

Observation of a New Universal Resistive Behavior of Two-Dimensional Superconductors in a Magnetic Field

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A new universal behavior has been observed in the zero-bias resistance of two-dimensional superconducting films of amorphous Mo-Ge in a perpendicular magnetic field. The behavior is thermally activated, with a *single* field-dependent activation energy $\sim H^{-2/3}$. Additionally, a continuous crossover between two different limiting behaviors is observed, with no evidence for a sharp phase transition. Although the observed universal behavior appears to depend crucially upon the existence of pinning, it is qualitatively different from conventional flux-flow or depinning phenomena.

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Because of recent theoretical developments, questions regarding the nature of phase transitions in two-dimensional (2D) disordered systems have grown rapidly in subtlety and richness. It is well known that thermal fluctuations play an important role in determining phase transitions in 2D. Two-dimensional phase transitions of the Kosterlitz-Thouless-Berezinskii^{1,2} type imply that a 2D solid should melt at a temperature T_m below the mean-field transition temperature T_c . However, the results of analogous random-field models³ point out that the existence of random background disorder in 2D can actually lead to the destruction of long-range order even as $T \rightarrow 0$, thereby destroying the melting transition as well.

The vortex phase of two-dimensional superconductors provides an interesting system for examining these new issues in 2D phase transitions. Recall that superconducting vortices are strongly interacting, with long-range repulsive forces.^{4,5} The application of the Kosterlitz-Thouless-Berezinskii theory to the vortex state of thin superconducting films predicts a melting of the vortex lattice via phase fluctuations at a melting temperature $T_m < T_c$, where T_c is the mean-field superconducting transition temperature at $H = 0$.^{6,7} The melting temperature T_m moves further below T_c with increasing sheet resistance R_{\square} , which can be systematically varied over a wide range.

The existence of pinning of the vortex lattice adds additional considerations. In principle, pinning provides a mechanism for applying shear to the vortices, and hence sensing their relative correlation.^{8,9} Also, random pinning produces distortions in the vortex "lattice" which, by analogy with the random-field models mentioned above, brings the very existence of the vortex lattice itself into question. Lastly, as the vortex density $n = H/\phi_0$ is uniquely determined by the

applied magnetic field H , experimentally it can be varied at will. Note that at fixed temperature in the absence of pinning, all lengths would scale with the vortex nearest-neighbor spacing $a \approx (\phi_0/H)^{1/2}$.

Our measurements were performed upon thin films of amorphous $\text{Mo}_{1-x}\text{Ge}_x$. These films have proven in other measurements to be of high quality, with no evident tendency to become discontinuous even at very small film thicknesses.¹⁰ The films were co-magnetron sputtered onto $a\text{-Si}_3\text{N}_4$ substrates, with a thin underlayer of $a\text{-Ge}$ and a protective overlayer of $a\text{-Si}$. The films described here were photolithographically patterned to produce four-terminal bridges of length ~ 2 mm and width $w = 48 \mu\text{m}$. Such dimensions meant that the effective penetration depth $\lambda^2/d \equiv \lambda_{\perp} \gg w$ throughout the region where the universal behavior is observed. Here, λ is the bulk penetration depth [$\lambda(0) \approx 3500 \text{ \AA}$] and d the film thickness ($d \ll \lambda$). Measurements made upon significantly wider bridges ($1016 \mu\text{m}$) indicated that edge pinning did not play an important role in our observations.

Table I lists the samples discussed here, along with some of their characteristics. These samples were

TABLE I. Characteristics of samples studied.

Sample No.	% Ge	R_{\square} (k Ω)	d (\AA)	ρ_N ($\mu\Omega\text{-cm}$)	T_c (K)	T_m (K)
81-598-7	30	0.25	75	187	4.4	2.8-3.4
83-578	21	0.16	102	167	6.1	4.4-5.1
83-580	21	0.29	58	168	5.0	3.0-3.7
83-582	21	0.47	36	170	4.0	1.9-2.5

quantitatively studied after a more qualitative study of a wider set of samples. Samples with higher R_{\square} had sufficiently suppressed T_c 's¹⁰ to make complete studies at high fields (and thus reduced temperatures) outside our capability. The estimated range for the lattice-melting temperature T_m was obtained by use of the results of Fisher.⁷

In the region of low reduced temperature $t = T/T_c$ and magnetic field $h = H/H_{c2}(0)$, the current-voltage curves of our films exhibit "classic" flux-flow behavior like that previously observed in bulk systems by other workers.¹¹ By classic behavior we specifically refer to the behavior where the electric field $E \approx 0$ until the current density J exceeds some critical current density J_c , beyond which a linear flux-flow resistivity $\rho_f = \partial E / \partial J$ arises. Such behavior is clearly seen in Fig. 1 at the lowest temperatures. Furthermore, for these low temperature and field levels, we observe that ρ_f is proportional to the field H and given within 10%–20% by the well-known Bardeen-Stephen result¹² for flux-flow dissipation due to eddy-current damping in the vortex cores, $\rho_f = 2\pi\xi^2(0)\rho H/\phi_0$. Note that this interpretation assumes that *all* the vortices participate in the motion. Here the normal-state resistivity ρ_N was measured directly, the zero-temperature superconducting coherence length $\xi(0)$ (~ 50 Å) was determined from the temperature dependence of the mean-field upper critical field H_{c2} obtained as described below, and ϕ_0 is the superconducting flux quantum. Finally, we note that the measured J_c 's are fairly substantial ($J_c > 10^4$ A/cm²), and the origin of this flux pinning is not known. Such large pinning strength argues against the collective (or lattice) pinning behavior of Larkin and Ovchinnikov¹³ for these films in this region.

As either the temperature or magnetic field is increased, the I - V curves progressively deviate from the classic behavior in two ways. First, as seen in Fig. 1,

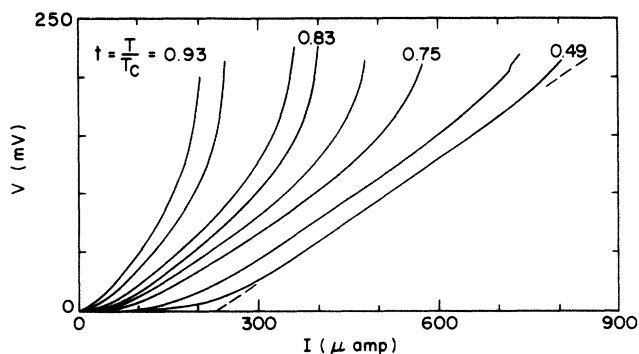


FIG. 1. The measured I - V curves at fixed field [$H = 2.5$ kOe, $h = H/H_{c2}(0) \approx 0.02$] for sample 81-598-7 at various temperatures. The reduced temperatures $t = T/T_c(H=0)$ are (from left to right): 0.93, 0.91, 0.87, 0.83, 0.80, 0.75, 0.60, and 0.49.

the "knee" at the critical current I_c becomes progressively less well defined with increasing temperature. (At all temperatures, the upward curvature at very high bias can be shown to be a result of sample heating.) Second, as is also apparent from Fig. 1, there is an increasing amount of linear zero-bias resistance $R_{I \rightarrow 0}$ with increasing temperature. Although this crossover from classic flux flow to a new high-temperature regime appears to occur continuously, the difference in the two limits is marked and clear. At low t and h the I - V curve displays classic behavior and the zero-bias resistance was too small for us to measure. In the regime at high t and/or h , the I - V 's deviate substantially from the classic behavior, and a finite $R_{I \rightarrow 0}$ is observed, upon which we shall now focus.

Figure 2 shows a plot of $\log R_{I \rightarrow 0}$ vs $1/T$. For fixed field the data are linear over the five decades in resistance which we were able to measure. Clearly some kind of thermally activated process is going on. From the different curves we see that the slopes of these curves decrease with increasing applied field. We also note that the linear behavior persists all the way to half the normal-state resistance R_N , i.e., as T approaches a

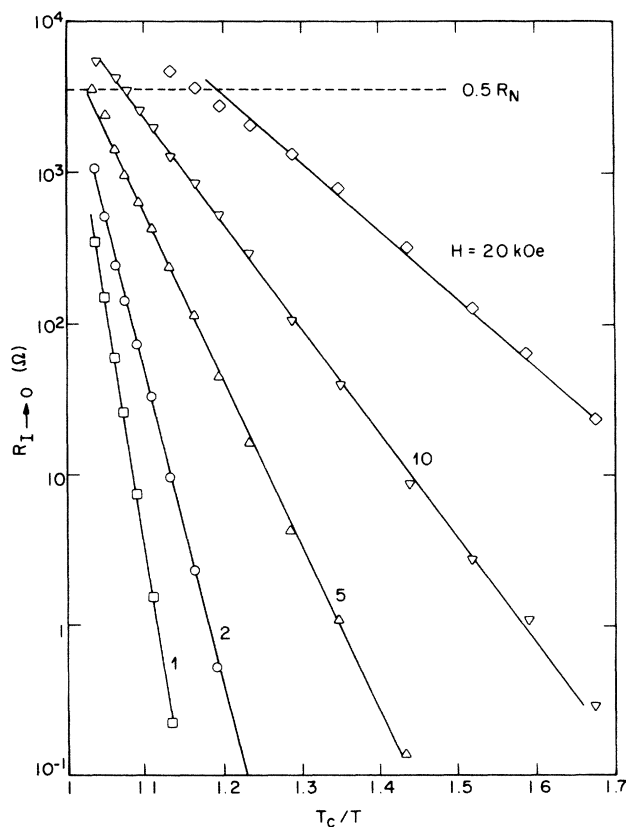


FIG. 2. The zero-bias resistance $R_{I \rightarrow 0}$ (logarithmic scale) vs T_c/T for different applied magnetic fields (sample 83-578). The dashed horizontal line corresponds to half the normal-state resistance. The intersection of the data at each field with this line is taken to be $T_{c2}(H)$.

characteristic temperature which we take to be the mean-field transition temperature $T_{c2}(H)$ for that field. With this identification of T_{c2} it is evident that the data of Fig. 1 can be represented in the functional form

$$R_{I \rightarrow 0}(H, T) = 0.5R_N \exp\{-T_0(H)[1/T - 1/T_{c2}(H)]\} \quad (1)$$

$$= 0.5R_N \exp\{[-T_0(H)/T_{c2}(H)][T_{c2}(H) - T]/T\}, \quad (2)$$

where $T_0(H)$ is the characteristic energy associated with the slopes of the curves in Fig. 1, and the thermally activated nature of the process is now explicitly demonstrated.

In Fig. 3, we examine the field dependence of the normalized characteristic energy $T_0(H)/T_{c2}(H)$. Shown are the data from three different samples with different R_{\square} , as listed in Table I. Note that for each sample $T_0(H)/T_{c2}(H)$ is observed to possess a power-law dependence behavior upon field, with an exponent very close to $-\frac{2}{3}$. Additionally, for fixed H , $T_0(H)/T_{c2}(H)$ is observed to be proportional to R_{\square}^{-1}

$$R_{I \rightarrow 0}(H, T, R_{\square})/0.5R_N = \exp\{-A[T_{c2}(H) - T]/H^{2/3}R_{\square}T\}. \quad (3)$$

In Fig. 4 we show the data for three of the samples listed in Table I after normalizing out the observed dependences. Note that all three samples follow this same universal behavior over the observed five decades of resistance. The fourth sample showed behavior in quantitative accord with that shown. For H measured in kilosterds and R_{\square} in kilohms, we obtain a value for the constant $A \approx 13$.

Let us now consider the meaning of our results. At low temperatures we observe classical flux-flow behavior with a well-defined critical current J_c . From the agreement of the observed classic flux-flow resistance

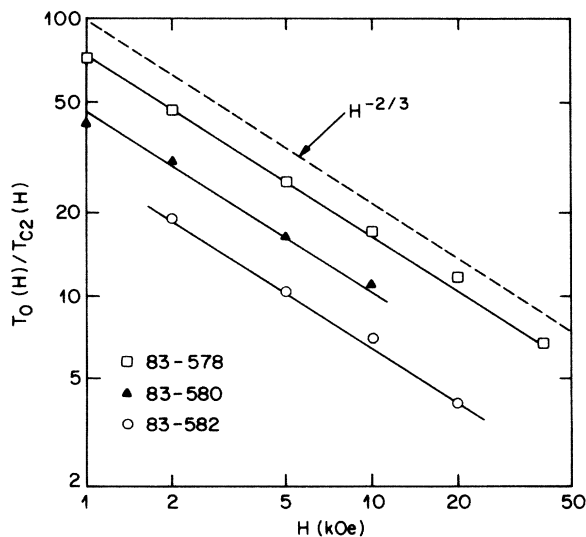


FIG. 3. The normalized zero-temperature activation energy $T_0(H)/T_c(H)$ vs applied magnetic field for three different samples. Note apparent $H^{-2/3}$ behavior.

(or proportional to d , as these films have constant resistivity $\rho = R_{\square}d$). Admittedly, on an empirical basis the universality of the observed behavior as a function of H and T is on firmer footing than of R_{\square} (or d). However, it is also more interesting as the R_{\square} (or d) behavior is as might be expected from previous flux-pinning results.

Having then unraveled the dependences of the activation energy upon all of H , T , and R_{\square} (or d), we can therefore write the zero-bias resistance as a universal function:

with the theory of Bardeen and Stephen, we conclude that in this regime the vortices are moving together uniformly. Using arguments of Larkin and Ovchinnikov¹⁴ for 2D superconductors in the weak-pinning limit together with the measured J_c , one can estimate the lattice correlation length ζ in units of lattice spacings (a) near $T=0$ as follows. The areal energy density for a phase distortion of the lattice on the length scale ζ is $\mu|\nabla\phi|^2 \approx \mu a^2/\zeta^2$, with μ the shear modulus for the lattice. The areal energy density gained by displacing the lattice "downhill" by one lattice constant in the presence of the critical current density J_c is $\phi_0 J_c d/ac$. Equating these two terms, we find $\zeta/a = [\mu ac/\phi_0 J_c d]^{1/2}$ for $T=0$. Following Conen and Schmid,¹⁵ we write $\mu = H\phi_0 d/64\pi^2\lambda^2$ (for no pinning at $T=0$). Using numbers appropriate for our low-temperature data in Fig. 1, we find $\zeta/a \approx 2-3$. This would indicate that we are in the strong-pinning limit, with only short-range correlations even in the low-temperature classic regime. Therefore, the data suggest that in the presence of strong random pinning, thermal fluctuations yield only a smooth crossover in the response of the vortex behavior with increasing temperature, rather than a sharp melting transition.

It is important to realize that the zero-bias resistance $R_{I \rightarrow 0}$ is qualitatively different from the high-bias flux-flow resistance R_f . In the flux-flow regime at high bias the vortices are actually pushed *en masse* across the sample, whereas in the limit of zero current we simply bias their natural diffusion. Furthermore, considering both the predictions of lattice melting and the existence of pinning, we may no longer *a priori* assume that all the vortices are uniformly participating in the motion (contrary to the high-bias classic flux-flow

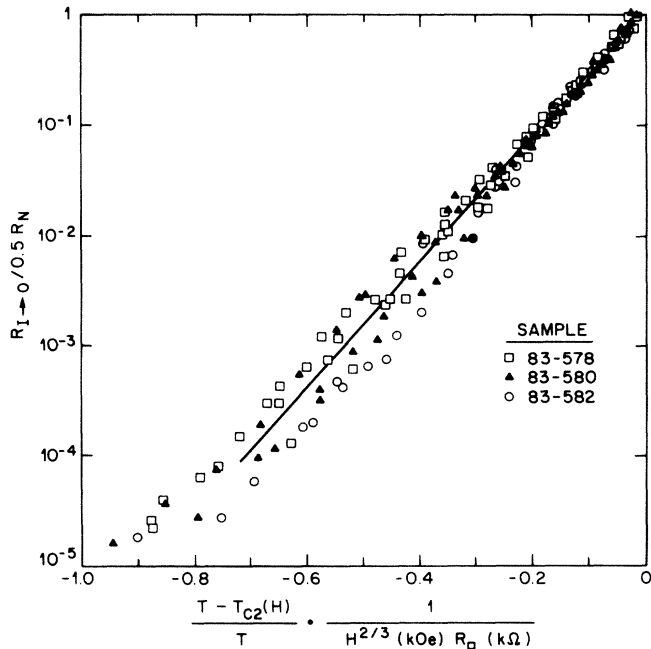


FIG. 4. Reduced zero-bias resistance $R_{I \rightarrow 0}(H, T) / 0.5R_N$ (logarithmic scale) vs $H^{-2/3}R_N^{-1} [T - T_{c2}(H)]/T$ for several magnetic field levels [$1 \leq HK = 40$ (kOe)] for the three different samples. Note clear universal behavior over almost five decades, with functional form as in Eq. (3).

behavior). Therefore, $R_{I \rightarrow 0}$ is a measure of the net vortex diffusion in a region dominated by temperature, strong vortex interaction, and pinning effects.

The fact that we observe thermally activated behavior in $R_{I \rightarrow 0}$ at fixed field is not necessarily surprising. That the activation energy should be proportional to $(T_{c2} - T)/T_{c2}$ is reasonable from the temperature dependence of the condensate. However, the observation of a *single*, universal activation energy for fixed H (as opposed to a distribution of activation energies) is unexpected for such a disordered system. It is interesting to note that in the weak-pinning limit,¹⁴ this could arise from the scale invariance for 2D systems, where for random pinning sites distributed on a scale finer than the correlation length, the pinning energy per correlated region [$(\mu a^2/\zeta^2)\zeta^2 = \mu a^2$] is independent of the region size. However, since we appear to be in the strong-pinning limit, the applicability of this analogy seems questionable. Finally, the observed $H^{-2/3}$ field dependence (or equivalently, $n^{2/3}$ vortex-density dependence) is particularly striking, and almost certainly will provide the key to unlocking the physics.

Last, it is interesting to contrast the situation here

with that of charge-density-wave (CDW) systems. There are similarities with regard to the existence of collective and depinning behavior.^{16,17} However, there are important differences.¹⁷ Strictly from dimensional arguments, fluctuations will play a much less important role in 3D CDW's than for 2D superconductors. Also, it is interesting to note that from an experimental point of view 2D superconductors have the unique feature that the vortex density $n = H/\phi_0$ can be varied at will.

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