Inhibition of Atomic Phase Decays by Squeezed Light: A Direct Effect of Squeezing

C. W. Gardiner
Physics Department, University of Waikato, Hamilton, New Zealand
(Received 22 November 1985)

By reduction of the electromagnetic field fluctuations in one quadrature phase, squeezed light can inhibit the phase decay of an atom. This gives three relaxation times: the usual longitudinal relaxation time and *two* different transverse relaxation times, which are inversely proportional to the variances of the two quadrature phases of the incident light. With sufficient reduction of one variance, the corresponding relaxation time can be made arbitrarily long. The two transverse decay times are observable in the spectrum of the fluorescent light, thus providing measure of the squeezing in the incident light.

PACS numbers: 42.50.Dv

The fundamental property of squeezed light is that of reduced quantum fluctuations in one quadrature phase. The early works of Yuen and Shapiro[1] and Caves,[2] in which squeezed states were first investigated, have been followed by much activity, which has been reviewed by Walls.[3] Squeezing has recently been observed by Slusher *et al.* in four-wave mixing[4] in confirmation of the predictions of Reid and Walls.[5] It is of interest to consider how physical effects of these reduced fluctuations might be detected, and in this paper we shall show that under the influence of squeezed light, atomic phase decays can be partially suppressed, and the consequent lengthened lifetime of a decaying atom can be observed in the spectrum of light fluorescing from such an atom.

One considers the effect of an incident *squeezed vacuum* on a two-level atom. Such a state can (in principle) be produced by a degenerate parametric amplifier, whose input is vacuum fluctuations. A detailed treatment of the output field of such a system has been carried out by Gardiner and Savage[6] and by Collett and Gardiner.[7]

Squeezing is in practice described by several parameters, and in a multimode situation, which we must use in the case of a traveling-wave input, one of the principal parameters is the bandwidth over which the light is squeezed. From Eq. (47) of Ref. 7 we can obtain the correlation functions of the output light from an ideal degenerate parametric amplifier. We assume that (a) the amplifier is in a single-ended cavity, (b) it operates around a frequency Ω, (c) the damping constant of the cavity is γ, (d) the amplification constant is a real, $\epsilon > 0$, and (e) the input is the vacuum fluctuations. Taking the Fourier transform of Eq. (47), we find that the correlation functions for $b(t)$ and $b^\dagger(t)$, the positive- and negative-frequency parts of the traveling-wave output field, evaluated at the location of the atom are stationary, and satisfy

$$\langle b(t)\rangle = \langle b^\dagger(t)\rangle = 0, \quad \langle b^\dagger(t)b(t')\rangle = [(\lambda^2-\mu^2)/8\mu\lambda](\lambda e^{-\mu|t-t'|} - \mu e^{-\lambda|t-t'|}),$$
$$\langle b(t)b(t')\rangle = [(\lambda^2-\mu^2)/8\mu\lambda](\lambda e^{-\mu|t-t'|} + \mu e^{-\lambda|t-t'|})e^{-i\Omega(t+t')}, \tag{1}$$

where

$$\lambda = \tfrac{1}{2}\gamma + \epsilon, \quad \mu = \tfrac{1}{2}\gamma - \epsilon. \tag{2}$$

If we consider the case of strong amplification and large damping, the exponentials can be replaced by delta functions, and we obtain

$$\langle b(t)\rangle = \langle b^\dagger(t)\rangle = 0, \quad \langle b^\dagger(t)b(t')\rangle = N\delta(t-t'), \quad \langle b(t)b^\dagger(t')\rangle = (N+1)\delta(t-t'),$$
$$\langle b(t)b(t')\rangle = Me^{-i\Omega(t+t')}\delta(t-t'), \quad \langle b^\dagger(t)b^\dagger(t')\rangle = M^* e^{i\Omega(t+t')}\delta(t-t'), \tag{3}$$

where

$$N = \tfrac{1}{4}(\lambda^2-\mu^2)(1/\mu^2 - 1/\lambda^2), \quad M = \tfrac{1}{4}(\lambda^2-\mu^2)(1/\mu^2 + 1/\lambda^2). \tag{4}$$

[The commutation relation $[b(t), b^\dagger(t')] = \delta(t-t')$ is implicit in (3); this is *not* exactly true, but is a reasonable approximation when only frequencies in a bandwidth about Ω which is small compared with Ω are considered. In order to avoid confusion, we emphasize that $b(t)$ is a *field* operator evaluated at the position of the atom. The Fourier components of $b(t)$ each correspond to a mode of the incoming field. See Refs. 6 and 7 for a detailed explanation.} This is what we have called squeezed white noise in Ref. 7. It will be valid to use (3) instead of (1) as long as the time constants of any system driven by the squeezed light are significantly longer than both $1/\lambda$ and $1/\mu$. In general, squeezed white noise does not require $M=M^*$, but a phase choise can always be made to fix M to be real and positive, and I shall do so in the remainder of this paper. By noting that $\langle [b(t)+\lambda b^\dagger(t)] \times [b^\dagger(t)+\lambda^* b(t)]\rangle$ is positive for all λ, one readily deduces that

$$|M|^2 \le N(N+1). \tag{5}$$

The equality is in fact achieved from the output of an ideal degenerate parametric amplifier, as can be deduced from Eq. (4), and indeed it is possible simultaneously to have $N \to \infty$. (How well these can be realized in practice is a matter of current investigation.)

If we choose M to be real and positive in this case, the quadrature phases are most conveniently chosen as

$$X(t)=[b^\dagger(t)e^{-i\Omega t}+b(t)e^{i\Omega t}]/2, \quad Y(t)=[b^\dagger(t)e^{-i\Omega t}-b(t)e^{i\Omega t}]/2i, \tag{6}$$

and the correlation functions are

$$\begin{aligned}\langle X(t)X(t')\rangle &= \tfrac{1}{2}(N+M+\tfrac{1}{2})\delta(t-t') \sim (N+\tfrac{1}{2})\delta(t-t') \text{ if } N\to\infty \text{ and } M=[N(N+1)]^{1/2},\\ \langle Y(t)Y(t')\rangle &= \tfrac{1}{2}(N+\tfrac{1}{2}-M)\delta(t-t') \sim 1/16N \text{ if } N\to\infty \text{ and } M=[N(N+1)]^{1/2}.\end{aligned} \tag{7}$$

Thus, in the limit considered in Eq. (7), we see that the fluctuations in the quadrature phase $Y(t)$ can be made as small as one pleases.

If one shines such light on a two-level atom whose transition frequency is Ω, the effect of the fluctuations on the Bloch vector will be felt only by that component of the Bloch vector in phase with $X(t)$; the component in phase with $Y(t)$ will feel fluctuations which can be arbitrarily small.

The decay of a two-level atom consists of two parts: polarization decay and inversion decay. The polarization decays as the result of field fluctuations gradually randomizing the phase of the Bloch vector, whereas the inversion decay is related to the radiation of photons into space.

If we squeeze the input light we can reduce the fluctuations in one quadrature phase, and that component of the polarization which is in phase with the low-noise quadrature phase will feel reduced fluctuations, and not decay as fast as the other, which will feel increased fluctuations.

The formalism developed by Gardiner and Collett[8] is well adapted to the treatment of this problem, since the properties (2) are those of quantum white noise, for which a formalism is developed in that paper. Let us consider a two-level system described by spin operators

$$S_z=\begin{pmatrix}1&0\\0&-1\end{pmatrix}, \quad S^+=\begin{pmatrix}0&1\\0&0\end{pmatrix}, \quad S^-=\begin{pmatrix}0&0\\1&0\end{pmatrix} \tag{8}$$

coupled to the electromagnetic field. The Hamiltonian is

$$\begin{aligned}H&=H_{\text{sys}}+H_{\text{em}}+H_{\text{int}},\\ H_{\text{em}}&=h\int_0^\infty d\omega\,\omega b^\dagger(\omega)b(\omega),\\ H_{\text{int}}&=ih\int_0^\infty K(\omega)[b^\dagger(\omega)S^- - b(\omega)S^+]d\omega,\\ H_{\text{sys}}&=\tfrac{1}{2}h\,\Omega S_z.\end{aligned} \tag{9}$$

In writing (9) I make the electric dipole approximation and the rotating-wave approximation. The dipole moment $K(\omega)$ is assumed smooth around $\omega=\Omega$, and Ω is sufficiently large that the lower limit of the integrals in (9) can be considered to be essentially $-\infty$. Collett and Gardiner derive a master equation for the atomic density operator ρ for the situation in which the incoming light is squeezed quantum white noise; this is (in a frame rotating at frequency Ω)

$$\begin{aligned}d\rho/dt=&\tfrac{1}{2}\gamma(N+1)(2S^-\rho S^+ - S^+S^-\rho-\rho S^+S^-)+\tfrac{1}{2}\gamma N(2S^+\rho S^- - S^-S^+\rho-\rho S^-S^+)\\ &-\tfrac{1}{2}\gamma M(2S^+\rho S^+ - S^+S^+\rho-\rho S^+S^+)-\tfrac{1}{2}\gamma M^*(2S^-\rho S^- - S^-S^-\rho-\rho S^-S^-).\end{aligned} \tag{10}$$

Here

$$\gamma=2\pi K(\Omega)^2. \tag{11}$$

This equation is valid provided the bandwidth of the squeezed white noise is substantially larger than γ. If we define

$$S^\pm=\tfrac{1}{2}(S_x\pm iS_y), \tag{12}$$

then we find equations of motion for the means of S_x, S_y, and S_z to be

$$\langle \dot{S}_x\rangle = -\gamma(N+M+\tfrac{1}{2})\langle S_x\rangle = -\gamma_x\langle S_x\rangle,$$
$$\langle \dot{S}_y\rangle = -\gamma(N-M+\tfrac{1}{2})\langle S_y\rangle = -\gamma_y\langle S_y\rangle, \quad (13)$$
$$\langle \dot{S}_z\rangle = -\gamma(2N+1)\langle S_z\rangle - \gamma = -\gamma_z\langle S_z\rangle - \gamma.$$

The quantities N and M can be varied independently in principle, though for the ideal squeezed input (such as is produced from a DPA), N and M are connected by the relation

$$M = [N(N+1)]^{1/2}.$$

We can see therefore that for large squeezing both γ_z and γ_x become large. In fact γ_x and γ_y are proportional to the coefficients of intensity of the respective quadrature phases of the input, as defined in Eq. (7). Since this noise can approach zero in the Y quadrature phase, it is clear that the decay constant γ_y can be as small as we please, while at the same time γ_x becomes extremely large. Thus in a time scale short compared with γ_y^{-1}, but substantially larger than γ_x^{-1} and γ_z^{-1}, we find

$$\langle S_x\rangle \to 0, \quad \langle S_z\rangle \to -1/(2N+1), \quad (14)$$

while $\langle S_y\rangle$ is essentially unchanged. Thus the projection of the original orientation of the Bloch vector on the direction of the low-noise quadrature phase is preserved—nevertheless the inversion $\langle S_z\rangle$ decays rapidly to its stationary value.

To detect this effect one can look at the spectrum of the fluorescent light. According to the quantum regression theorem, and standard methods connected with it,[9] the stationary correlation functions of the atom are

$$\begin{aligned}
\langle S^+(t)S^-(0)\rangle &= \tfrac{1}{2}[N/(2N+1)][\exp(-\gamma_x t)+\exp(-\gamma_y t)],\\
\langle S^-(t)S^+(0)\rangle &= \tfrac{1}{2}[(N+1)/(2N+1)][\exp(-\gamma_x t)+\exp(-\gamma_y t)],\\
\langle S^+(t)S^+(0)\rangle &= \tfrac{1}{2}[(N+1)/(2N+1)][\exp(-\gamma_x t)-\exp(-\gamma_y t)],\\
\langle S^-(t)S^-(0)\rangle &= \tfrac{1}{2}[N/(2N+1)][\exp(-\gamma_x t)-\exp(-\gamma_y t)].
\end{aligned} \quad (15)$$

The above has been formulated in the case that there is one input and one output channel. In the case of interaction of light with an atom, this is realistic if we view the channels as corresponding to the various partial waves, since only the electric dipole partial wave (corresponding to the electric dipole approximation) interacts appreciably with the atom. If the experiment can be carried out with a squeezed electric dipole wave, then the output would be of the same kind. We will not attempt a consideration of how this might be achieved in practice, though schemes involving parabolic mirrors come to mind immediately.

In this case the amplitude correlation function of the fluorescent light (whose Fourier transform is the spectrum) is computed from the boundary condition

$$b_{\rm out}(t) = \sqrt{\gamma}S^-(t) + b(t), \quad (16)$$

which is derived in Ref. 6. We find

$$\langle b^\dagger_{\rm out}(t)b_{\rm out}(0)\rangle = N\delta(t) + \gamma u(t)\langle[S^+(t),NS^-(0)-MS^+(0)]\rangle + \gamma u(-t)\langle[NS^+(t)-MS^-(t),S^-(0)]\rangle + \gamma\langle S^+(t)S^-(0)\rangle \quad (17)$$

[where $u(t)=0,\ t<0;\ u(t)=1,\ t>0$]. This correlation function contains elements arising from the input correlation function $[N\delta(t)]$ and elements from the atom itself—arising because the fluorescent light consists of a radiated part and a reflected part. With use of (11) and (13), the correlation function becomes

$$\langle b^\dagger_{\rm out}(t)b_{\rm out}(0)\rangle = N\delta(t) + \tfrac{1}{2}[\gamma M/(2N+1)][\exp(-\gamma_y t)-\exp(-\gamma_x t)], \quad (18)$$

which displays the characteristic decay constants.

The spectrum obtained by the Fourier transform of (18) consists of a flat background, from $\delta(t)$, plus a negative peak of width γ_x (which will be very broad), and finally a positive peak of width γ_y, which can be as narrow as we please. Such narrowing is an effect which can only be produced by some reduction of noise, i.e., squeezing. The narrowed width is in fact directly proportional to the squeezing, and thus provides a measurement of squeezing.

One might prefer to use plane waves as an input, but in fact most of the squeezing effect is then lost. For I characterize the state of the three-dimensional input field by operators $b_i(\mathbf{n},t)$ such that

$$\begin{aligned}
[b_i(\mathbf{n},t),b_j^\dagger(\mathbf{n}',t')] &= \hat{\delta}(\mathbf{n},\mathbf{n}')\delta(t-t')\delta_{ij},\\
\langle b_i^\dagger(\mathbf{n},t)b_j(\mathbf{n}',t')\rangle &= \overline{N}_l(\mathbf{n},\mathbf{n}')\delta(t-t')\delta_{il}\delta_{jl},\\
\langle b(\mathbf{n},t)b(\mathbf{n}',t')\rangle &= \overline{M}_l(\mathbf{n},\mathbf{n}')\delta(t-t')\delta_{il}\delta_{jl}.
\end{aligned} \quad (19)$$

Here $\mathbf{n}$ and $\mathbf{n}'$ are unit vectors, $\hat{\delta}(\mathbf{n},\mathbf{n}')$ is the delta function on the unit sphere, and the operators $b_i(\mathbf{n},t)$ are Fourier transforms with respect to the frequency of free-wave input destruction operators with a given direction of propagation $\mathbf{n}$. The indices i and j are polarization indices, and l is the particular polarization which is squeezed. The functions $\bar{N}_l$ and $\bar{M}_l$ characterize the squeezing, and are in principle arbitrary.

The annihilation operator corresponding to a square normalized wave function $f_l(\mathbf{n})$ is

$$A_l(t)=\int d^3n\, f_l^*(n) b_l(\mathbf{n},t). \tag{20}$$

The case in which only this direction is squeezed is given by

$$\bar{N}_l(\mathbf{n},\mathbf{n}')=f_l^*(\mathbf{n})f_l(\mathbf{n}')N_l,$$
$$\bar{M}_l(\mathbf{n},\mathbf{n}')=f_l^*(\mathbf{n})f_l(\mathbf{n}')M_l, \tag{21}$$

since in this case

$$\langle A_l^\dagger(t)A_l(t')\rangle=\delta(t-t')N_l,$$
$$\langle A_l(t)A_l(t')\rangle=\delta(t-t')M_l, \tag{22}$$

and similar averages corresponding to all orthogonal functions vanish.

The corresponding parameters to N_l and M_l in (22) which would arise from the operator $B(t)$ corresponding to a wave function $g(\mathbf{n})$ are obtained by multiplying N and M in (22) by $|\int d^3n\, f_l^*(\mathbf{n})g(\mathbf{n})|^2$.

This can be a considerable reduction. For example if $f_l(\mathbf{n})$ corresponds to an almost plane wave which includes only directions $\mathbf{n}$ confined to a solid angle Ω with equal amplitude, while $g(\mathbf{n})$ corresponds on electric dipole wave, i.e., one which is of order of magnitude $(4\pi)^{-1/2}$ on the unit sphere, then this overlap factor is of order of magnitude $\Omega/4\pi$ which is vanishingly small as $\Omega\to 0$, corresponding to a plane wave. In order, therefore, to utilize squeezing effectively, Ω must be as close to 4π as possible.

If the experiment is done with an incoming wave that is not a perfect electric dipole, then the effect of multiplying M and N by the factor $\epsilon=|\int dn\, f_l^*(n) \times g(n)|^2$ is to yield

$$\gamma_x\to\gamma[\epsilon(N+M+\tfrac{1}{2})+(1-\epsilon)\tfrac{1}{2}],$$
$$\gamma_y\to\gamma[\epsilon(N-M+\tfrac{1}{2})+(1-\epsilon)\tfrac{1}{2}]. \tag{23}$$

The term proportional to $1-\epsilon$ in both these expressions represents the input of vacuum fluctuations from those modes in the electric dipole wave other than the input mode. Unless ϵ is very close to 1, these can completely mask the effect. The production of a squeezed electric dipole incident wave will therefore be an important component of the experimental verification of this effect, unless it is found possible to set up a genuine one-dimensional experiment. This would involve the manufacture of an appropriate waveguide, at whose termination one would locate the two-level atom. This might be difficult at visible wavelengths, though it might be possible at the wavelengths corresponding to transitions in Rydberg atoms.

In practice the light will not be perfectly delta correlated, but the analysis will be valid provided the input squeezing bandwidth is larger than γ_x, γ_y, and γ_z. This can be achieved in principle in four-wave mixing.[10] It will therefore be relevant to find ways of computing the effects of finite squeezing bandwidth.

I would like to thank Craig Savage for conversations which led me to consider this problem, and Dan Walls for his enlightment on certain points.

[1]H. P. Yuen, Phys. Rev. A **13**, 2226 (1976); H. P. Yuen and J. H. Shapiro, IEEE Trans. Inf. Theory **24**, 657 (1978), and **26**, 78 (1980).

[2]C. M. Caves, Phys. Rev. D **26**, 1817 (1982).

[3]D. F. Walls, Nature (London) **306**, 141 (1983).

[4]R. E. Slusher, L. W. Hollberg, B. Yurke, J. C. Mertz, and J. F. Valley, to be published.

[5]M. D. Reid and D. F. Walls, Phys. Rev. A **31**, 1622 (1985).

[6]C. W. Gardiner and C. M. Savage, Opt. Commun. **50**, 173 (1984).

[7]M. J. Collett and C. W. Gardiner, Phys. Rev. A **30**, 1386 (1984).

[8]C. W. Gardiner and M. J. Collett, Phys. Rev. A **31**, 3761 (1985).

[9]C. W. Gardiner, *Handbook of Stochastic Methods* (Springer, Berlin, 1983), Chap. 10.

[10]D. F. Walls and A. S. Lane, private communication.