Rayleigh Scattering and Weak Localization

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A radiative-transfer equation describing the propagation of light in a disordered medium is presented. The equation includes the effects of weak localization and polarization of the light. The polarization of the light in coherent backscattering from a disordered medium is discussed. It is found that for circularly or elliptically polarized light the interference in the backward direction is partially destructive. For linearly polarized light the depolarized scattered component does not contain coherent backscattering, in qualitative accord with observations.

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Recently the coherent backscattering of light by a disordered medium has been observed by Van Albada and Lagendijk[1] and Wolf and Maret.[2] This effect has been discussed theoretically by a number of authors: Watson,[3] Golubentsev,[4] Anderson,[5] Akkermans and Maynard,[6] and others. Akkermans and Maynard have given a particularly clear discussion in terms of the theory of Anderson localization.[7] Previous work on the localization of classical waves concentrated on the behavior of the diffusion constant.[8,9] Experiments with light enable us to explore in more detail the multiple scattering due to the disordered medium. The theoretical discussions up to now have mainly dealt with scalar waves. In the experiments interesting polarization effects are observed and it is the object of this Letter to discuss the effects of polarization on coherent backscattering from a disordered medium. It should be noted that the term "coherent" is used here because the scattered waves in the backward direction have phases which are related. It does not imply that the scattered wave is coherent with the incident wave.

We suppose that the disordered medium can be described by a dielectric constant $1+\epsilon'(r)$ where ϵ' is a random variable with zero mean and correlation function $\langle\epsilon'(r_1)\epsilon'(r_2)\rangle=\Delta\delta(r_1-r_2)$. The extension to the case where the particles responsible for the scattering are anisotropic is given below. The electric field $\mathbf{E}$ of the light satisfies the wave equation

$$\{\nabla^2+k^2[1+\epsilon'(r)]\}\mathbf{E}(r)=0,\quad \nabla\cdot\mathbf{E}=0. \tag{1}$$

We have omitted a term $\nabla[\mathbf{E}\cdot\nabla\ln(1+\epsilon')]$ which is small if the variation of the disorder is slow compared with the wavelength $2\pi/k$ of the light. The scattering is purely elastic. A convenient way to approach the problem is to derive a transport equation from (1). This has been discussed by Watson,[3] Tatarski,[10] and others. The transport equation is of the form of the radiative transfer equation.[11] The radiation field at a point $\mathbf{R}$ can be characterized by the four intensities $J_{ij}(\mathbf{R},\mathbf{s})$ $(i,j=1,2)$, where $\mathbf{s}$ is a unit vector giving the direction of propagation of the light which together with the directions 1 and 2 form an orthogonal set. The J_{ij} are related to the usual Stokes parameters[11] by

$$J_{11}=I_l,\quad J_{22}=I_r,\quad J_{12}=J_{21}^*=\tfrac{1}{2}(U-iV). \tag{2}$$

The radiative transfer equation satisfied by J is

$$(\mathbf{s}\cdot\nabla_R+\alpha)J_{ij}(\mathbf{R},\mathbf{s}) =\int\sigma_{ijmn}(\mathbf{s},\mathbf{s}')J_{mn}(R,\mathbf{s}')\,d^3s'. \tag{3}$$

The scattering matrix, including weak localization effects, i.e., sum of maximally crossed diagrams,[12] is

$$\sigma_{ijmn}(\mathbf{s},\mathbf{s}') =\sigma_0[\delta_{im}(\mathbf{s})\delta_{jn}(\mathbf{s})+\delta_{in}(\mathbf{s})\delta_{jm}(\mathbf{s})f(\theta)], \tag{4}$$

where $\delta_{im}(\mathbf{s})=\delta_{im}-s_is_m$. The first term is the well-known Rayleigh scattering with $\sigma_0=\pi\Delta k^{d+1}/2(2\pi)^d$ and the second term arises from weak localization effects, and

$$f(\theta)=\frac{d\beta^2}{[2(1+\cos\theta)]^{1/2}\{\beta+[2(1+\cos\theta)]^{1/2}\}}, \tag{5}$$

where d is the dimensionality, θ is the angle between $\mathbf{s}$ and $\mathbf{s}'$, and $\beta=\lambda/2\pi l$, where l is the mean free path. $f(\theta)$ is sharply peaked in the backward direction, where $|\pi-\theta|\sim\lambda/l$ and gives rise to the coherent backscattering. If Eq. (3) is solved to first order in $f(\theta)$ then (3) gives results equivalent to those of weak-localization theory, i.e., the sum of ladder graphs and maximally crossed graphs is correctly included. These same graphs are taken into account in the calculation of the electrical conductivity[13] and diffusion constant.[8,9] Conservation of energy requires that $\alpha=\int d^3s\sigma_0[1+f(\theta)]$. This diverges in two dimensions and a cutoff is necessary. We consider $d=3$.

The form of the scattering in (3) is simply illustrated by considering a light beam being scattered from $\mathbf{s}'$ to $\mathbf{s}$ by a small piece of material (see Fig. 1). The scattering is described by

$$J_{ij}(\mathbf{s})=\sigma_{ijmn}(\mathbf{s},\mathbf{s}')J'_{mn}(\mathbf{s}'). \tag{6}$$

The incident and scattered light are described by Stokes parameters $I'_{\parallel}$, $I'_{\perp}$, U', V' and $I_{\parallel}$, $I_{\perp}$, U, V, respectively, and from (6) (in $d=3$)

$$I_{\parallel}=\sigma_0[1+f(\theta)]\cos^2\theta I', \quad I_{\perp}=\sigma_0[1+f(\theta)]I',$$
$$U=\sigma_0[1+f(\theta)]\cos\theta\, U', \tag{7}$$
$$V=\sigma_0[1-f(\theta)]\cos\theta\, V'.$$

In these results the leading terms are exactly those for Rayleigh scattering. The backscattering for the first three parameters interferes constructively while for V it is destructive. V is nonzero if the incident beam is circularly or elliptically polarized. It would be interesting to observe this effect.

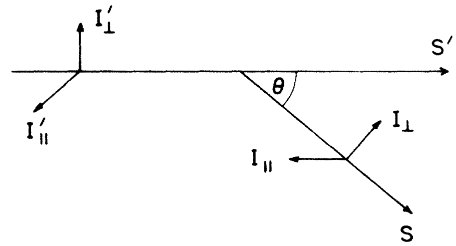

FIG. 1. Scattering of light from $\mathbf{s}'$ to $\mathbf{s}$. $I_{\perp}'$, $I_{\perp}$ and $I_{\parallel}'$, $I_{\parallel}$ are the intensities perpendicular to and in the scattering plane, respectively.

The generalization of (3) to the case where the particles responsible for the scattering have an anisotropic polarizability is straightforward. The cross section is given by

$$\sigma_{ijmn}(\mathbf{s},\mathbf{s}')=\frac{\sigma_0}{1+2\gamma}\{(1-2\gamma)\delta_{im}(\mathbf{s})\delta_{jn}(\mathbf{s})+\gamma\delta_{ij}(\mathbf{s})\delta_{mn}+[\gamma+(1+2\gamma)f(\theta)]\delta_{in}(\mathbf{s})\delta_{jm}(\mathbf{s})\}, \tag{8}$$

where $\gamma=(a-b)/(3a+2b)$ and $a=\alpha_1^2+\alpha_2^2+\alpha_3^2$, $b=\alpha_1\alpha_2+\alpha_2\alpha_3+\alpha_3\alpha_1$, and $\alpha_1,\alpha_2,\alpha_3$ are the principal polarizabilities of the scattering particles. The relations replacing (7) are now given by

$$I_{\parallel}=\frac{\sigma_0}{1+2\gamma}\{[\cos^2\theta+\gamma\sin^2\theta+(1+2\gamma)f(\theta)]I_{\parallel}'+\gamma I_{\perp}'\}, \quad I_{\perp}=\frac{\sigma_0}{1+2\gamma}\{[1+(1+2\gamma)f(\theta)]I_{\perp}'+\gamma I_{\parallel}'\},$$
$$U=\frac{\sigma_0}{1+2\gamma}\{1-\gamma+(1+2\gamma)f(\theta)\}U', \quad V=\frac{\sigma_0}{1+2\gamma}\{1-3\gamma-(1+2\gamma)f(\theta)\}V'. \tag{9}$$

One interesting feature of these results is that if the incident light is polarized perpendicular to the scattering plane, i.e., $I_{\parallel}'=U'=V'=0$, $I_{\perp}'\neq 0$, then

$$I_{\parallel}\sim\gamma I_{\perp}', \quad I_{\perp}\sim[1+(1+2\gamma)f(\theta)]I_{\perp}'. \tag{10}$$

Thus the depolarized part of the scattering does not show coherent backscattering. This is in agreement with the consideration of Van Albada and Lagendijk[1] and also in qualitative agreement with the experimental observations. A more detailed comparison with experiments, properly including multiple scattering effects, requires solutions of (3). This will be given elsewhere including a derivation of (3).

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