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Could There Be a Planck-Scale Unitary Bootstrap Underlying the Superstring?

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It is proposed that the superstring and its spectrum may not be fundamental but may themselves correspond to an approximate solution to a self-consistent dynamical theory built from the basic principles of S-matrix unitarity, analyticity, and crossing. In particular, it is shown that a simple approximate dual unitary scheme based on these principles does generate linear Regge trajectories and selects closed (rather than open) strings.

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Superstring theory has recently shown considerable promise as a way of unifying all the known particle interactions.[1-3] It has been stressed by Witten, among others, that one of the deepest remaining problems is to "find the fundamental principle that impels it."

String theories first attracted general attention over ten years ago when they were found to be equivalent to dual-resonance models (DRM), which were themselves solutions to a program for finding consistent zero-width-resonance–dominated S matrices constrained by analyticity, crossing symmetry, and vertex factorization.[4] Unitarity is violated in such a partial "bootstrap," but can be built up perturbatively through higher-order loop diagrams.

An alternative view is to regard DRM themselves as approximations to full dual unitary bootstrap (DUB) amplitudes, which have finite-width resonances and already contain at least a subset of loop diagrams.[5,6] (DUB) Regge trajectories and amplitudes are generated dynamically from infinite sets of coupled integral equations, with each member of the DUB mass spectrum arising as a bound system of all allowed combinations of members of the same spectrum. If such a DUB amplitude is found to have approximately linear Regge trajectories and to be dominated by relatively narrow resonances at low energies, it can clearly be approximated by a DRM amplitude, and hence by a string, since both (DUB and DRM) amplitudes then satisfy the constraints on which DRM's are based, at least approximately. This would also, in effect, explain why a string arises in the first place.

One should get the same final (unitary) amplitude starting from either (DUB or DRM) amplitude if one could correctly bring in all loop corrections in each case, at least in a domain where such a program is meaningful. The dual-topological unitarization framework[5,7] was an example of such a program in the case of planar DUB amplitudes. Simple approximate four-dimensional (4D) DUB calculations did in fact generate linearly rising leading Regge trajectories[8] and relatively narrow low-energy resonances[9] dynamically and therefore did lead to (open) strings as approximations. Of course, as is well known, a complete anomaly-free 4D string theory with finite or renormalizable loop corrections has never been found. However, DUB schemes can be readily generalized to higher dimensions for both planar and nonplanar amplitudes. We shall see that we again generate linearly rising leading Regge trajectories within simple approximate calculations. Coupling calculations are much more difficult and have not as yet been carried out for $D>4$; on the basis of experience with $D=4$, however, we conjecture that they should again lead to relatively narrow low-energy resonances. We are then again led to (open or closed) strings as approximations.

In order to be able to identify such strings with the ones currently discussed in the superstring program, we have to check if the latter can have couplings strong enough to sustain (and arise from) a unitary bootstrap. If we use the equations of Kaplunovsky,[10] for example, we find that quantum (or loop) effects are of the order

$$g_{10}^2 \Lambda^6 \gtrsim g_{10}^2 M_{\rm str}^6 \sim g_4^2 (M_{\rm str}/M_{\rm comp})^6,$$

where $M_{\rm comp}$ and $M_{\rm str}$ are the compactification and string-tension mass scales, Λ is an effective cutoff, and g_{10} and g_4 are the 10D and 4D gauge couplings. Thus, if we are to have even a marginal mass-scale hierarchy $M_{\rm str} \gtrsim M_{\rm comp}$ of the type required for the usual simple supersymmetry compactification program, and if $g_4^2 \sim \alpha_{\rm GUT} \sim 10^{-2}$ (GUT denotes grand unified theory), as required by $\Lambda_{\rm QCD} \sim 100$ MeV, the quantum effects can certainly be of the order of unity and therefore large enough to sustain a bootstrap. For example, $\alpha_{\rm GUT} = 10^{-2}$ and $M_{\rm str} = 2M_{\rm comp}$ gives $g_{10}^2 \Lambda^6 \gtrsim 0.64 \sim 1$.

The requirement that we have a superstring which is tachyon and anomaly free and finite in at least some perturbative sense has narrowed the possible theories to a small number of open[2] and closed[3] strings in ten dimensions. The latter may themselves arise as particular solutions to the 26D closed bosonic string.[11] It would be desirable to find further fundamental restrictions and perhaps eventually reduce the options to a unique theory. If there is a unitary bootstrap underlying the superstring it should hopefully be able to provide such restrictions; and we do indeed find, as we shall see below, that at least in the present simple approximation scheme, only a nonplanar bootstrap can consistently satisfy our equations, whereas a planar one does not. Since the former should give rise to closed strings[4] as approximations, whereas the latter should provide the dominant dynamics generating open ones,[4,8] the present (precompactification) scheme effectively selects the closed superstring.

Let us begin with the planar bootstrap, which involves summation of all possible planar graphs. For a given process $12 \rightarrow 34$ at moderate t, this is approximately equivalent to the infinite ladder sum of Fig. 1,[8] where s,t,u are the usual Mandelstam variables and ξ represents all the other independent (angular) variables for a given dimension D. In Figs. 1(b), 1(c), . . . the ladder exchanges must themselves have the form of the entire sum of Fig. 1, and the masses of the vertical-line exchanges $(a, \ldots)$, $(b, \ldots)$, $(c, \ldots)$, . . . must be bounded to avoid double counting, say, between Figs. 1(a) and 1(b). We will formally associate a coupling parameter ϕ with each of these latter exchanges.

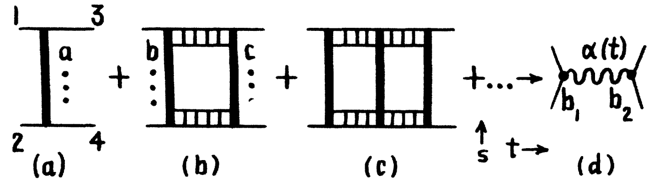

FIG. 1. Approximation to the sum of all possible planar graphs for moderate t.

If we then take the Mellin transform of the s-channel absorptive part $A(s,t,\xi)$ of Fig. 1,

$$A_j(t,\xi) = \int_0^\infty ds\, \nu^{-j-1} A(s,t,\xi), \tag{1}$$

where ν is the usual crossing-symmetry variable $\frac{1}{2} \times (s-u)$, or

$$\nu = s + \tfrac{1}{2}(t - \textstyle\sum m_i^2), \tag{2}$$

and construct a $[1,\bar{N}]$ Padé approximant for the expansion of A_j in ϕ, we obtain,[8] for a given t, ξ,

$$A_j = W_j / [1 - \int_{y_0}^\infty dy\, y^{-j-1} G(y)], \tag{3}$$

where the m_i and S_i are the masses and spins of the external particles, $y = \nu/\nu_a$, ν_a is Eq. (2) at $s = s_a$, and W_j is the transform (1) of Fig. 1(a), which we will approximate by

$$W(s,t,\xi) = \Gamma(t,\xi)\delta(s-s_a) + b(t,\xi)\nu^{\alpha(t)}\theta(s-\bar{s})\theta(s_0 - s), \tag{4}$$

with $\theta(x) = 1$ for $x > 0$, and zero otherwise. We are assuming that the lowest exchange a comes from the ground-state particles z, together perhaps with possible background coming from low-s zz, zzz, . . . exchange; the latter would shift the effective mass $\sqrt{s_a}$ of a away from the mass m_z of z. The Regge term $b\nu^\alpha$ takes into account all the higher $(s > \bar{s})$ contributions to $(a, \ldots)$ in the sense of average Regge-resonance finite-energy sum-rule duality relating Figs. 1(a) and 1(d):

$$\int_0^{\bar{s}} ds [\Gamma\delta(s-s_a) - b\nu^\alpha \theta(\nu)]\nu^{N-S_m} = 0 \tag{5}$$

for a given t, ξ, where $S_m = \max(S_1 + S_2, S_3 + S_4)$, N is an integer $\geqslant 0$, and $S_m - N \geqslant 0$ for the zeroth-moment sum rule.

Equation (3) becomes exact for a factorizable model, even for Padé $[1,\bar{N}] = [1,1]$, in which case the denominator integral is just the Mellin transform of Fig. 1(b) divided by W_j. Such factorizability should be a reasonable approximation because of the average duality between vertical-line exchanges like $(b, \ldots)$ and factorizable t-channel Regge behavior.[8]

If we now require the vanishing of the demonimator in Eq. (3) to give a (Regge) j-plane pole at $j = \alpha(t)$ with residue b, we obtain, using Eqs. (4) and (5),[8]

$$\bar{y}^{\alpha+N+1-S_m}(\alpha+N+1-S_m)^{-1} = C \equiv \ln(\bar{y}/y_0) + \left[\int_{y_0}^\infty dy\, G(y) y^{-\alpha-1} \ln y\right]\left[\int_{y_0}^\infty dy\, G(y) y^{-\alpha-1}\right]^{-1}. \tag{6}$$

In the case of 4D planar graphs it is well known that two-Reggeon Amati-Fubini-Stanghellini (AFS) singularities are absent on the physical sheet.[12] This gives a rapid falloff for large s (or y) of Figs. 1(b), . . . and hence of $G(y)y^{-\lambda}$, which should therefore be peaked in y with an appropriate constant λ. The same result can be readily seen for sums of Feynman graphs for general D by use of Feynman α-parameter methods to analyze their large-s behavior, which then arises, e.g., from end-point contributions of loop integrals as long as D is such that individual graphs converge.[12] If they do not, the absence of AFS singularities should still persist for any well-defined sums of graphs by analytic continuation in D.

The peaking of $y^{-\lambda}G(y)$ permits us to set[8]

$$\ln y \approx \ln y_1, \tag{7}$$

within Eq. (6), with an α-independent integrand peak position $y = y_1$, and hence an α-independent $C = \ln(y_1\bar{y}/y_0)$. We will assume that the dynamics requires y_1 (and hence s_1) to take on the lowest value capable of giving a solution for α from Eq. (6), which then gives

$$\alpha(t) = S_m - N - 1 + (e-1)/\ln(\nu_1/\nu_0). \tag{8}$$

This minimum y_1 ($= y^{-e-1}y_0$), which is also the only one giving a unique α, corresponds to the highest s-channel vertical-line production multiplicity when we expand Eq. (3) in powers of its denominator integral and take its inverse Mellin transform to obtain $A(s,t,\xi)$.[8,13] In a string picture this would be equivalent to maximization of the breaking of the string as it is stretched. Since the energy of an unbroken string rises rapidly with length, such maximal breaking corresponds to a minimization of energy in the s channel.

Since s_1 and s_0 are bounded, positive, and (presumably) monotonic in t, and since $s_1 > s_0$, Eq. (8) gives a linear $\alpha(t)$ for large t. Now if the mass or masses making up $(b, \dots)$ and $(c, \dots)$ in Fig. 1(b) are approximated by a single effective mass m_x, the threshold of this graph should be roughly equal to $s = s_0$ if we are to avoid double counting between Figs. 1(a) and 1(b). We therefore have $s_0 \approx 4m_x^2$. Now in the case of Fig. 1(b), for example, the AFS high-s behavior arising from the intermediate state cut by line I of Fig. 2(a) begins to be canceled by higher intermediate states, such as the one cut by line II of Fig. 2(b), at a threshold $s \approx 9m_x^2$.[14] We might therefore expect the peaking of Fig. 1(b) to occur below this at

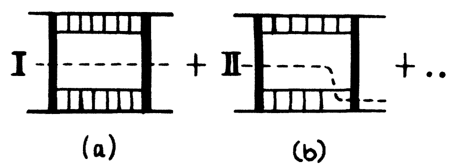

FIG. 2. Different intermediate-state contributions to Fig. 1(b), represented by the sets of lines cut by the dashed lines I, II, ...

$s = s_1 \lesssim 9m_x^2$, or $s_1/s_0 \lesssim \frac{9}{4}$. Equation (8) then gives

$$\alpha(\nu = s) \gtrsim S_m - N + B, \tag{9}$$

with $B = 1.12$. This result does not appear to be modified much by finite-peak-width corrections to the approximation (7). If, for example, we take

$$y^{-\lambda}G(y) \propto \theta(u+f) - \theta(u-f),$$

where $u = (y-y_1)/(y_1-y_0)$ and $0 < f < 1$, we recover Eq. (7) in the limit $f \to 0$. But even in the maximum-width limit $f = 1$ we obtain Eq. (9), with $B = 0.94$.

All consistent anomaly-free tachyon-free superstrings so far have $0 \leqslant J \leqslant 2$ ground states (z) with mass $m_z = 0$.[1-3] If we apply Eq. (9) to $zz \to zz$, for example, we find that, since N is an integer and $S_m - N \geqslant 0$ in Eq. (5), and since $B \approx 1$, we cannot have any zz channels which would be able to generate simultaneously all the correct $0 \leqslant J \leqslant 2$ ground states coupled to this channel, as required. It is not possible for $\alpha(0) = \alpha(\nu = s) = 0$ to be consistent with Eq. (9), for instance. We conclude that planar bootstraps (and hence open strings) are excluded by our scheme.

In the case of the nonplanar bootstrap (which leads to closed strings), nonplanar graphs play an important role from the beginning. Figure 1 can be generalized in the usual way to deal with such graphs, and we are again led to Eqs. (1)–(2), but this time for modified absorptive parts A and with

$$N \to N + n, \tag{10}$$

since we can now have either $n = 0$ or $n = 1$ for the lowest-moment sum rule (5), depending on the signature of the output $\alpha(t)$.[14] AFS-type singularities are now present on the physical sheet and should lead to the effective large-s behavior $s^{\alpha_{AFS}}$ for Fig. 1(b), etc. We will assume that this persists down to moderate s and take[15]

$$G(y) \simeq H y^{\alpha_{AFS}} \theta(y - y_B) \tag{11}$$

in Eqs. (3) and (6). Equations (3) and (10) now give

$$[\alpha(t) + 1 + N + n - S_m]^{-1} \bar{y}^{\alpha(t)+1+N+n-S_m} = \ln(\bar{y}y_0/y_B) + [\alpha(t) - \alpha_{AFS}(t)]^{-1}. \tag{12}$$

where α_{AFS} is associated with Fig. 1(b), etc., and not with the full amplitude.

Since $\alpha_{AFS}(t)$ is itself calculable from $\alpha(t)$, Eq. (12) is a "functional" equation for $\alpha(t)$ which cannot have a solution unless $s_B = s_0$ and $\bar{s}$ is given a moderate t dependence in such a way that $\alpha(t)$, and hence $\alpha_{AFS}(t)$, are linear in t. With $\bar{y} = \exp\{1/[\beta + \nu_a/(\bar{s}_\infty - s_a)]\}$, we obtain[15]

$$\alpha(t) = S_m - N - n + c + 2\alpha'\nu_a, \tag{13}$$

with $c = \beta \ln f - 1$, $2\alpha' = \ln f/(\bar{s}_\infty - s_a)$, and $\ln f = f - 1/(1 - \alpha'_{\rm AFS}/\alpha')$. Strictly speaking, this derivation breaks down at $\nu_a \approx 0$, since the Mellin transform (1) leads to a spurious singularity at $\nu_a = 0$ with our approximations. However, by simultaneously considering more than one process and using the above derivation only in the (overlapping) t regions where it is valid for a given process, one can argue that an equation like Eq. (13) should apply for general t.[8]

As is well known, for a given linear $\alpha(t)$, the intercept $\alpha_{\rm AFS}(0)$ depends on the dimensionality D, but the slope result $\alpha'_{\rm AFS} = \alpha'/2$ does not.[4] We therefore obtain $\ln f = f - 2 = 1.147$. With $\beta = 1/\ln f$, for example, $c = 0$ and we have a moderately slow t dependence for $\bar{s}$. A consistent superstring-spectrum solution with $\alpha(\nu = s) = S_m - N - n$ and $s_a = m_z^2 = 0$ is then possible.

An even slower t dependence for $\bar{s}$ can be obtained with $\beta = \frac{1}{2}$, which gives $c = -0.4265$. A consistent superstring-spectrum solution with $\alpha(\nu = s) = S_m - N - n$ and $m_z^2 = 0$ is again possible, but this time with $\alpha' s_a = 0.2133$. In other words, the exchange a must include some zz, zzz, . . . background in addition to a. Such a background is, of course, absent in a string theory, but we must remember that the string is merely an approximation to a more fundamental theory here.

In conclusion, we see that, at least within the context of the present dynamical framework, a superstring-type infinitely rising Regge-trajectory spectrum arises quite naturally. We also saw that it selects a nonplanar solution and hence a closed-loop string. This suggests that an underlying bootstrap, where each member of the spectrum is a bound system of all allowed combinations of members of the same spectrum, may both justify a string spectrum in the first place, and select the correct specific theory, hopefully uniquely. But, clearly, much better calculations will be needed to confirm this, taking full account of the details of the loop integrals of Figs. 1(b), 1(c), etc., and calculating couplings at the same time. Finally, the bootstrap may help in unraveling the compactification problem; this was not even attempted here but will be needed to establish proper contact with experiment.

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