Polarization Effects in Exclusive Hadron Scattering

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Measured helicity nonconservation in $\pi^- p \to \rho^- p$ and in pp elastic scattering indicates that higher-twist contributions are $\frac{1}{10}-\frac{1}{3}$ the size of the leading-twist amplitudes, and that the relative phase between certain pp amplitudes is at least 16°. The reported levels of helicity nonconservation are therefore consistent with leading-twist perturbative QCD.

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There has been considerable controversy regarding whether perturbative QCD (PQCD) can be used to calculate amplitudes for exclusive scattering at accessible values of momentum transfer. Power-law scaling[1] has set in by $|t| \sim 5$ GeV^2 for the proton form factor, and the residual Q^2 dependence observed in the new, very-high-precision SLAC data[2] is consistent with that estimated by use of leading-twist PQCD and the Cernyak-Zhitnitsky[3] wave function. The correct power-law dependence on energy is also observed for the other reactions tested[4]: Compton scattering, photoproduction, meson-baryon scattering in several flavor channels, and pp scattering. (See Ref. 1 for details.) The observation of the expected power-law energy dependence, in many different reactions, is strong circumstantial evidence that leading-twist PQCD dominates in the regime $|t| > 5$–10 GeV^2. In addition, a few of the exclusive processes which have been measured at large momentum transfer have been calculated in PQCD. For these, the pion form factor,[5] the proton magnetic form factor,[3,6] and $\gamma\gamma \to \pi^+\pi^-$,[7] the agreement between the data and the PQCD predictions is good. The other exclusive reactions which have been calculated in PQCD are $\gamma\gamma \to B\bar{B}$ [8] and Compton scattering.[9] The data for these processes are in rough agreement with the QCD predictions, but are inadequate to provide a detailed test, by virtue of covering a small kinematic region or having a marginal momentum transfer. Unfortunately, while good data exist for meson-baryon scattering, substantive complications arise in the evaluation of the PQCD amplitudes for these more complicated reactions. In addition to the technical difficulty of the calculations, which involve tens to hundreds of thousands of high-order diagrams, integrated in six to eight dimensions, it is necessary to deal with the probable Sudakov suppression of the regions of Landshoff singularities.[4]

In view of the difficulty of obtaining theoretical predictions for exclusive hadron scattering in the more complicated reactions, it is fortunate that measurements of helicity nonconservation can provide a direct experimental means of determining whether leading-twist PQCD is applicable to these processes.[10] The leading-twist contributions to scattering amplitudes are helicity conserving,[11] since PQCD is chirality conserving. Therefore helicity nonconservation, which necessarily involves a mass scale such as the quark mass or confinement scale, arises only in higher twist. Thus, the leading contribution to helicity-flip observables comes from the interference between the lowest-twist part of the amplitude and higher-twist pieces. (Here we are assuming the validity of the operator-product expansion, i.e., that terms of higher twist are of decreasing importance, which is necessary for any PQCD analysis to work.) Consequently, the magnitude of helicity-flip observables relative to nonflip observables reflects the size of nonleading-twist contributions to the amplitude relative to leading-twist contributions.[10] If these nonleading-twist contributions are found to be small compared to the leading-twist contributions, our analysis based on the assumption of the validity of the operator-product expansion is consistent, and PQCD is applicable to these reactions. The two helicity-nonconserving observables we will employ are the spin analyzing power in pp scattering, and the density-matrix element $\rho_{1,-1}$ in $\pi^- p \to \rho^- p$ scattering. We will find in the analysis below that the helicity-flip amplitudes are small relative to the nonflip amplitudes, i.e., that leading twist dominates nonleading twist. This demonstrates the phenomenological consistency of applying PQCD to these hadronic reactions, and yields the first direct estimate of the relative importance of the higher-twist terms.

The analyzing power in pp elastic scattering, denoted A, has been measured[12] recently at $p_{\text{lab}} = 28$ GeV/c and $(p_t)^2 = 6.5$ (GeV/c)^2, i.e., $s = 54$ GeV^2 and $t = -6.5$ GeV^2. The researchers actually report what are said to be three independent measurements of A: $(51 \pm 17)\%$, $(22 \pm 26)\%$, and $(13.5 \pm 10.5)\%$, which they average to give $(24 \pm 8)\%$. Since the true uncertainty in this procedure may be greater than the nominal 3σ, results are given below in terms of $A/0.24$, to facilitate modification of the analysis if the accepted value of A changes in the future. A can be written in terms of helicity amplitudes as[13]

$$A = -\text{Im}[\phi_5^*(\phi_1 + \phi_2 + \phi_3 - \phi_4)]/d\sigma, \quad (1)$$

where

$$d\sigma = (|\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2 + |\phi_4|^2 + 4|\phi_5|^2)/2. \quad (2)$$

ϕ_1, ϕ_3, and ϕ_4 are helicity conserving ($++ \to ++$, $+- \to +-$, and $+- \to -+$, respectively) and are thus present in leading twist. ϕ_2 and ϕ_5 are helicity nonconserving ($++ \to --$ and $++ \to +-$, respectively) and are thus nonzero by virtue of higher-twist effects such as quark masses, hadron size, and "intrinsic" p_t of the quarks inside hadrons.

In order to estimate the size of ϕ_5 from Eqs. (1) and (2), we need information on the relative sizes of ϕ_1, ϕ_2, ϕ_3, and ϕ_4. We will assume that the other helicity-nonconserving amplitude, ϕ_2, can be neglected.[14] The measurement of A is at $\theta_{\text{c.m.}} = 45°$, and the spin-transfer amplitude, ϕ_4, vanishes for small angle. Furthermore, because of the identity of the two final protons, $\phi_3(\theta) = -\phi_4(\pi - \theta)$. At 45°, ϕ_4 should lie between $-\phi_3$, its value at 90°, and zero, its value at 0°.

We can further constrain our estimates of the relative values of the allowed ϕ's by looking at another polarization observable,[13]

$$A_{nn} = \text{Re}(\phi_1\phi_2^* - \phi_3\phi_4^* + 2|\phi_5|^2)/d\sigma. \tag{3}$$

A_{nn} is measured in pp scattering at large momentum transfer ($t \sim -12$ GeV^2) to be[15] 0.6 ± 0.04 at $\sim 90°$ in the center-of-mass system. In order to extract relative values of ϕ_1, ϕ_3, and ϕ_4 to use in the determination of ϕ_5 from the 45° data on A, it would be best to have comparable 45° data on A_{nn}. However, the availabale data on A_{nn} are at $p_{\text{lab}} = 11.75$ GeV/c, so that the 45° point is at too low a value of $|t|$ to be relevant to the PQCD regime. Nevertheless A_{nn} at 90° does provide useful information, since one would not expect rapid variation of the ratio ϕ_3/ϕ_1 between 90° and 45°. At 90° we know that $\phi_3 = -\phi_4$ and so we can use (3) to find that $|\phi_3| = 0.9|\phi_1|$ at 90°, assuming $\phi_2 = \phi_5 = 0$. What about smaller angles? If we insist that $|t| > 5$ GeV^2, the $p_{\text{lab}} = 11.75$ GeV/c data on A_{nn} can be used down to $\theta_{\text{c.m.}} = 60°$, where it is about half as large as at 90°, i.e., 0.3. Thus ϕ_3 and ϕ_4 have opposite signs, as expected. Now by use of $|\phi_3| = 0.9|\phi_1|$ as at 90°, we conclude that $\phi_4 = -\frac{1}{3}\phi_3$ at 60°, which is very reasonable according to the arguments given above.

In the evaluation of ϕ_5 below, we take six cases for the relative magnitudes of ϕ_1, ϕ_3, and ϕ_4 inspired by the arguments of the preceding paragraphs and constrained by the data on A_{nn}: (i) ϕ_3 and ϕ_1 equal, or (ii) $\phi_3 = -0.9\phi_1$, with (a) $\phi_4 = -\phi_3$, (b) $\phi_4 = -\frac{1}{2}\phi_3$, or (c) $\phi_4 = 0$. The conclusions are insensitive to this choice.

Before proceeding with the evaluation of ϕ_5, it is worth pausing to stress that the fact that $|\phi_4/\phi_3| \sim \frac{1}{3}$ has interesting physical implications. If the spin transfer amplitude ϕ_4 is comparable in magnitude to the non-spin-transfer amplitudes ϕ_1 and ϕ_3, the pp scattering amplitude must get significant contributions from diagrams with quark exchange. This is because gluon exchange does not transfer helicity, and so only contributes to ϕ_4 at 60° by the gluon exchange process at 120°. Since gluon-exchange diagrams are proportional to several powers of t^{-1}, and since the ratio of $t(60°)/t(120°) \sim \frac{1}{3}$, gluon exchange cannot give rise to spin-transfer amplitudes comparable to non-spin-transfer amplitudes at 60°. When data on A_{nn} become available at higher energy, so that we have smaller-angle points at sufficiently large t to be relevant, say $|t| > 5$ GeV^2, the value of $|\phi_4/\phi_3|$ will provide an even more powerful means of determining the importance of pure gluon exchange relative to quark exchange.

Returning to the determination of ϕ_5, we need an estimate of the phase between ϕ_5 and $\phi_1 + \phi_3 - \phi_4$, which we call η below. In Born approximation the quark scattering amplitudes are real. A phase arises, first of all, because higher-order (but leading twist) logarithmic corrections are in general not purely real; phases from this effect are presumably $O(\alpha_s)$. Secondly, the integration over the quark wave functions generates a phase in all but a few simple cases. Carrying out the integrations for Compton scattering (the simplest case where a phase arises from other than higher-order perturbation theory contributions) Maina[9] found large phases. Possibly more germane to the baryon-baryon case, which has a Landshoff pinch in many Born-approximation diagrams, is the fact that the Landshoff pinch gives a purely imaginary contribution [cf. Eq. (9.11) of Mueller[4]]. Moreover, when the argument of the logarithm in the Sudakov factors is spacelike, as it is in most cases, that also leads to a phase.[16] Therefore it is incorrect to presume that the phase will be small when it is finally calculated in PQCD. Let us assume for the sake of definiteness that η is 45°. The results given below can easily be scaled by the reader for a different choice of phase.

We can now estimate ϕ_5. In all six cases mentioned above for the values of ϕ_3 and ϕ_4 relative to ϕ_1, $|\phi_5|$ lies in the range 0.16–0.20 times $(A/0.24) \times [\sin(\sqrt{2}\eta)]|\phi_1|$. Taking the nominal value 0.24 for A, and $\eta = 45°$, gives the nonleading-twist amplitude $\phi_5 \sim \frac{1}{5} - \frac{1}{6}$ as large as the leading-twist amplitude ϕ_1.

As a result of the dependence of A on the relative phase η, and the present theoretical uncertainty in the relative signs and magnitudes of the leading-twist contributions, it is not possible to obtain an upper bound on ϕ_5. However, one can obtain a lower bound on its magnitude relative to $(|\phi_1|^2 + |\phi_3|^2 + |\phi_4|^2)^{1/2}$, which we shall denote as R, by assuming that it is 90° out of phase with $(\phi_1 + \phi_3 - \phi_4)$, denoted as C below, and maximizing C for fixed R. That is a simple problem in three-dimensional geometry, and one finds $C_{\max} = \sqrt{3}R$. This then yields the lower bound, $|\phi_5| > 0.07(A/0.24)R$. Alternatively, one can maximize the coefficient of the sine of the relative phase in Eq. (1), called η above, and conclude thereby that η is greater than $\arcsin(2A/\sqrt{3})$, which is 16° for $A=0.24$, with the minimum occurring for $|\phi_5| = R/2$.

Proceeding now to $\pi^- p \rightarrow \rho^- p$ scattering, the helicity-nonconserving observable we shall analyze is the density-matrix element $\rho_{1,-1}$ measured by Heppelman *et al.*[17] at 90° (c.m.) for incident-pion momentum of 9.9 GeV/c ($t = -9.7$ GeV^2). In general, the density-matrix elements can be written

$$\rho_{i,j} = \sum A^i_{m,n} A^{j*}_{m,n}, \qquad (4)$$

where i,j are possible ρ helicities and the summation is over initial and final proton helicities, m and n. Helicity conservation requires that $m = n + i$ and $m = n + j$. The experiment determines

$$\rho_{1,1} = 0.44 \pm 0.15, \quad \rho_{0,0} = 0.12 \pm 0.30,$$

$$\rho_{1,-1} = 0.32 \pm 0.10, \quad \mathrm{Re}\rho_{1,0} = -0.01 \pm 0.05.$$

If we neglect helicity-nonconserving contributions to processes which have allowed helicity-conserving contributions, we can use $\rho_{1,1}$ to determine $A^1_{1/2,-1/2} = 0.66 \pm 0.11$. From $\rho_{0,0}$ we have $A^0_{1/2,1/2} = 0$ within errors. Then using $\rho_{1,-1}$ and the value of $A^1_{1/2,-1/2}$ determined above, we find 0.24 ± 0.09 for the helicity-nonconserving amplitude $A^{-1}_{1/2,-1/2}$.

Since the spin wave functions of the π^- and ρ^- are orthogonal, it is not surprising[18] that $\rho_{0,0}$ is very small compared to $\rho_{1,1}$. Therefore, a reasonable measure of the magnitude of $A^{-1}_{1/2,-1/2}$ relative to a leading-twist amplitude is $A^{-1}_{1/2,-1/2}/A^1_{1/2,-1/2} = 0.36 \pm 0.15$. Improving the error bars[19] on this result will be very helpful, since it is difficult to interpret a $2\frac{1}{2}$ standard deviation effect. In any case, the nominal result of $\sim\frac{1}{3}$ for the relative magnitude of nonleading-twist amplitudes is consistent with dominance of the leading-twist amplitudes in cross sections, as discussed below.

It is worth noting that since the helicity-nonconserving observables discussed above arise from interference terms between leading- and nonleading-twist amplitudes, they should vanish only as $1/\sqrt{s}$, for fixed t/s. This can be tested. A corollary of this observation is that as $|t|$ is decreased, for fixed s/t, the importance of the nonleading amplitudes which we found above do not increase very rapidly. For instance, at $|t| = 5$ GeV^2 their relative values should be 1.4 times larger.[20]

Finally, we must check whether the estimates presented here for the nonleading-twist contributions at $t \sim -10$ GeV^2 are consistent with the experimental success of the leading-twist power laws. For this, consider the effect of nonleading-twist contributions which are 36% of the leading amplitudes at $s = 20$ GeV^2 and $t = -10$ GeV^2 (90°) on the apparent power-law energy dependence of meson-baryon cross sections, which asymptotically should scale[4] as s^{-8}. Fitting the energy dependence of the 90° cross section over the s range 20–40 GeV^2, assuming a nonleading contribution $\sim (0.36)^2(20/s)^9$, would lead to an apparent power of -8.1 Since the experimental determination of the exponent has an uncertainty of at least ± 0.2, this discrepancy could not be observed, and would not be significant in view of the remark of Ref. 4. This exercise demonstrates the assertion in the introduction, that interference phenomena such as helicity nonconservation must be used to observe the effects of nonleading-twist terms.

To summarize, we have used polarization data to estimate the magnitude of nonleading-twist effects in exclusive hadron scattering at $|t| \sim 6.5$–10 GeV^2. Taking the pp data at face value, we find that the relevant nonleading-twist amplitude is at least 0.07 times the square root of the sum of the squares of the leading-twist amplitudes, and that the phase between that nonleading-twist amplitude and a certain linear combination of the leading-twist amplitudes is at least 16°. Our best estimate, assuming a 45° relative phase, is that the nonleading-twist amplitude is less than or about $\frac{1}{5}$ the typical leading amplitude. We also discover that the spin-transfer amplitude at 60° is $\sim\frac{1}{3}$ the spin-nontransfer amplitudes, indicating that quark exchange probably contributes significantly to pp scattering amplitudes. From the $\pi^- p \rightarrow \rho^- p$ data, we find that the magnitude of the relevant higher-twist amplitude in that process is 0.36 ± 0.15 compared to the typical leading amplitude. Thus for both pp and πp scattering, the *Ansatz* that the amplitudes are given by perturbative QCD is internally consistent, and the leading-twist approximation should be accurate to $\sim 10\%$ for quantities such as the magnitude and energy dependence of differential cross sections.

The analysis presented here yields two significant conclusions, even though the error bars on the data are large. It strengthens the case for applying perturbative QCD to these and other exclusive scattering processes in the region $|t| \sim 5$–10 GeV^2, and it demonstrates vividly how much information can be extracted from the polarization data which we have employed. If experimentalists can push these measurements so that they cover a greater angular region at large $|t|$, with improved errors and systematics, that will be very useful for unraveling the dynamics of exclusive hadron scattering in the PQCD regime.

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[19]In arriving at the error estimates on the amplitudes I have added uncertainties in quadrature. It would be desirable to refine this procedure, since the systematic errors in the ρ's are correlated.

[20]Of course, at sufficiently small t, the nonperturbative effects become large enough that it is not meaningful to attempt to describe the situation in terms of leading and nonleading twist. In that regime one expects exponential rather than power-law dependence on momentum transfer.