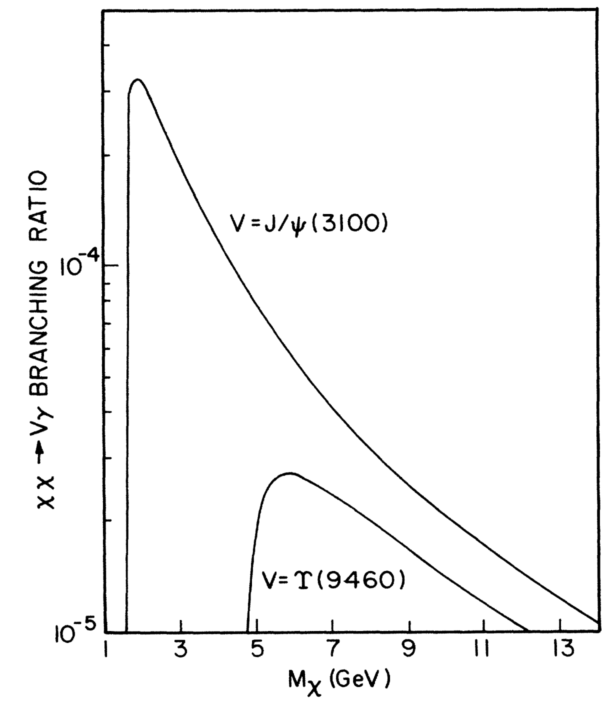

FIG. 2. The branching ratio $\sigma(\chi\chi \to V\gamma)/\sigma(\chi\chi \to Q\bar{Q})$, as given in Eq. (13) of the text, for $V = J/\psi$ and Υ.

A comparison with the results of Srednicki, Theisen, and Silk[2] shows a reduction in the calculated branching ratio for J/ψ production by a factor which ranges from 10 to 15 for M_χ of a few gigaelectronvolts and rapidly increases to about 100 for $M_\chi > 10$ GeV. We also see that contrary to an assertion made in Ref. 2, the production of *b*-quarkonium and *t*-quarkonium is even more suppressed.

We can now redo the gamma-ray line-flux estimates based on the reaction $\chi\chi \to (J/\psi)\gamma$, using the same astrophysical numbers as Ref. 2. Taking[2] $\sigma(\chi\chi \to Q\bar{Q})v_{\rm rel} = 3\times10^{-27}\Omega_\chi^{-1}$ cm^3 sec^{-1} and, correspondingly, with the $\chi\chi \to (J/\psi)\gamma$ branching ratio crudely parametrized as $2\times10^{-4}[(3\text{ GeV})/M_\chi]^2$, the gamma-ray line flux is

$$F_{\rm line} \simeq \sigma(\chi\chi \to (J/\psi)\gamma)v_{\rm rel}R(\rho_\chi/M_\chi)^2(4\pi\text{ sr})^{-1} \simeq (6\times10^{-10}\text{ cm}^{-2}\text{ sec}^{-1}\text{ sr}^{-1})[(3\text{ GeV})/M_\chi]^4, \tag{14}$$

with $\Omega_\chi = 0.1$ for the mean mass density of χ particles in units of the closure critical density, and with $\rho_\chi = 0.3$ GeV/cm^3 and $R = 40$ kiloparsecs the mean mass density and effective radius of the halo. Observation of a line flux of this magnitude, over and above a steeply falling diffuse cosmic gamma-ray background, is probably not possible with the U.S. National Aeronautics and Space Administration Gamma-Ray Observatory, with a quoted energy resolution of 15% (cf. the discussion in Ref. 2). However, for $M_\chi \lesssim 4$ GeV or so, it may be possible to resolve this line with a second-generation experiment using the currently planned space-borne superconducting-magnet spectrometer facility[10,11] with an energy resolution of 1% or better.

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