Ultrasonic Attenuation in Clean Anisotropic Superconductors

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We calculate the longitudinal ultrasonic attenuation α of clean anisotropic superconductors. The temperature dependence of α depends not only on the density of states and the sound orientation, but also on $R = T_c/2T_F$, where T_c and T_F are the transition and Fermi temperatures. Peaks in $\alpha(T)$ can occur for certain orientations if $R \neq 0$. The asymptotic low-temperature regime is only reached for $T \ll RT_c$.

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Heavy-electron superconductors have excited substantial interest,[1] partly because it has been suggested that the electron pairing is of odd parity.[2] Direct probing of the pairing parity is extremely difficult,[3] but thermodynamic and transport measurements can help determine whether the gap is anisotropic. As $T \to 0$, only low-energy quasiparticles are excited and the zeros of the gap are probed. If no zeros occur, the quasiparticles are exponentially suppressed at low temperatures.

The low-temperature specific heat (if it is dominated by the electronic component) is proportional to an integral over the quasiparticle density of states. Recent experiments[4] on UPt_3 indicate that at low temperatures, $C \sim AT + BT^2$, which might suggest that the system is in a gapless polar state,[5] though other interpretations are possible.[4] Similar results and interpretations for the NMR $1/T_1$ data in UBe_{13} have been presented.[6] The thermal conductivity in UPt_3 has also been measured, but a simple interpretation consistent with the other two measurements mentioned does not appear obvious.[4]

Bishop *et al.*[7] have found that the longitudinal ultrasonic attenuation α in UPt_3 is proportional to T^2 at low temperatures T down to ~ 100 mK ($T \sim 0.2T_c$). They interpret this result as evidence for a gap with a line of zeros, or a polarlike state. However, Rodriguez[8] claims that $\alpha \sim T^2$ corresponds to an axial-like state, with the gap vanishing at points in k space. The nonexponential temperature dependence probably indicates that the gap is anisotropic, but this disagreement over whether the gap nodes are points or lines limits the information available. Since odd-parity pairing may be inconsistent with lines of nodes of the gap,[9] resolution of this question can provide a vital clue to the nature of the pairing. In addition, $\alpha(T)$ just below T_c is qualitatively different from Bardeen-Cooper-Schrieffer (BCS) behavior.[7,10] Therefore, it is of interest to calculate carefully the ultrasonic attenuation for anisotropic superconductors.

Here we calculate $\alpha(T)$ for anisotropic superconductors in the clean limit $ql \gg 1$, where q is the sound wavelength and l is the quasiparticle mean free path, in contrast to the calculations in Refs. 7 and 8. The two regimes are experimentally distinguishable since $\alpha \sim \omega$ when $ql \gg 1$ and $\alpha \sim \omega^2$ for $ql \ll 1$.[11] We consider the clean limit for its simplicity and for its possible experimental relevance (as described below).[12]

Examining the clean (rather than hydrodynamic) limit substantially simplifies the calculation of α, but the extremely large quasiparticle effective mass m^* in heavy-electron systems significantly affects the quasiparticle energy dispersion with an anisotropic gap, causing $\alpha(T)$ to have novel features. Also, the large m^* corresponds to a sufficiently small Fermi velocity v_F that it may be of the same order of magnitude as the speed of sound v_s. However, this latter complication does not qualitatively change the results.

Our major conclusions for the clean limit are as follows:

(1) The attenuation is strongly anisotropic at all temperatures.

(2) As $T \to 0$ a polarlike state corresponds to an $\alpha(T) \sim T$ while a T^2 dependence results from an axial-like state.

(3) The attenuation depends sensitively on $R \equiv m^* T_c/k_F^2 \sim T_c/2T_F$, where k_F and T_F are the Fermi wave vector and temperature, respectively. This ratio measures the importance of gap variation relative to kinetic energy changes. The asymptotic low-temperature regime is not reached until $T \ll RT_c$, and since we expect $R \sim 0.1$,[1] the asymptotic regime has not been experimentally attained. Explicit numerical calculations show that both axial and polar states may exhibit an apparent T^n behavior with $n \geqslant 2$ over a large temperature range for certain orientations and R values.

(4) For $R \neq 0$, a peak in $\alpha(T)$ appears for certain orientations of the sound wave with respect to the gap. This peak becomes more pronounced as R increases, and is more prominent in the simple polar state than in the simple axial state.

We assume that the normal quasiparticle energy at wave vector k is $\epsilon_k = k^2/2m^* - E_F$ (with E_F the Fermi energy), where the effective mass m^* is assumed to be isotropic and temperature independent below T_c. In the superconducting state, the quasiparticles have energy $E_k = \{\epsilon_k^2 + |\Delta_k|^2\}^{1/2}$, where Δ_k is the appropriate order parameter for the state in question. Although many different states are possible for anisotropic pair states in a crystal, we only consider the p-wave polar and axial (Anderson-Brinkman-Morel) states, in which $|\Delta_k| = \Delta(T)|\cos\theta_k|$ and $\Delta(T)\sin\theta_k$, respectively. (The angle between $\mathbf{k}$ and the fixed $\hat{\mathbf{z}}$ axis is θ_k. The direction of $\hat{\mathbf{z}}$ is arbitrary here, but it is actually chosen by the crystal.) We have used the weak-coupling expressions for the temperature dependence of Δ and have ignored spin-orbit coupling, strong-coupling effects, and Fermi-liquid corrections that may be substantial in these materials. Although these approximations are severe, we expect the asymptotic low-temperature behavior to reflect only the density of states. Near T_c, improving our approximations may quantitatively change the results, but the qualitative features should remain.

We have also neglected the effects of pair breaking and of collective excitations of the order parameters that may couple to the sound wave. Although pair-breaking effects are at least as strong as in a BCS superconductor, they vanish as $q \to 0$. Collective modes are only important very near T_c.[13] Although other contributions to α_s are possible, we consider here only the electronic contribution. Since for UBe_{13} and UPt_3 the electronic contribution to the thermal conductivity was shown to be more important than the phononic contribution,[4] we assume the same to be true of the ultrasonic attenuation. These mechanisms are expected to have very different magnetic-field dependences. In addition, a phononic mechanism is not expected to exhibit substantial α_s anisotropy, contrary to these results.

Our starting point is Eq. (64) of Balian and Werthamer,[14] in which α_s for a pure isotropic p-wave state is dominated by the sound-wave scattering off the quasiparticles. The longitudinal α_s for a sound wave of wave vector $\mathbf{q}$ and frequency $\omega_q = v_s|q|$ obeys, as $q \to 0$,

$$\alpha_s \propto \omega_q^2 \sum_k (\epsilon_k/E_k)^2 \frac{\partial f}{\partial E_k} \delta(E_{k+q} - E_k - \hbar\omega_q), \tag{1}$$

where ϵ_k and E_k are defined above, and $f(E)$ is the Fermi function. Equation (1) can be shown to be valid for the polar and axial states as well, by assumption of the appropriate form of the p-wave pairing interaction. In this clean limit, conservation of momentum $(\mathbf{k}' = \mathbf{k} + \mathbf{q})$ and conservation of energy $(E_k = E_{k'} - \hbar\omega_q)$ place severe restrictions on the allowed values of momentum $\mathbf{k}$ for the quasiparticles that are scattered by the sound wave. In the "light fermion" limit $v_s/v_F \to 0$, this implies $\mathbf{q}\cdot\nabla_k E_k = 0$. If $R \to 0$, then states with $\mathbf{k} \perp \mathbf{q}$ are sampled.[15]. For nonvanishing R, the quasiparticle energies depend on the direction of $\mathbf{k}$, yielding different constant-energy surfaces. We note that particle-hole symmetry (valid when $R = 0$) no longer holds. There are substantial flat regions of the quasiparticle spectrum over which one must integrate, leading to enhanced attenuation, especially for certain directions.

To see this and other features explicitly, we evaluate the azimuthal integral in Eq. (1), and find

$$\alpha \propto \omega_q \sum_{\substack{s=\pm 1\\ \sigma=\pm 1}} \int_0^\infty d\tilde{\epsilon} \int_{-1}^1 dx (df/d\tilde{E})(\tilde{\epsilon}^2/\tilde{E}) \Theta(g_s) g_s^{-1/2}(\tilde{\epsilon}, x), \tag{2}$$

where $x = \cos\theta_k$, $\tilde{\epsilon} = \epsilon/T$, $\tilde{E} = E/T$, $\Theta(z)$ is the Heaviside step function, and $g_s(\tilde{\epsilon}, x)$ is given by

$$g_s(\tilde{\epsilon},x) = \tilde{\epsilon}^2(\sin^2\theta_q - x^2) + 2s\tilde{\epsilon}[r\sigma x\tilde{E}\cos\theta_q - x^2(1-x^2)B] + B^2x^2(1-x^2)(x^2 - \cos^2\theta_q) - r^2\tilde{E}^2 + 2\sigma\tilde{E}xrB(1-x^2)\cos\theta_q, \tag{3}$$

where $\cos\theta_q = \hat{\mathbf{q}}\cdot\hat{\mathbf{z}}$, $B = \Delta^2(T)R/TT_c$, $r = v_s/v_F$, $E^2 = \epsilon^2 + \Delta^2(x,T)$, and $\Delta^2(x,T) = x^2\Delta^2(T)$ for the polar state and $(1-x^2)\Delta^2(T)$ for the axial state. The polar and axial states differ only in the x dependence of $\tilde{E}$. The relevant parameters are r, R, θ_q, and T.

We first evaluate $\Delta(T)$ using a weak-coupling approximation analogous to that used in BCS theory:

$$1 = \lambda N(0) \int_{-1}^1 dx \frac{\Delta^2(x,T)}{\Delta^2(T)} \int_0^{\omega_0} d\epsilon \frac{\tanh(E/2T)}{E}. \tag{4}$$

The cutoff parameter ω_0 only appears in the expression for T_c, which is found in the usual way by our setting $\Delta(T_c) = 0$ in Eq. (4). The parameters λ and $N(0)$ are the appropriate p-wave pairing interaction strength and the normal-state single-spin particle density of states, respectively. The temperature dependence of $\Delta(T)$, found from Eq. (4), is plotted in the inset of Fig. 1. At $T = 0$, $\Delta(0) = PT_c$, where $P = 2.461$ for the polar, 2.029 for the axial, and 1.764 for the BCS states, respectively. The weak-coupling approximation is questionable, but one expects Δ to be roughly T in-

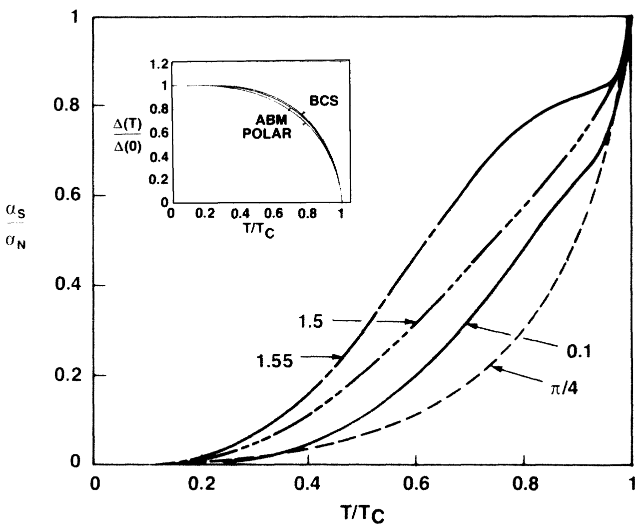

FIG. 1. Plot of α_s/α_n vs T/T_c for the axial state with $R=0.5$, for various values of θ_q. Inset: Plot of $\Delta(T)/\Delta(0)$ vs the reduced temperature T/T_c for the polar, axial, and BCS states.

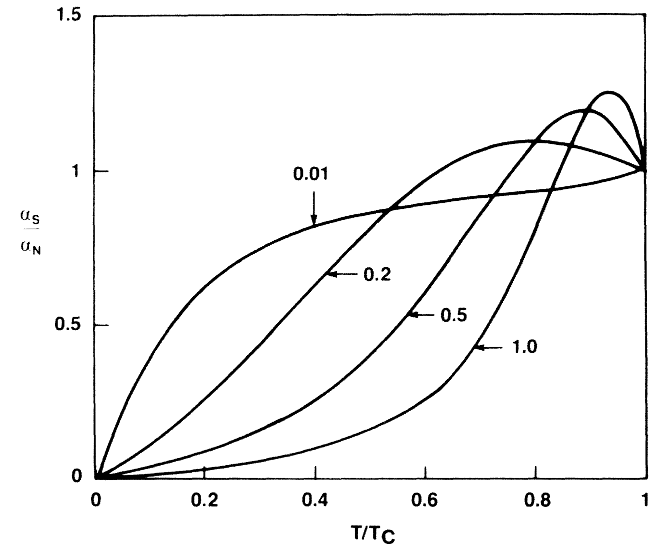

FIG. 2. Plot of α_s/α_n vs T/T_c for the polar state at $\theta_q=0.1$, for different values of R.

dependent as $T\rightarrow 0$ and to decrease sharply as $T\rightarrow T_c$.

As $T\rightarrow 0$, the attenuation α obeys $\alpha_s/\alpha_n = A_p(T/T_c)$ and $A_a(T/T_c)^2$ for the polar and axial states, respectively. The parameters A_p and A_a can be found for arbitrary v_s, but their dependence on this parameter is not very important. As $v_s\rightarrow 0$,

$$A_p = 2\pi f_1(\theta_q)E(f_1(\theta_q))/3P\sin\theta_q, \tag{5}$$

where $f_1(z)=(1+R^2\cot^2 z)^{-1/2}$ and $E(z)$ is the standard elliptic integral, and

$$A_a = \pi^2 f_2^3(\theta_q)/6RP^2\sin\theta_q, \tag{6}$$

with $f_2(z)=(1+R^{-2}\cot^2 z)^{-1/2}$. A_p diverges as $\theta_q\rightarrow 0$, since for $r\rightarrow 0$, $\alpha_s/\alpha_n=1$ at this angle for the polar state. For the axial state, $A_a\rightarrow 0$ as $\theta_q\rightarrow 0$ or π, resulting in $\alpha_s \propto T^4$ at those points. For $\theta_q\rightarrow \pi/2$, A_a diverges, as α is linear in temperature for this special angle. When a finite angular resolution is taken into account and $R\neq 0$, $\alpha\propto T^2$ for the axial state and $\alpha\propto T$ for the polar state in any orientation. We may also integrate Eq. (2) analytically for the polar state with $\theta_q=0$ and $v_s=0$. We find a logarithmic divergence which is removed when v_s is finite.

The attenuation for intermediate temperatures was calculated numerically. Figures 1–3 indicate that the asymptotic low-temperature limit is not reached until $T \ll T_c^2/T_{\mathrm{F}}$, and so an apparent T^2 (or T^n for $n\geqslant 2$) variation over a broad temperature range can occur for both axial and polar states. Also, the additional contributions caused by the gap variation induce peaks in the attenuation for certain values of θ_q and R. For the axial state, peaks only occur when $R>0.5$ very near $\theta_q=\pi/2$, but for the polar state a peak occurs close to $\theta_q=0$ for any finite R. This result is reasonable, since the gap varies rapidly only near the poles for an axial state, while it changes quickly in the entire equatorial region for the polar state. Increasing R makes the peaks more prominent. For the polar state, at a fixed $\theta_q \ll 1$, as R increases, the peak moves toward T_c, becomes sharper, and persists for larger values of θ_q. For fixed R, the peak is most prominent near $\theta=0$, though a peak also occurs near $\theta=\pi/2$ when $R>0.3$. However, gaps in real materials may be more complicated than the simple forms used here, and the presence of a peak does not necessarily imply that the state is purely polar. We have also calculated the attenuation when $r=v_s/v_{\mathrm{F}}$ is nonzero; up to $r\sim 0.1$, the curves are basically unaffected.

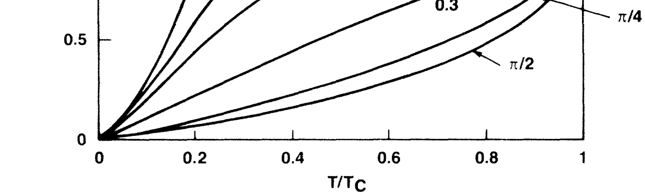

FIG. 3. Plot of α_s/α_n vs T/T_c for the polar state with $R=0.1$ for various values of θ_q.

In conclusion, we have shown that low-temperature ultrasonic attenuation measurements may not provide unambiguous information on the density of states of an anisotropic superconductor because variations of the gap lead to a new temperature scale that is much less than T_c. The attenuation can exhibit structure near T_c if the gap variation is of the same order of magnitude as the kinetic energy change when the momentum of a quasiparticle is altered. The effect is amplified not only when the gap increases in magnitude but also when its angular variation is large. The peaks are enhanced when substantial changes in the gap occur over large regions of reciprocal space.

Impurity scattering modifies our results substantially, but the clean limit may be experimentally relevant at low temperatures if the elastic scattering dominates and its rate is roughly proportional to the quasiparticle density of states, which becomes small at low energies and hence low temperatures.[12] In UPt_3, $ql \cong 0.1$ at T_c, and so at low temperatures the clean limit could conceivably apply. More importantly, the clean limit may be attained by an increase in the sound frequency or by use of very clean samples, allowing experimental test of our calculations over the entire temperature range.

Peaks in the longitudinal ultrasonic attention of UBe_{13} and UPt_3 have been observed,[10,16] but experimentally $ql \ll 1$ at T_c. Measurements at much higher frequencies would reveal whether the effects discussed here apply to the experimental situation near T_c. It would be interesting to know if impurity scattering would completely eliminate these peaks and shoulders, or if some remnants of them might persist. Detailed calculations of the role of impurity scattering are presently underway.[17]

Our main point is that an essential new parameter $R \sim T_c/2T_F$ enters into the calculation of the longitudinal ultrasonic attenuation for materials with highly anisotropic gaps. For heavy-fermion materials $R \sim 0.1$, and cannot be neglected. Even in the presence of scattering, we expect R to alter substantially the ultrasonic attenuation. An important consequence of $R \neq 0$ is that the asymptotic regime $T \ll RT_c$ has not yet been attained experimentally. Therefore, a definitive conclusion about the nodal structure of the gap is extremely difficult to make on the basis of existing data.

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