## Elastic Properties of Charge-Density-Wave Conductors: ac-dc Electric Field Coupling

L. C. Bourne, M. \$. Sherwin, and A. Zettl

Department of Physics, University of California, Berkeley, Berkeley, California 94720 (Received 21 October 1985; revised manuscript received 14 February 1986)

We have measured Young's modulus Y and internal friction  $\delta$  of the charge-density-wave  $(CDW)$  conductors TaS<sub>3</sub> and NbSe<sub>3</sub>, in the presence of combined ac and dc electric fields. In the pinned CD% state, the elastic properties are sensitive to even small ac-field amplitudes. In the sliding-CDW state, sharp harmonic and subharmonic anomalies are observed in Y and  $\delta$ ; these coincide with Shapiro-type interference in the electronic response. Our experiments demonstrate that internal degrees of freedom (incoherence) of the CDW condensate are intimately tied to the dc- and ac-field-induced elastic anomalies.

PACS numbers: 72.15.Nj

It has been clearly established that the anomalous low-field ac conductivity and nonlinear dc conductivity of low-dimensional materials such as  $TaS_3$  and NbSe<sub>3</sub> are the result of collective-mode charge-density-wave (CDW) dynamics.<sup>1</sup> Recently, it was demonstrated<sup>2, 3</sup> that bulk elastic properties of CDW conductors are modified by applied dc electric fields  $E_{dc}$  exceeding the threshold field  $E_T$  for the onset of nonlinear electronic conduction. In particular, an increasing CDW drift velocity results in a decrease in Young's modulus Y and a corresponding increase and eventual saturation of internal friction  $\delta$  of the crystal. No detailed model of CDW elasticity has made predictions<sup>4</sup> consistent with these experimental findings.

We have investigated the bulk elastic properties of the CDW materials orthorhombic  $TaS_3$  and  $NbSe_3$ , in the presence of combined ac and dc electric driving fields. In the limit of zero ac-field amplitude, our results are in accord with previous measurements.<sup>2, 3</sup> In the limit of zero dc field, we find that  $Y$  is modified by the applied ac field, even in the range of very small ac amplitude where the CDW remains always pinned. In the regime of combined ac and dc electric fields, sharp anomalies are observed in Y and  $\delta$ ; these coincide with Shapiro-type interference<sup>5-7</sup> in the electroni response. We present a model of CDW transport and elasticity which includes coupling of the CDW dynamics to the underlying lattice dynamics. A computer solution of the model for ac, dc, and combined ac and dc drive fields yields behavior for elastic constants of the crystal in agreement with our experimental findings.

Our experiments were performed on single crystals of NbSe<sub>3</sub> and orthorhombic TaS<sub>3</sub> grown by conventional vapor transport methods, of typical dimensions 1 mm × 2  $\mu$ m × 5  $\mu$ m. The samples were mounted with silver-paint contacts in a two-probe "clampedclamped" configuration. Flexural resonances were excited by capacitive coupling to the crystal, and the mechanical response was monitored by an rf detection technique using a 600-MHz carrier. The crystal formed part of a phase-locked loop which allowed simultaneous recording of the frequency  $\omega$ , and amplitude A, of the mechanical resonance. The Young's modulus of the crystal was obtained from the resonance frequency using the relation  $Y=D\omega_r^2$ , where D is a constant which depends on boundary conditions and crystal dimensions and density.<sup>2</sup> Changes in  $1/A_r$ were directly related to changes in  $\delta$ . Typical resonant quality factors for crystals used in these experiments were a few hundred. Our measurement system also allowed simultaneous measurement of the differential resistance  $dV/dl$  of the sample. On occasion, signal averaging of the data was performed to improve the signal-to-noise ratio. The samples were cooled in a liquid-nitrogen gas-flow cryostat, and the gas pressure inside the sample chamber was held at 5 Torr to reduce Joule self-heating of the crystals. Details of the experimental technique will be presented elsewhere.

Figure 1 shows Y,  $\delta$ , and  $dV/dl$ , as functions of dc bias, for a  $TaS_3$  crystal in the CDW state. Data for various amplitudes of the applied ac field at frequency  $\omega_{\text{ac}}/2\pi = 1$  MHz are shown. Curves A correspond to no applied ac field, and here the drop in Y and rapid increase in  $\delta$  for dc bias values exceeding  $E_T$  are consistent with previous studies.<sup>2,3</sup> As the ac field amplitude is increased from zero, Fig. 1 shows that  $E_T$ , as determined from the initial break in the  $dV/dl$  curve, is decreased to a lower value  $E_T$ , again consistent with previous conductivity studies.<sup>5-7</sup> Curves B, C, and D in Fig. 1 demonstrate that the depression in  $E_T$  is reflected in the behavior of Y and  $\delta$  as well. Both Y and  $\delta$  begin to deviate at  $E_T$  from their zero-dc-bias values (measured with ac applied), and thus, even with an applid ac field,  $Y$  and  $\delta$  are insensitive to dc bias until the CDW becomes depinned and assumes a finite dc drift velocity at  $E'_T$ . Figure 1 also shows that  $\delta$  saturates more quickly, as a function of dc bias, with increasing ac field amplitude. On the other hand, Fig. 1 demonstrates that the effect of an ac field on Y and  $\delta$  is not just to rescale the effective dc bias field  $E_{dc}$ . For example, curves  $A-D$  show that, even for zero dc bias, an increase in ac amplitude  $E_{ac}$  results in a decrease in Y and an increase in  $\delta$ , although  $dV/dl$  remains un-



FIG. 1. Y,  $\delta$ , and  $dV/dl$  as functions of dc bias in TaS<sub>3</sub>. A 1-MHz ac field of amplitude  $E_{ac}$  is also present.

changed. It should be noted that curves  $A-D$  in Fig. 1 have, within each group, not been displaced with respect to one another.

The detailed behavior of Y,  $\delta$ , and  $dV/dl$  in the presence of an ac field alone (zero dc bias) is shown in Fig. 2. Here, for low-amplitude ac excitation, the CDW remains always pinned. Within experimental resolution,  $\delta$  is roughly independent of ac amplitude for  $E_{ac} < E_T$ . On the other hand, Y changes smoothly as ac amplitude is increased from zero, with no evidence for a threshold behavior. Hence, in the pinned CDW state, the elastic properties of the crystal are influenced even by low-amplitude ac dynamics of the CDW condensate. As  $E_{ac}$  is increased from zero to  $E_T$ , there is a relative decrease in Y of approximately  $5 \times 10^{-5}$ . For  $E_{ac} = 2E_T$ , an increase in  $\delta$  of roughly  $2 \times 10^{-5}$  is observed.

An interesting consequence of superposed ac and dc electric fields in CDW conductors is Shapiro-step electronic interference, where sharp "peaks" are observed in  $dV/dl$  whenever the frequency of the ac field matches a harmonic or subharmonic of the intrinsic narrow-band noise frequency.<sup>5-7</sup> Shapiro-step interference is present in the  $dV/dl$  traces of Fig. 1, but somewhat difficult to identify due to the relatively poor resolution. Figure  $3(a)$  shows the results of a very sensitive measurement of  $dV/dl$ , Y, and  $\delta$  in the presence of combined ac and dc electric fields for a different  $TaS_3$  crystal. The dominant (fundamental) Shapiro step is identified in the lower  $dV/dl$  trace with an arrow. Related anomalies are clearly observed in Y



FIG. 2. Y,  $\delta$ , and  $dV/dl$  as functions of ac field amplitude in  $TaS<sub>3</sub>$ . No dc bias field is present. A low-amplitude field softens the crystal lattice.

and  $\delta$  at the same values of dc bias. Figure 3(a) indicates that, during Shapiro-step electronic interference,  $Y$  and  $\delta$  tend to the values characteristic of the pinned CDW state. This follows the behavior of  $dV/dl$  on the Shapiro step. Figure 3(b) shows similar interference data for a  $NbSe<sub>3</sub>$  crystal in the upper CDW state. Here, the interference effect in  $dV/dl$  is far more dramatic than that for  $TaS_3$ , and indeed the corresponding anomalies in Y and  $\delta$  are striking. Both harmonic anomalies (identified by  $n = 1$ , see Ref. 5) and subharmonic anomalies (identified by nonintegral values of n, see Ref. 6) are observed in Y and  $\delta$ , as



FIG. 3. Y,  $\delta$ , and  $dV/dl$  as functions of dc bias. (a) TaS<sub>3</sub> with  $\omega_{ac}/2\pi = 1$  MHz,  $E_{ac}/E_T = 1.3$ . (b) NbSe<sub>3</sub> with  $\omega_{ac}/$  $2\pi = 2$  MHz,  $E_{\text{a}}/E_T = 3.8$ . The arrows identify interference structure for  $TaS_3$ .

well as in  $dV/dl$ . Measurements on other TaS<sub>3</sub> and  $NbSe<sub>3</sub>$  crystals indicate that there exists a direct correlation between the "strength" of the Shapiro-step electronic interference (i.e., the height of the peak in  $dV/dl$ , during interference, relative to the low-field resistance) and the size of the anomalies in Y and  $\delta$ during interference. For "complete" electronic mode locking,<sup>7</sup> we expect Y and  $\delta$  to attain exactly their zero field,  $E_{dc} = E_{ac} = 0$ , values.

Figures  $3(a)$  and  $3(b)$  clearly demonstrate that the detailed behavior of Y and  $\delta$  in the depinned CDW state does *not* depend only on the average CDW drift velocity. If that were the case, no anomalies (other than at best slight "plateau" structure) would be observed in  $Y$  and  $\delta$  during Shapiro-step interference. Hence, ac-dc interference corresponds to more than simple electronic locking of the CDW drift velocity; there result profound changes in the elastic properties of the CDW crystal as well. As we discuss below, this appears to be related to a loss of internal degrees of freedom for the CDW condensate during interference, as has been suggested to account for unusual broadband-noise suppression on mode-locked Shapiro steps.<sup>7</sup>

Although a variety of models<sup>1</sup> have been advanced to account for the unusual properties of CDW conductors, most have been applied only to the electronic response. A difficulty in applying such models to the elastic response is that they in general treat the impurity pinning potential as immobile, and hence exclude the possibility of elastic-mode interactions between the underlying lattice and the dynamics of the CDW condensate. No model of CDW elasticity has been successful in accounting for the observed changes in Y and  $\delta$  due to CDW depinning. Coppersmith<sup>4</sup> has considered the situation of a rigid CDW sliding through a deformable lattice. Although an anisotropic shift in the velocity of sound is obtained, the predicted effects are orders of magnitude smaller than elastic changes determined (at low frequency) by experiment.<sup>2,3</sup> This indicates that the CDW must be treated as deformable. Brill and Roark<sup>2</sup> suggest that the changes in Y and  $\delta$ , due to CDW depinning, may reflect CDW-domainwall relaxation, with the relaxation time decreasing with increasing CDW current. On the other hand, the large magnitude of the observed changes in  $\delta$  would appear to rule out simple relaxation processes. Mozurkewich, Chaikin, Clark, and  $Gruner<sup>3</sup>$  have proposed that when the CDW is pinned to the lattice, its stiffness will add to the crystal stiffness, while a depinned CDW will not contribute to crystal stiffness. This approach yields an (order of magnitude) estimate that, changes in  $Y$  upon depinning should be comparable to CDW stiffness (computed, for example, from the Fukuyama-Lee model<sup>9</sup>), in rough agreement with experiment.<sup>2, 3</sup>

Our experimental results appear to have a simple physical interpretation which points to the importance of inclusion of internal degrees of freedom in the description of elastic properties of CDW materials. Figures 1 and 2 demonstrate clearly that lattice softening in  $TaS_3$  is not solely a result of CDW depinning and increasing CDW drift velocity. The strong  $E_{ac}$ dependence on Y for a pinned CDW shows that an ac field tends to decouple the CDW from the lattice. If an additionally applied dc field  $E_{dc}$  is increased past  $E'_T$ , the CDW slides, adding further to the decoupling process. Naively, one might therefore expect that, on a mode-locked Shapiro step (where the CDW velocity is determined strictly by the external ac field frequency), the CDW dynamics would fully *decouple* from the underlying lattice, thereby allowing Y and  $\delta$  during mode lock to attain their high-field (saturated) values. As previously discussed and demonstrated by Figs.  $3(a)$  and  $3(b)$ , however, exactly the *opposite* effect occurs in  $TaS_3$  and  $NbSe_3$ , i.e., on a model-locked step, the CDW couples *more* strongly to the lattice, and  $Y$ and  $\delta$  tend to their low-field values. This effect implies that it is in fact CDW incoherence which is responsible for the dramatic changes in  $Y$  and  $\delta$  due to CDW depinning.

On a mode-locked step, the electronic CDW drift velocity is fixed by the frequency  $\omega$  of the external ac electric field, with  $\omega >> \omega_r$ . The anomalies in Y and  $\delta$ indicate that the mode-locked CDW inhibits distortion of the lattice, presumably through a frictional coupling between the CDW and lattice. The connection between CDW electronic mode locking and elastic anomalies is well illustrated by a simple extension of the Grüner-Zawadowski-Chaikin  $(GZC)$  model<sup>10</sup> of CDW transport, where the CDW is represented by a rigid particle in a periodic potential. The GZC equation of motion for the CDW is

$$
m d2r/dt2 + \gamma dr/dt + QV \sin(Qx) = e [Edc + Eac \cos(\omegaac t)],
$$
\n(1)

where m is the CDW mass,  $\gamma$  the CDW damping constant,  $Q = 2\pi/\lambda$  with  $\lambda$  the CDW wavelength, and V the strength of the impurity pinning potential. Equation (1) describes well the electronic Shapiro-step interference effects,<sup>5</sup> but cannot describe any elastic properties, as the lattice and CDW are here considered fully rigid. Allowing for CDW internal degrees of freedom and lattice elasticity, Eq. (1) may be extended to a simple set of coupled equations for the CDW and the lattice,

$$
m^*d^2r/dt^2 + \gamma_C d(r - x)/dt + k_C r + eE_T \sin[2k_F(r - x)] = e[E_{dc} + E_{ac} \cos(\omega_{ac}t)],
$$
\n(2)

$$
M_L d^2x/dt^2 + \Gamma_L dx/dt + \gamma_C d(x - r)/dt + K_L x + eE_T \sin[2k_F(x - r)] = F \cos(\omega_r t),
$$
\n(3)

where  $r$  and  $x$  are respectively the positions of the CDW center of mass and lattice.  $m^*$  is the total CDW effective mass,  $M_L$  the lattice mass,  $\gamma_C$  and  $\Gamma_L$  respectively the total CDW damping and internal lattice friction, and  $k_F$  is the Fermi wave vector.  $k_C$  and  $K_L$ parametrize respectively the total elasticity of the CDW and underlying lattice, and  $F\cos(\omega_r t)$  is the mechanical force applied to the lattice which drives the mechanical force applied to the lattice which drives the resonance in the displacement x at frequency  $\omega_r$ .<sup>11</sup> In the limit of infinite lattice mass and a loss of internal degrees of freedom, i.e.,  $M_L \rightarrow \infty$ ,  $k_C$ ,  $K_L \rightarrow 0$ , Eqs. (2) and (3) reduce directly to Eq. (1), the rigidparticle GZC model.

In the framework of Eqs. (2) and (3), the behaviors of the elastic constants have a simple interpretation. For example, with  $E_{ac} = 0$  and finite  $E_{dc} < E_T$ , both  $k_c$ and  $K_L$  contribute to Y (there being an approximate constraint  $r \approx x$ ) and hence Y is large. For  $E_{dc} >> E_T$ , only  $K_L$  contributes, and hence Y is small. Similarly, during mode locking (finite  $E_{ac}$  and  $E_{dc}$ ),  $\langle \dot{r} - \dot{x} \rangle$  is a constant, and hence  $k_c$  and  $K_L$  again contribute and Y is large. Indeed, detailed computer solutions<sup>12</sup> of Eqs. (2) and (3) show behavior for  $dV/dl$ , Y, and  $\delta$  in striking qualitative agreement with the experimental results reported here for  $TaS_3$  and  $NbSe_3$ , both for the cases of individually applied ac and dc electric fields, and combined ac and dc fields (mode locking). This supports further our conclusion that CDW incoherence is central to the behavior of CDW elastic properties, and that the unusual elastic anomalies associated with CDW depinning cannot be treated in any model in which  $\langle \dot{r} \rangle$ , the CDW drift velocity, alone dictates Y and  $\delta$ .

Finally, we note that the strong coupling here implied between elastic and electronic response properties, and internal degrees of freedom, may have ramifications on the correct interpretation of Shapiro-step electronic interference in CDW conductors.

We thank Dr. S. Coppersmith for stimulating discus-

sions. This research was supported in part by National Science Foundation Grant No. DMR 84-00041, and an IBM Faculty Development Award (A.Z.). One of us (A.Z.) also received funding from the Alfred P. Sloan Foundation. L.B. acknowledges support from a National Science Foundation Fellowship, and M.S. acknowledges support from an AT&T Bell Laboratories Ph.D. Scholarship.

<sup>1</sup>For a review, see G. Grüner and A. Zettl, Phys. Rep. 119, 117 (1985),

2J. %. Brill and %. Roark, Phys. Rev. Lett. 53, 846 (1984).

36, Mozurkewich, P. M. Chaikin, W. G. Clark, and G. Grüner, in Charge Density Waves in Solids, edited by Gy. Hutiray and J. Solyom (Springer, New York, 1985) p. 353.

4S. N. Coppersmith, Ref. 3, p. 206; see also S. N. Coppersmith and C. M. Varma, Phys. Rev. 8 30, 3566 (1984); Y. Nakane and S. Takada, J. Phys. Soc. Jpn. 54, 977 (1985).

5A. Zettl and G. Grüner, Phys. Rev. B 29, 755 (1984).

 ${}^{6}R$ . P. Hall and A. Zettl, Phys. Rev. B 30, 2279 (1984); S. E. Brown, G. Mozurkewich, and G. Grüner, Solid State

Commun. 54, 23 (1985).  $7M.$  S. Sherwin and A. Zettl, Phys. Rev. B 32, 5536

(1985).

SL. Bourne and A. Zettl, unpublished.

<sup>9</sup>H. Fukuyama and P. A. Lee, Phys. Rev. B 17, 535 (1978).

<sup>10</sup>G. Grüner, A. Zawadowski, and P. M. Chaikin, Phys. Rev. Lett. 46, 511 (1981).

 $11$ We here regard Eqs. (2) and (3) as phenomenological extensions of Eq. (1), where the CDW and lattice stiffness have been "lumped" into single parameters  $k_c$  and  $k_l$ , respectively. On a firmer theoretical footing, Eqs. (2) and (3) may be derived from a sine-Gordon model with discretized CDW and lattice units (M. S. Sherwin and A. Zettl, unpublished). Also note that, in order for the CDW to slide continuously through the lattice and to preserve the periodicity of the pinning potential seen by the CDW,  $k<sub>C</sub>$  in Eq. (2) is restricted to respond only to ac excitations.

 $12$ Sherwin and Zettl, Ref. 11.