Sensitivity of the Conductance of a Disordered Metal to the Motion of a Single Atom: Implications for $1/f$ Noise

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We show that the conductance of metals is sensitive to the motion of a single scattering center. At zero temperature and in two dimensions, the motion of one strong scattering center induces changes of the conductance of order e^2/h, independent of sample size. We discuss the implications of this result for room-temperature $1/f$ noise in disordered metals and we predict an anomalous low-temperature $1/f$ noise in metallic glasses, due to two-level systems.

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Recently it has been found both experimentally[1-3] and theoretically[4-9] that the electrical conductance in disordered metals exhibits fluctuations as the magnetic field B or the chemical potential μ is varied. The rms magnitude of these fluctuations for a given sample is the same as that from sample to sample and is of order $\delta G \approx e^2/h$, independent of the degree of disorder, the sample size, and spatial dimensions, provided the temperature is low enough that the inelastic scattering length L_{in} is larger than any of the sample dimensions. $L_{\text{in}} = (D\tau_{\text{in}})^{1/2}$ is the distance an electron diffuses during the inelastic scattering time τ_{in} if D is the diffusion constant. It is to be emphasized that such fluctuations are not time-dependent noise. Instead, the conductance $G(\mu,B)$ is a deterministic, albeit fluctuating, function of its arguments, for a given realization of the impurity configuration.

The surprisingly large conductance fluctuation leads naturally to the following question: How sensitive is the conductance of a given metal to a small change in the impurity configuration? We provide an answer to this question in this Letter and our main result can be summarized very simply as follows. Suppose we have a small metallic sample of size L. By metallic we mean that the elastic mean free path l is small compared with the sample length L, which is much less than the localization length. At low temperature such that the inelastic diffusion length L_{in} is large compared to L, if *one* impurity atom is moved a distance $\delta r > k_F^{-1}$, the conductance of the sample will change by the order $\delta G_1 \approx e^2/h$ in one and two dimensions, and $\delta G_1 \approx (e^2/h)(l/L)^{1/2}$ in three dimensions. In one and two dimensions, we have the novel result that the conductance fluctuations induced by moving a *single* scattering center are as large as those produced by changing the entire sample! Small ($\sim 0.5\%$) discrete conductance jumps have been observed in quasi one-dimensional metal-oxide-semiconductor field-effect transistors and attributed to the filling of individual trap states in the oxide which affects the electron mobility.[10] Our present result predicts quantitatively the magnitude of the conductance fluctuation and, more importantly, shows that it is not simply a finite-size effect observable only in ultrasmall samples.

Our result can be understood by a simple physical argument. Conductance is proportional to the quantum-mechanical transmission probability through the sample,[11] which in turn can be understood in terms of interference between classical Feynman paths through the sample.[6] For a disordered material, the Feynman paths are random walks with step size l. The number of sites visited by each Feynman path that crosses the sample is $(L/l)^2$. In two dimensions this means that each path visits a finite fraction of the sites, or, alternatively, a finite fraction of all Feynman paths pass through a site. The motion of a single strong scatterer will alter the phase of all the Feynman paths passing through that site. The resulting interference will be completely altered so that the motion of a single strong scatterer has the same effect as altering the entire impurity configuration. In $d \geq 2$ dimensions, a fraction $(L/l)^{2-d}$ of the Feynman paths passes through a given site. With the additional assumption that the contribution to the transmission amplitude due to this fraction of the paths is statistically independent from the contribution of the remaining paths, we conclude that the change in the conductance is of the order $\delta G_1 = (e^2/h)(L/l)^{(2-d)/2}$. This last assumption is by no means obvious, because we know that correlations between Feynman paths are required to produce the universal conductance fluctuations.[6,9] Nevertheless, it appears that sufficient correlation has been built into the partial summation over the Feynman paths passing through a given site that this argument produces the correct answer. Finally we note that in quasi one-dimensional systems, each Feynman path actually visits a given site many times, so that the motion of a weak scatterer, or a slight motion of a strong scatterer,

can accumulate sufficient phase shift to change the conductance by e^2/h.

The quantitative calculation is a straightforward extension of our previous model.[5] We assume our system to be a general rectangular box in d dimensions, with a volume $\Omega = L_x \cdots L_{d-1} L_z$. The current flows in the z direction, which is extended to $\pm\infty$ by attaching ideal "leads." The electrons are treated as noninteracting quasiparticles; we have shown elsewhere[6] that to leading order in $(k_F l)^{-1}$, interaction effects are accounted for by the introduction of a τ_{in} due to electron scattering. Let $\{\mathbf{r}_i\}$ $(i = 1, \ldots, N_i)$ be the positions of the impurities. We consider the conductance change induced by moving *one* impurity scattering center a distance $\delta\mathbf{r}$, i.e.,

$$(\delta G_1)^2 = \langle (G\{\mathbf{r}_1, \ldots, \mathbf{r}_{N_i}\} - G\{\mathbf{r}_1 + \delta\mathbf{r}, \ldots, \mathbf{r}_{N_i}\})^2 \rangle, \quad (1)$$

where G is the system's conductance. The average in Eq. (1) is taken over the random distribution of impurity positions $\{\mathbf{r}_i\}$. Let us first assume that $L_{in} > L_x, \ldots, L_{d-1}, L_z$. Equation (1) can be evaluated and we work to leading order in the perturbing potential. It is easy to show that the Feynman rule is to insert a new vertex connecting the two conductivity loops into the diagrams of Ref. 5. For example, Fig. 1(d) is the most important modification of Fig. 1(b) of Ref. 5. The new vertex is denoted by a diamond shape and, for a momentum transfer $\mathbf{k}$, is given by

$$V_1(\mathbf{k}) = (u^2/N_i)\,\mathrm{Re}[1 - \exp(i\mathbf{k}\cdot\delta\mathbf{r})], \quad (2)$$

where u^2 is the impurity scattering strength related to the elastic scattering time by $\tau^{-1} = 2\pi n(\epsilon_F) u^2$, $n(\epsilon_F)$ being the density of states per spin.

We note that the effect of introducing the vertex V_1 is to introduce an extra diffusion pole in the evaluation of the diagram so that the result is more singular by a factor L^2. Combining this with the vertex [Eq. (2)] which is proportional to $N_i^{-1} \sim L^{-d}$ gives the size dependence $\delta G_1^2 \sim L^{2-d}$ discussed earlier. Similar modifications of other diagrams in Ref. 5 give contributions of the same form. We give the result for several geometries accurate up to a numerical constant.

For hypercubes, i.e., $l < L_x = \ldots = L_{d-1} = L_z = L < L_{in}$,

$$(\delta G_1)^2 \approx (e^2/h)^2 (\Omega/N_i l^d)(L/l)^{2-d} \alpha(k_F \delta r), \quad (3)$$

where

$$\alpha(x) = 1 - (\sin\tfrac{1}{2}x/\tfrac{1}{2}x)^2. \quad (4)$$

Usually the individual impurity scatters strongly, in which case the factor $(\Omega/N_i l^d) \approx 1$. If $k_F \delta r \gtrsim 1$, then $\alpha \approx 1$, and Eq. (3) gives the results quoted in the introduction.

If more than a single atom is moved, the effect on Eq. (3) is simply additive as long as $\delta G_1 \ll e^2/h$. Of course, in general δG_1 is bounded by e^2/h, which is the rms change for a complete rearrangement of the impurity configuration. This bound is important for the motion of more than one impurity in $d \geq 2$ and for $d = 1$. Mathematically this bound appears as a breakdown of our perturbative treatment of the vertex V_1, since repeated insertion of the diamond vertex in Fig. 1(d) will generate a series in powers of $\alpha(L/l)^{2-d}$.

For thin films ($L_x \ll L_y, L_z$), if $l < L_x$, the film thickness, we replace the factor $(L/l)^{2-d}$ in Eq. (3) by l/L_x. For wires ($L_x \sim L_y \ll L_z$), if $l < L_x, L_y$, we make the corresponding replacement by $L_z l/L_x L_y$. This enhancement factor is due to multiple visits of a Feynman path to a given site. Thus in wires even small atomic displacement ($k_F \delta r \ll 1$) can produce the saturation effect of $\delta G_1 \approx e^2/h$.

We test Eq. (3) for $d = 2$ by numerical calculations of G for a nearest-neighbor tight-binding model with random site energies (the Anderson model).[4,5] In Fig. 1(a) we show the rms fluctuation of the sample con-

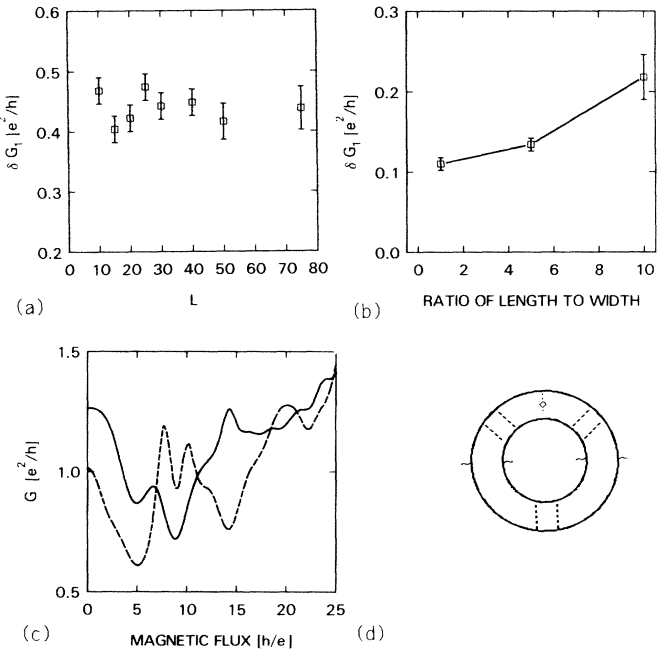

FIG. 1(a) The rms deviation of the conductance (in units of e^2/h) of an $L \times L$ Anderson model ($W/V = 4$) upon interchange of the local energy of two sites. The rms deviation for a complete configuration change is 0.86 (Ref. 5). (b) δG_1 of an $M \times L$ Anderson model ($M = 25$, $W/V = 1$) as the site energy of a single site is changed by $W/2$ plotted vs the aspect ratio L/M. (c) $G(B)$ for 40×40 Anderson model vs B in units of h/e normal to the sample. Solid and dashed curves correspond to impurity configurations which differ only by the interchange of a pair of sites. (d) A typical diagram that contributes to $(\delta G_1)^2$. Dashed lines denote impurity averaging and the diamond denotes the vertex [Eq. (2)] which represents the motion of a single impurity.

ductance δG_1, after an interchange of on-site energies of one pair of sites, averaged over different realizations of the random energies, as a function of the sample sizes. The simulated conductance fluctuation $\delta G_1 \approx 0.45(e^2/h)$ is independent of the sample size, in agreement with the above analytic result. In Fig. 1(b) we show how δG_1 is increased as a strip is lengthened, a demonstration of the effect of multiple visits in a quasi one-dimensional geometry. In Fig. 1(c) we plot the "magnetofingerprints," i.e., the function $G(B)$ for two configurations which differ only by the interchange of a single pair. The drastic effect produced by interchanging two sites out of 1600 is evident.

We next consider the effect of finite temperature. We can divide the metal into boxes bounded by $L_{\rm in}$, e.g., its volume $\Omega_{\rm box}$ is $L_{\rm in}^d$ for hypercube and $L_{\rm in}^2 L_x$ for thin films. At finite temperature, we have to average the conductance change within each box over an electron energy spread of order kT.[4,6,9] It is easy to show, following Ref. 5, that as a function of electron energy, the conductance change in a box of size $L_{\rm in}$ is correlated over an energy scale of $\hbar\tau_{\rm in}^{-1}$. Provided that $kT > \hbar\tau_{\rm in}^{-1}$, we have to reduce the results of Eq. (3) by the factor $(kT\tau_{\rm in}/\hbar)^{-1}$.[12] The conductance change δG_{1T} due to the motion of one impurity in the sample is given by combining the δG conductance of each box classically in series and parallel, and we conclude that $\delta G_{1T} = \delta G_1 (L_{\rm in}^2/L_z^2)[f(\hbar/kT\tau_{\rm in})]^{1/2}$, where $f(x) = x$ for $x < 1$ and $f(x) = 1$ for $x > 1$.

The above picture is applicable as long as $L_{\rm in} > l$ which is the same as $\tau^{-1} > \tau_{\rm in}^{-1}$. For sufficiently disordered materials, this relation holds up to very high temperature.[13] For example, at room temperature, $\tau_{\rm in}^{-1}$ due to electron-phonon scattering is $\approx kT/\hbar$ and the above inequality holds as long as $k_F l < \epsilon_F/kT$. Furthermore, the temperature reduction factors have a relatively slow power-law dependence on temperature, so that the effect of quantum coherence manifested through the sensitivity to impurity configuration may be observable at room temperature! This is in contrast to the magnetofingerprint itself, which is unobservable at higher temperatures because the magnetic field necessary to change the conductance in a box becomes too large.[6]

An obvious application of these ideas is to the phenomenon of $1/f$ noise. Already, the experimental observation of discrete conductance jumps in small metal-oxide-semiconductor field-effect transistors has been related to $1/f$ noise in large devices.[10,14] Now we can make the relationship more general and quantitative. A standard model for $1/f$ noise is that it measures the spectrum of resistance fluctuation, i.e., $S_R(\omega) = \int dt \langle \delta R(t) \delta R(0) \rangle e^{i\omega t}$.[15] Resistance fluctuations are thought to be due to thermally activated motion of defects. Given any model of defect motion, $S_R(\omega)$ can be determined. As an illustration, let us take a model in which each defect hops back and forth between two sites with a time scale t. If $t \gg \tau_{\rm in}$, and if the hopping distance is uncorrelated with t, we can write

$$S_R(\omega) = R^2 \left(\frac{(\delta G)^2}{G^2} \right) \int \frac{2t}{\omega^2 t^2 + 1} p(t)\, dt, \tag{5}$$

where $p(t)$ is the distribution of the defect motion time. If the motion is thermally activated with a broad distribution of activation energies, one obtains an approximately $1/f$ spectrum by standard arguments.[15] The magnitude of the noise is proportional to the fluctuation of the total conductance,

$$\frac{(\delta G)^2}{G^2} = \frac{(\delta G_{\rm box})^2}{G_{\rm box}^2} \frac{\Omega_{\rm box}}{\Omega} f(\hbar/kT\tau_{\rm in}). \tag{6}$$

The factor $\Omega/\Omega_{\rm box}$ comes from combining the conductance fluctuation of each box classically. $\delta G_{\rm box}$ is given by Eq. (3) multiplied by $N_{\rm box}$, the number of active defects in each box during the experimental time scale, and $\delta G_{\rm box}$ is bounded by e^2/h. We note that Eq. (6) implies dimensional crossover when $L_{\rm in}$ exceeds the film's thickness or wire diameter, leading to an enhancement of the noise over the usual inverse volume dependence.

Equations (5) and (6) relate the magnitude of the noise to a microscopic model of defect motion. To our knowledge, the issue of how the motion of a few defects out of 10^{20} atoms can lead to observable resistance noise has not been adequately addressed previously. It is remarkable that according to our theory, the room-temperature $1/f$ noise in disordered metals owes its observability to properties of quantum diffusion on a length scale short compared with $L_{\rm in}$.

Finally, the unusual sensitivity of conductance to atomic motion motion leads us to predict a novel low-temperature $1/f$ noise in metallic glasses. At low temperatures the conventional defect migration mechanism is frozen out. However, in metallic glasses we expect the two-level systems (TLS) to persist down to very low temperatures. A standard estimate of the density of experimentally accessible TLS, n_T, is 10^{19} to 10^{20} eV^{-1} cm^{-3}. The inelastic diffusion length $L_{\rm in}$ in metallic glasses such as PdSi or NiP is probably of the order of 1000 Å at 100 mK. The elastic mean free path l in such materials should be no more than a few angstroms. Let us consider a thin film. Assuming that all the TLS within energy kT contribute to the fluctuation during the experimental time scale, we can estimate the total conductance fluctuation by combining Eq. (6) with the thin-film version of Eq. (3) and multiplying by the number of TLS within each box $(n_T kT L_x L_{\rm in}^2)$, to obtain

$$\left(\frac{\delta G}{G} \right)^2 = (k_F \delta r)^2 \left(\frac{n_T}{n(\epsilon_F)} \right) \left(\frac{L_{\rm in}}{L} \right)^2 (k_F L_x)^{-2}, \tag{7}$$

where we have assumed $kT > \tau_{in}^{-1}$ and the first factor comes from $\alpha(k_F\delta r)$ when $k_F\delta r << 1$. Taking $L \approx 10^4$ Å, $L_x = 100$ Å and assuming $\delta r \approx 0.1$ Å and $n(\epsilon_F) \approx 10^{23}$ eV^{-1} cm^{-3}, we estimate $\delta G/G \approx 10^{-6}$. For wires we expect even larger effects. An interesting feature of result (7) is that the noise level is predicted to increase with decreasing temperature. This is because the reduction of the number of TLS is offset by the thermal average [f in Eq. (6)] and the increasing L_{in} gives rise to greater sensitivity to atomic motion. Ultimately the noise level saturates when $L_{in} \geq L$ or when there are no active TLS left in the finite sample.

The integrated noise power is given by $\int d\omega\, S_R(\omega) = R^2(\delta G/G)^2$, with $(\delta G/G)^2$ given by Eq. (7). The power spectrum itself is constructed with use of Eq. (5) and the result from TLS theory, $t = t_0 \exp[(V/V_0)^{1/2}]$ where $V_0 = h^2/(2m\delta r^2) \approx 1$ eV, $t_0 \approx 1/\omega_D$, and V is the height of the potential barrier separating the two metastable states of a given TLS. By standard arguments,[15] we see that if V is broadly distributed on the scale of V_0, which is a rather reasonable assumption, one should have $S_R(\omega) \propto 1/\omega$, i.e., the power spectrum of TLS-produced conductance noise should be $1/f$ in form.[16] One also notices that if the number of active tunneling centers ($= n_T kT\Omega$), is of order unity, which is the case for ultrasmall samples at very low temperature, one should be able to see the actual "glitches" when a single tunneling event takes place.

In this Letter we have focused on the application of our theory to $1/f$ noise. There are clearly many other potentially important applications. Examples include these: (a) Study the time scale of spin-glass and other glassy phenomenon such as thermal depinning of charge- and spin-density waves.[17] One can study the effect of thermal annealing on the magnetofingerprint of such glassy objects. (b) Study the radiation damage of small films and wires. At low flux, it should be possible to observe glitches in the conductance as individual α particles damage the sample. Another possibility is to study samples in which an occasional β decay changes the nuclear charge on a single atom which should lead to a substantial change in the scattering potential. (c) Study optical and other classical-wave analogs to the quantum diffusion problem.

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Note added.—After the completion of this work, we received a paper by Altshuler and Spivak[18] who also pointed out the sensitivity of the conductance fluctuation to small configuration changes and applied the theory to the magnetic field dependence of the spin-exchange scattering in spin-glasses.

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