# Tests of Time-Reversal Symmetry in Compound-Nucleus Reactions

D. Boosé, H. L. Harney, and H. A. Weidenmüller
*Max-Planck-Institut für Kernphysik, Heidelberg, Federal Republic of Germany*
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Using a generating function with both commuting and anticommuting variables, we calculate perturbatively the violation of the principle of detailed balance due to breaking of time-reversal symmetry in a statistical nuclear reaction. We apply this result to the best data and establish upper bounds for the strength of, and the spreading width associated with, the symmetry-breaking part of the Hamiltonian. We argue that the spreading width is the fundamental parameter of the theory. A way of further improving the upper bounds is suggested.

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The observation of $CP$ nonconservation in the $K$-$\bar{K}$ system[1,2] has led to high-precision experiments testing detailed balance in compound-nucleus (CN) reactions.[3,4] The interpretation of these experiments has suffered from the lack of an adequate statistical theory of CN reactions which would connect the observable with the time-reversal-symmetry–breaking (TRSB) part of the nuclear Hamiltonian. The solution of this problem is reported in this Letter. It is based on a recent advance[5] in the statistical theory of nuclear reactions. We present an expression for the amount of violation of the law of detailed balance in terms of the strength of the symmetry-breaking part of the Hamiltonian. (Lack of space does not allow us to give the derivation in detail.[6]) We discuss the experimental limits in the light of our result. We argue that TRSB in nuclei should be parametrized in terms of a spreading width, and we suggest that the most promising region to perform detailed-balance experiments is that of weakly overlapping CN resonances.

The most stringent upper limit obtained so far[4] for TRSB in CN reactions yields $|X| < 10^{-6}$. Here $X$ is the deviation from unity of the observable

$$|R_{ab}|^2 = [\langle \sigma_{ab}\sigma_{ba}\rangle - \langle \sigma_{ab}\rangle\langle \sigma_{ba}\rangle][\langle \sigma_{ab}\rangle\langle \sigma_{ba}\rangle]^{-1}, \quad (1)$$

which measures the correlation between the cross section $\sigma_{ab}$ of a CN reaction from channel $a$ to channel $b$ (with $b \neq a$), and the cross section $\sigma_{ba}$ of the inverse reaction. The angular brackets indicate an average over a suitable energy interval. The theoretical effort is directed towards expressing $|R_{ab}|^2$ in terms of the TRSB part of the nuclear Hamiltonian.

To this end we model the statistical $S$ matrix $S_{ab}(E)$ in its dependence on energy $E$ in analogy to Ref. 5 and Weidenmüller.[7] We denote the open channels by $a,b,c,\ldots$. The CN levels are modeled as $N$ orthonormal bound states labeled $\mu, \nu, \ldots$; we eventually take the limit $N \to \infty$. The levels are coupled to each other by the nuclear Hamiltonian with matrix elements $H_{\mu\nu} = H^*_{\nu\mu}$, and to the channels via matrix elements $W_{a\mu} = W^*_{\mu a}$. We go beyond the work of Refs. 5 and 7 because TRSB does not allow us to choose the matrices $H$ and $W$ real as was done there. The unitary $S$ matrix has the form[8]

$$S_{ab}(E) = \delta_{ab} - 2i\pi \sum_{\mu\nu} W_{a\mu}[(EI - H + i\pi M)^{-1}]_{\mu\nu} W_{\nu b}, \quad (2)$$

where $I$ is the unit matrix. The Hermitian matrix $M$ has elements $M_{\mu\nu} = \sum_a W_{\mu a} W_{a\nu}$. We neglect the dependence of the matrix $W$ on energy. Without TRSB this dependence does not destroy detailed balance. Our neglect therefore only implies that $\Gamma^{(u)}$ given below is approximate, the error being of the order $\Gamma/\Delta E$ where $\Gamma$ is the average CN width and $\Delta E$ the energy over which $W$ changes significantly. With[9] $\Gamma = 45$ keV and $\Delta E \sim 1$ MeV given by threshold or shell effects, we find that $\Gamma/\Delta E$ is negligible.

In Eq. (2) we have omitted direct reactions and the shape-elastic scattering phase shifts. As to the former, the experiment has been set up in such a way as to minimize their contribution. Concerning the latter, phase factors do not affect the value of $|R_{ab}|^2$. We have likewise suppressed the shift functions in the propagators. Again, this simplification does not affect the structure of the result but only the form of the transmission coefficients.[5]

The statistical properties of the $S$ matrix are embodied in those of the Hamiltonian matrix $H$. We assume[5,7] that $H$ is a member of an ensemble of random matrices. To allow for TRSB, we decompose $H$ into two parts,

$$H = H^{(o)} + H^{(u)}. \quad (3)$$

Here, $H^{(o)}$ preserves time-reversal symmetry and is drawn from the Gaussian orthogonal ensemble (GOE), while $H^{(u)}$ breaks this symmetry and is drawn

from the Gaussian unitary ensemble (GUE). We assume $H^{(o)}$ and $H^{(u)}$ to be uncorrelated, to have zero mean values, and to have second moments given by (the angular brackets denote the ensemble average)

$$\langle H^{(o)}_{\mu\nu} H^{(o)}_{\mu'\nu'} \rangle = \lambda^2 N^{-1} (\delta_{\mu\mu'}\delta_{\nu\nu'} + \delta_{\mu\nu'}\delta_{\nu\mu'}), \quad (4a)$$

$$\langle H^{(u)}_{\mu\nu} H^{(u)*}_{\mu'\nu'} \rangle = 2\alpha_0^2 \lambda^2 N^{-2} \delta_{\mu\mu'}\delta_{\nu\nu'}. \quad (4b)$$

Equation (4a) has the form used previously[5,7] with $\lambda$ a strength parameter of dimension energy. The average GOE level spacing $d$ is given in terms of $\lambda$ by $d \propto \lambda N^{-1}$. The real dimensionless parameter $\alpha_0 N^{-1/2}$ measures the relative strength of the GUE versus that of the GOE. It must be chosen[10,11] to be of order $N^{-1/2}$. Indeed, the dimensionless parameter $|R_{ab}|^2$ in Eq. (1) can depend on the strength of $H^{(u)}$ only via the dimensionless combination $\alpha_0^2 \lambda^2 N^{-2}/d^2$. This ratio is independent of $N$ only if $\alpha_0$ does not depend on $N$.

The fact that the matrix $W$ is Hermitian but cannot be chosen real has no bearing on the value of $|R_{ab}|^2$. Indeed, both Eqs. (4) are invariant under real orthogonal transformations in the space of CN levels. Except for trivial overall phase factors (which cancel in $|R_{ab}|^2$), both the one-point functions $\langle S_{ab}(E) \rangle$ and the two-point functions considered below can depend on $W_{\mu a}$ only via the orthogonally invariant form $\sum_\mu W_{a\mu} W_{\mu b}$. In keeping with Nishioka and Weidenmüller,[12] we assume that this Hermitian matrix in channel space has been diagonalized by a unitary transformation. With

$$\sum_\mu W_{a\mu} W_{\mu b} = \delta_{ab} \sum_\mu W_{a\mu} W_{\mu a} = \delta_{ab} \sum_\mu |W_{\mu a}|^2,$$

we see that the one- and two-point functions depend nontrivially only on the moduli of the matrix elements $W_{\mu a}$. We therefore take $W_{\mu a}$ to be real in the sequel.

We calculate $|R_{ab}|^2$ to lowest nonvanishing (= first) order in $\alpha_0^2$, and for the case of strongly overlapping CN resonances, where the number $\Lambda$ of open channels is large, $\Lambda \gg 1$. Using the fact that $\sigma_{ab}$ is the product[5,7] of two $S$-matrix elements, and that for $\Lambda \gg 1$ the elements of the $S$-matrix are Gaussian distributed,[13] we deduce from Eq. (1) that $R_{ab}$ is the ratio of two two-point functions,

$$R_{ab} = \langle S_{ab}(E) S^*_{ba}(E) \rangle \langle S_{ab}(E) S^*_{ab}(E) \rangle^{-1}. \quad (5)$$

In keeping with the random-matrix model, we have replaced the energy average of Eq. (1) by the ensemble average over the joint probability density of $H^{(o)}$ and $H^{(u)}$.

Use of the GOE in Eq. (4a) to simulate the local fluctuation properties is well justified, and forms also the basis of the work of Ref. 11. Even in the presence of secular changes, all available evidence in nuclei (isolated levels, doorways) shows that the fluctuation properties are consistent[14,15] with the GOE. The same holds true for the fluctuation properties of the Sinai billiard.[15] As for the GUE, the recent analysis[16] of the quantized Sinai billiard with magnetic field assigns to it a similarly universal role in the presence of TRSB. Secular changes of $\lambda$ or $\alpha_0$ in Eqs. (4) lead to small corrections of $\Gamma^{(u)}$.

We expand the $S$-matrix elements in Eq. (5) to second order in $H^{(u)}$ and take the GUE average of numerator and denominator with the help of Eq. (4b). The second-order terms cancel mutually in the ratio $R_{ab}$ and therefore need not be considered. We also use that for $\alpha_0 = 0$, we have[7]

$$S_{ab} S^*_{ab} = S_{ab} S^*_{ba} = T_a T_b / \sum_c T_c.$$

This is the Hauser-Feshbach formula with transmission coefficients $T_a = 1 - |\langle S_{aa} \rangle|^2$ and for $\langle S_{ab} \rangle = \delta_{ab} \langle S_{aa} \rangle$. To first order in $\alpha_0^2$, we find

$$R_{ab} \simeq 1 + 2\alpha_0^2 \lambda^2 (Y_{ab} - Y_{ba}) [T_a T_b / \sum_c T_c]^{-1} \quad (6)$$

with the four-point functions $Y_{cd}$ given by

$$Y_{cd} = 4\pi^2 N^{-2} \sum_{\kappa\mu\nu\rho\sigma\tau} W_{\kappa a} W_{\rho b} \langle G^+_{\kappa\mu} G^-_{\mu\nu} G^+_{\rho\sigma} G^-_{\sigma\tau} \rangle W_{\nu d} W_{\tau c}. \quad (7)$$

The average extends over the GOE only and the propagators, denoted by $G^{(\pm)} = (EI - H^{(o)} \pm i\pi M)^{-1}$, are symmetric, $G^{(\pm)}_{\mu\nu} = G^{(\pm)}_{\nu\mu}$. We must calculate the GOE four-point functions of Eq. (7). This goes beyond the problem solved in Ref. 5. We confine ourselves to the term of lowest order in an asymptotic expansion in powers of $(\sum T_c)^{-1}$, the same approximation which yields the Hauser-Feshbach term in Eq. (6), and valid for $\Lambda \gg 1$. Even in this approximation the calculation is very involved and favorably carried out with the help of Grassmann variables.[5] It yields $R_{ab} = R$ independent of $a,b$ for all $a \neq b$. The quantity $R$ can be written in two different ways. One way is

$$R = 1 - 2\alpha_0^2 (\sum_c T_c)^{-1}. \quad (8)$$

Alternatively, we may introduce the average CN decay width $\Gamma = d/(2\pi) \sum T_c$, and the spreading width $\Gamma^{(u)} = 2\pi (2\alpha_0^2 \lambda^2 / N^2)/d$ associated with the GUE Hamiltonian $H^{(u)}$. Taking for $d$ the expression $d = \pi \lambda N^{-1}$ valid at the center of the GOE spectrum, we obtain

$$R = 1 - \tfrac{1}{4} \Gamma^{(u)}/\Gamma. \quad (9)$$

Equation (8) shows that $R$ is independent of the location of the energy $E$ within the GOE spectrum (for transmission coefficients chosen constant throughout the spectrum). Equation (9), suggested in similar form twenty years ago by Ericson,[17] shows that $R$ differs from unity by a term which is proportional to

the ratio of the CN lifetime $\hbar/\Gamma$ versus the time $\hbar/\Gamma^{(u)}$ it takes to mix any two close-lying CN levels because of the presence of $H^{(u)}$. This is intuitively reasonable. The negative sign in Eqs. (8) and (9) is implied by the Schwarz inequality.

In using our result to interpret the data, we must choose between the forms (8) and (9). This raises the following question: Which of the two quantities $\alpha_0^2$ and $\Gamma^{(u)}$ is the natural choice for parametrizing $R$ in the sense that the value of the parameter used depends weakly on energy $E$ and mass number $A$? The question is nontrivial since the ratio $\alpha_0^2/\Gamma^{(u)}$ depends on the level spacing and thus strongly on both $E$ and $A$. We have no unequivocal answer and defer the question to the end of this Letter, first discussing the data in the light of both parametrizations (8) and (9).

The authors of Ref. 4 expressed their experimental result in the form $R^2=1-4\xi^2$. For the CN $^{28}$Si the quantity $\xi$ was found to be smaller than $5\times10^{-4}$ with a confidence level of 80%. With $\Gamma=45$ keV taken from Ref. 9 this implies

$$\Gamma^{(u)}\leq 9\times10^{-2}\ \text{eV}. \tag{10}$$

Using the Fermi-gas level density of Gilbert and Cameron[18] and the level-density parameter from Vonach and Hille,[19] we find for a CN with spin 3 that $d=9.6$ keV. This and $\Gamma=45$ keV imply $\sum_c T_c=29$, and hence

$$\alpha_0^2\leq 7.3\times10^{-6}. \tag{11}$$

This result compares favorably with a recent analysis of level fluctuations by French *et al.*[11]. When expressed[20] in our notation, the result of Ref. 11 reads $\alpha_0^2\leq10^{-1}$. Converting the result of Ref. 11, by help of the GOE relation $\Gamma^{(u)}/\alpha_0^2=4d/\pi$, into an upper limit for $\Gamma^{(u)}$ (a quantity meaningful also for isolated levels), we obtain $\Gamma^{(u)}\leq1$ eV. Here, we have used $4d/\pi\simeq10$ eV as a rough approximation[21] to the average spacing of the levels analyzed in Ref. 11. Again the result (10) of the detailed-balance experiment compares favorably with the result from statistical spectroscopy.

The very different roles played by the two parameters are brought to light most clearly if we express the results[1,2] for the $K$-$\bar{K}$ system in terms of $\alpha_0^2$ and $\Gamma^{(u)}$. We take for the level spacing $d$ the mass difference $3.51\times10^{-12}$ MeV between $K_L$ and $K_S$, and interpret the probability amplitude $|\epsilon|=2.3\times10^{-3}$ for mixing of the two configurations as the ratio of the matrix element of the TRSB part of the interaction versus $d$. With $\alpha_0^2=\frac{1}{2}\pi^2\epsilon^2$ we obtain $\alpha_0^2=2.6\times10^{-5}$ while $\Gamma^{(u)}$ is of the order $10^{-10}$ eV. Certainly the concept of a spreading width in the $K$-$\bar{K}$ system is highly questionable. However, the figures just given underline the difference between the parameters $\alpha_0^2$ and $\Gamma^{(u)}$: If $\alpha_0^2$ is the natural parameter, then the result from the $K$-$\bar{K}$ system is comparable to the upper limit on TRSB set by the detailed-balance experiment. This is not so if $\Gamma^{(u)}$ is the natural parameter.

A strong hint that $\Gamma^{(u)}$ is the relevant parameter is furnished by recent analyses[22,23] of a large set of data on isospin-symmetry breaking in CN reactions. While a plot of the rms Coulomb matrix elements shows a strong dependence on both mass number and excitation energy of the CN, the associated spreading widths—defined in analogy to $\Gamma^{(u)}$—vary by less than a factor of 10 around their mean value. This can be understood semiquantitatively on the basis of a sum rule. The isospin symmetry broken by the Coulomb force is, of course, of a type different from time-reversal symmetry. Nonetheless, the theory of symmetry breaking in CN reactions has so many points in common in both cases that we strongly feel that $\Gamma^{(u)}$ is the parameter which ought to be used.

Taking this point of view, we note that the most promising choice for a further improvement of the experimental limit on TRSB in nuclei is a case with a small decay width $\Gamma$ which means that $d$ and/or $\sum_c T_c$ should be small. The analysis of the data may then require an extension of our theory: For $\Gamma\lesssim d$, the distribution of the $S$-matrix elements is no longer Gaussian. Even for $\alpha_0^2=0$, the terms appearing on the right-hand side of Eq. (1) then contain the GOE four-point function, and for the terms of first order in $\alpha_0^2$, the six-point function is needed. Work on this problem is under way.[24]

In conclusion, we have established a novel and rigorous theoretical framework for connecting experimental bounds on violation of the law of detailed balance with the underlying TRSB part of the Hamiltonian. We have analyzed the best existing data in this framework. We have suggested a way of further improving the upper bounds on $\Gamma^{(u)}$.

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[1]J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Lett. **13**, 138 (1964).

[2]K. Kleinknecht, Annu. Rev. Nucl. Sci. **26**, 1 (1976).

[3]A. Richter, in *Interaction Studies in Nuclei,* edited by H. Jochim and B. Ziegler (North-Holland, Amsterdam, 1975), p. 191.

[4]E. Blanke, H. Driller, W. Glöckle, H. Genz, A. Richter, and G. Schrieder, Phys. Rev. Lett. **51**, 355 (1983).

[5]J. J. M. Verbaarschot, H. A. Weidenmüller, and M. R. Zirnbauer, Phys. Rep. **129**, 367 (1985).

[6]Details may be found in D. Boosé and H. A. Weidenmüller, to be published.

[7]H. A. Weidenmüller, Ann. Phys. (N.Y.) **158**, 120 (1984).

[8]C. Mahaux and H. A. Weidenmüller, *Shell-Model Approach to Nuclear Reactions* (North-Holland, Amsterdam, 1969).

[9]B. W. Allardyce, W. R. Graham, and I. Hall, Nucl. Phys. **52**, 239 (1964).

[10]H. S. Camarda, Phys. Rev. C **13**, 2524 (1976).

[11]J. B. French, V. K. B. Kota, A. Pandey, and S. Tomsovic, Phys. Rev. Lett. **54**, 2313 (1985).

[12]H. Nishioka and H. A. Weidenmüller, Phys. Lett. **157B**, 101 (1985).

[13]D. Agassi, H. A. Weidenmüller, and G. Mantzouranis, Phys. Rep. **22C**, 145 (1975).

[14]H. L. Harney, Phys. Rev. Lett. **53**, 537 (1984).

[15]O. Bohigas and M. J. Giannoni, in *Mathematical and Computational Methods in Nuclear Physics,* edited by J. S. Dehesa, J. M. G. Gomez, and A. Pollis, Lecture Notes in Physics Vol. 209 (Springer-Verlag, Berlin, 1984).

[16]M. V. Berry and M. Robnik, to be published; O. Bohigas, private communication.

[17]T. E. O. Ericson, Phys. Lett. **23**, 97 (1966).

[18]A. Gilbert and A. G. W. Cameron, Can. J. Phys. **43**, 1446 (1965).

[19]H. Vonach and M. Hille, Nucl. Phys. **A127**, 289 (1969).

[20]Note that the quantities $\alpha$ and $\Lambda$ of Ref. 11 are related to $\alpha_0$ by $\alpha_0=\alpha N^{1/2}$ and $\Lambda=\alpha_0^2\pi^{-2}$.

[21]This is clearly only a rough estimate. If—as we suggest below—$\Gamma^{(u)}$ is indeed the physical parameter of primary interest, the data of Ref. 11 should be reanalyzed.

[22]E. Kuhlmann, Z. Phys. A **322**, 527 (1985).

[23]H. L. Harney, A. Richter, and H. A. Weidenmüller, to be published.

[24]E. Lanza and P. Pereyra, private communication.