Double-Helix Current Drive for Tokamaks

M. J. Dutch, A. L. McCarthy, and R. G. Storer
School of Physical Sciences, The Flinders University of South Australia, Bedford Park, South Australia 5042, Australia
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Double sets of $m = 1$ helical coils fed by appropriately phased radio-frequency currents are shown theoretically to be capable of driving both toroidal and poloidal currents in a toroidal plasma by means of the nonlinear Hall effect. It is shown that total toroidal current is optimized when the ratio of the plasma circumference to the pitch length of the helical coils is 1.35.

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Recent experiments have shown that toroidal currents can be driven in a plasma by rf or higher-frequency currents in a double set of helical coils or in a large-area helical array.[1–5] These results are important for tokamak research because they offer the possibility of continuous current drive, overcoming the limitations of normal inductive current drive.

In the experiments of Dutch and McCarthy[5] a double helix of $m = 1$ coils (see Fig. 1) completely spans the toroidal plasma vessel. These are driven by two separate (~ 5.5 MW, 330 kHz) rf generators with a phase difference of $\pi/2$. The rf fields of the structure thus progress in the poloidal and toroidal directions. The dominant spatial harmonic of this field has $m = 1$. There was a steady toroidal magnetic field. A vertical magnetic field was provided for toroidal MHD equilibrium. A steady driven torodial current of up to 1 kA and a steady poloidal current were observed.

In this paper we present a theoretical and computational analysis of this mode of current drive based on Maxwell's equations and Ohm's law with the inclusion of the nonlinear Hall term. As pointed out by Thonemann, Cowhig, and Davenport[6] in their foundation paper on the interaction of traveling magnetic fields with ionized gas, the origin of the driving force on the electrons is understandable in macroscopic terms since the changing magnetic field induces screening electron currents which interact with the fields from the external coils to produce a force in the direction in which these fields move. The situation is analogous to an induction motor. The helical geometry complicates this simple picture. In our calculations we obtain explicit expressions for the radial distribution of the toroidal and poloidal components of the current density for a range of plasma parameters including resistivity, rf field amplitudes, and external coil geometry. These are integrated to give the total driven currents, and computed results are used to show which choice of parameters maximize the driven currents.

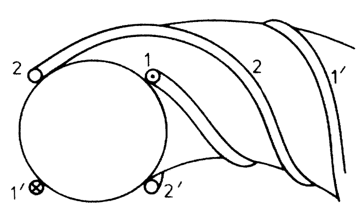

FIG. 1. Schematic diagram showing the $m = 1$ double-helix structure used to drive torodial and poloidal currents.

Our study is made in a frequency range such that the ions are not brought into motion by the moving magnetic fields but the electrons are. This requires that the radio frequency ω and the electron-ion collision frequency ν_{ei} lie between the ion and electron cyclotron frequencies (referred to the rf field strength). This moving–magnetic-field current-drive technique does not rely on the use of wave-particle interaction effects which act only on a subgroup of electrons within the electron velocity distribution. Rather, it uses a nonlinear force which acts on the entire electron gas. In a system where an electron current is driven by a moving magnetic field, one might imagine that the ions would be set into motion by electron-ion collisions and, in a completely isolated plasma, both ions and electrons would ultimately move in step with the field (causing the net current to vanish in time). This possibility is common to all rf current-drive schemes. In practice, some ion-momentum relaxation mechanism, such as charge exchange or the counterfeeding of fuel gas, must exist to ensure a net current. Hugrass[7] has shown that an ion-momentum relaxation mechanism with effective frequency as low as $\nu_{ei} m_e / m_i$ is sufficient to ensure substantial net current.

The analysis presented here may be related to the ideas of helicity injection,[8] but it is more straightforward to regard it as an extension of the successful rotating magnetic field theory and experiments.[9] We consider the simplest physical MHD model, developed by Hugrass and Jones,[9] for nonlinear Hall-effect current drive, taking the generalized Ohm's law to be

$$\mathbf{E} - (ne)^{-1}\mathbf{J}\times\mathbf{B} = \eta\mathbf{J} \tag{1}$$

for a plasma of uniform density n and scalar resistivity η. The Hall term $(\mathbf{J}\times\mathbf{B})$ provides a mechanism for current drive independent of the usual inductive current drive in tokamaks. A tensor resistivity could be used, but in the regime of greatest interest (i.e., that in which the Hall term dominates) the exact form of the resistivity is of little consequence. The analysis will be carried out for arbitrary values of η and n; however, it will be shown that the well-known skin effect which inhibits the penetration of rf fields into a plasma when the resistive term dominates is completely replaced by a physical process which allows penetration of the fields when the Hall term dominates in Ohm's law. We consider, in this model, an rf frequency range $eB_\omega/m_i < \omega < eB_\omega/m_e$ (i.e., ω lies between the ion and electron cyclotron frequencies referred to the vacuum rf field strength, B_ω) so that the plasma can be regarded as being composed of a mobile electron fluid and a uniform immobile ion population, as discussed in detail by Hugrass.[7] As will be seen in our analysis, where an arbitrary steady field is introduced, the ion cyclotron frequency in the steady field plays no role in the current-drive mechanism nor in defining the frequency range required.

The large–aspect-ratio limit is taken, in which a cylindrical plasma of radius a is wound with a series of helical rf coils of pitch length l. We define $k=2\pi/l$ and introduce the natural helical coordinate system related to the coil structure, with unit vectors $\hat{\mathbf{r}}$, $\hat{\chi}$, and $\hat{\zeta}$, where $\chi=m\theta+kz$, $\hat{\zeta}=\hat{\mathbf{r}}\times\hat{\chi}$, and (r,Θ,z) are the usual cylindrical coordinates.[10] Here we have retained a general poloidal number m although, in the experiments with double helical coils, $m=1$ so that $\hat{\zeta}$ is parallel to the coil windings.

The distribution of externally applied currents can be approximated by the continuous distribution $\hat{\zeta} I_e \operatorname{Re}\{\exp[i(\omega t+m\Theta+kz)]\}\delta(r-a)/a$ where I_e is a measure of the strength of the rf currents in the helical coils.

The fields and currents satisfy Maxwell's equations and Ohm's law, with the Hall term [Eq. (1)]. In general, the solutions can be Fourier analyzed in time; however, it was shown by Hugrass[11] in a similar context that the general steady-state solution is given sufficiently accurately by the zero- and first-order harmonic terms. Thus, after waiting for transients to die out, we can write any of the field quantities current ($\mathbf{J}$), electric field ($\mathbf{E}$), or magnetic field ($\mathbf{B}$) in the form

$$\mathbf{B}(r,\chi,t)=\mathbf{B}_0(r)+\tfrac{1}{2}\mathbf{b}(r,\chi,t)+\tfrac{1}{2}\mathbf{b}^*(r,\chi,t), \quad (2)$$

where $\mathbf{b}(r,\chi,t)$ and the other lower-case quantities are proportional to $e^{i(\omega t+\chi)}$. In the plasma region, Ohm's law [Eq. (1)] implies

$$\mathbf{E}_0-(ne)^{-1}(\mathbf{J}_0\times\mathbf{B}_0+\tfrac{1}{4}\mathbf{j}\times\mathbf{b}^*+\tfrac{1}{4}\mathbf{j}^*\times\mathbf{b})=\eta\mathbf{J}_0, \quad (3)$$

$$\mathbf{e}-(ne)^{-1}(\mathbf{j}\times\mathbf{B}_0+\mathbf{J}_0\times\mathbf{b})=\eta\mathbf{j}. \quad (4)$$

The solution to these equations should be matched to the external fields. The magnetic field produced by the applied currents, in the absence of plasma, can be written in terms of Bessel functions (for $0<r<a$) as

$$\mathbf{B}_{\text{ext}}=B^0_{\text{ext}}\hat{\mathbf{z}}+B_\omega[-2iI_1'(kr)\hat{\mathbf{r}}+2(k^2+m^2/r^2)^{1/2}I_1(kr)/k\,\hat{\zeta}]e^{i(\omega t+\chi)}, \quad (5)$$

where B^0_{ext} represents a steady toroidal field and B_ω, which depends on the pitch of the coils, is the magnitude of the rf field on the axis.

If there are no imposed external steady electric fields ($\mathbf{E}_0=0$), then a physically realistic particular solution to the set of nonlinear equations can be obtained when there is no radial oscillating current, i.e., when $j_r=0$. The r component of Ampere's law then implies that $b_\zeta=0$ and $j_\chi=0$ and, from Eq. (3), $J_{0\zeta}=0$. If we express the vector operators in terms of the helical coordinates,[10] the r component of Faraday's law implies

$$k_0e_\zeta=-\omega b_r, \quad (6)$$

where $k_0=k_0(r)=(k^2+m^2/r^2)^{1/2}$, and hence the ζ component of Eq. (4) gives

$$e_\zeta-(ne)^{-1}J_{0\chi}(k_0/\omega)e_\zeta=\eta j_\zeta. \quad (7)$$

Thus e_ζ, j_ζ, and b_r must all be in phase. The χ component of Eq. (3) then leads to

$$\eta J_{0\chi}=-(2ne)^{-1}\operatorname{Re}(j_\zeta b_r^*)$$
$$=(2ne)^{-1}(k_0/\omega)j_\zeta e_\zeta^*. \quad (8)$$

Elimination of $J_{0\chi}$ and j_ζ from Eqs. (7) and (8) gives the following closed nonlinear equation for e_ζ:

$$k_0\frac{\partial}{\partial r}\left[\frac{1}{k_0^2 r}\frac{\partial}{\partial r}(k_0re_\zeta)\right]-k_0^2e_\zeta=\frac{i\omega\mu_0}{\eta}\frac{e_\zeta}{[1+\frac{1}{2}(k_0/\omega ne\eta)^2|e_\zeta|^2]} \quad (9)$$

in the plasma $(0<r<a)$.

Matching to the appropriate external field allows us to express the boundary condition for e_ζ at $r=a$ in terms of Bessel functions (for the case $m=1$):

$$\left[e_\zeta\frac{\partial}{\partial r}\left(\frac{K_1'(kr)}{k_0}\right)-\frac{\partial e_\zeta}{\partial r}\left(\frac{K_1'(kr)}{k_0}\right)\right]_{r=a}=\frac{2i\omega B_\omega e^{i(\omega t+\chi)}}{k^2a}. \quad (10)$$

We can then use Eqs. (7) and (8) to recover the steady plasma current $J_{0\chi}$ in terms of e_ζ, viz.,

$$J_{0\chi}=\frac{ne\omega}{k_0}\left[\frac{\frac{1}{2}(k_0/\omega ne\eta)^2|e_\zeta|^2}{1+\frac{1}{2}(k_0/\omega ne\eta)^2|e_\zeta|^2}\right]. \quad (11)$$

The solutions are completely specified in terms of three dimensionless parameters: $\lambda = a/\delta$, which is the ratio of plasma radius to the classical skin depth, $\gamma = eB_\omega/m_e\nu_{ei} = B_\omega/ne\eta$, which is the ratio of the electron cyclotron frequency to the electron-ion collision frequency, and $\kappa = 2\pi a/l$, which is the ratio of the plasma circumference to the pitch length of the helical windings. Analytic solutions are possible,[11] not only for the linear limit ($\gamma \to 0$), which corresponds to the classical skin effect, but also for the strongly nonlinear limit ($\gamma >> \lambda$) where the Hall term dominates in Ohm's law and leads to full penetration of the rf fields into the plasma. Physically, the penetration of the fields is enhanced because the frequency seen by the electrons moving with the fields approaches zero. In this case Eq. (9) reduces to

$$k_0\frac{\partial}{\partial r}\left[\frac{1}{k_0^2 r}\frac{\partial}{\partial r}(k_0 r e_\zeta)\right] - k_0^2 e_\zeta = 0, \tag{12}$$

which has the solution [with use of Eq. (10)]

$$e_\zeta = (i\omega B_\omega/k_0)I_1'(kr)e^{i(\omega t+\chi)}. \tag{13}$$

Hence, in this limit (i.e., $k_0/\omega ne\eta >> 1$)

$$J_{0\chi} = ne\omega r/(1+k^2r^2)^{1/2}, \tag{14}$$

which can be represented in terms of a toroidal component

$$J_{0z} = ne\omega kr^2/(1+k^2r^2), \tag{15}$$

and a poloidal component

$$J_{0\theta} = ne\omega r/(1+k^2r^2). \tag{16}$$

The total toroidal current is

$$I_{0z} = ne\omega\pi a^3/\kappa[1-\kappa^{-2}\ln(1+\kappa^2)]. \tag{17}$$

The parameters which define the experimental arrangements are primarily the pitch of the helical windings and the strength of the rf fields required. A most important conclusion is illustrated in Fig. 2, which shows the total toroidal current as a function of the inverse pitch, κ, for the strongly nonlinear case with maximum penetration of the rf fields. This current has a maximum when $\kappa = ka = 1.35$, thus defining the optimal experimental arrangement. If the pitch is too small, the rf fields cancel; if the pitch is too large, only poloidal currents are driven.

Figure 3 shows the radial distribution of the normalized toroidal and poloidal current densities for various values of the field penetration. The degree of field penetration is determined by the ratio γ/λ and here λ is held constant at 10.0 while γ is increased by an increase of the strength of the rf field. The increased penetration results in a larger fraction of the maximum possible current, $\alpha_\theta = I_{0\theta}/I_{0\theta(\max)}$ and $\alpha_z = I_{0z}/I_{0z(\max)}$, to be driven. We use α_θ and α_z to label the curves, which were obtained by a numerical solution of Eq. (9). Note that, even for maximal penetration, the current profiles are hollow. This result can be explained physically by noting that the rf screening currents have only ζ components, and as r tends to zero $\hat{\zeta} \to \hat{z}$, so that at $r=0$ the time-averaged nonlinear Hall-effect term has no component along $\hat{z}$. Figure 4 shows the variation of the normalized total driven currents α_θ and α_z as a function of γ, i.e., increasing rf field strength. It can be seen that, in all cases, as the rf current is increased, a stage is reached where both the poloidal and toroidal current reach their maximum values corresponding to Hall-effect

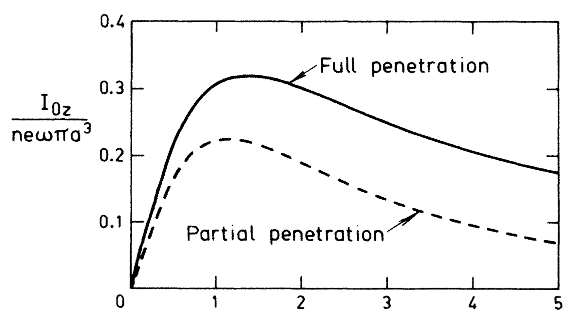

FIG. 2. Variation of the normalized toroidal current with κ (the normalized reciprocal pitch length). For full penetration, optimal current drive occurs with $\kappa = 1.35$.

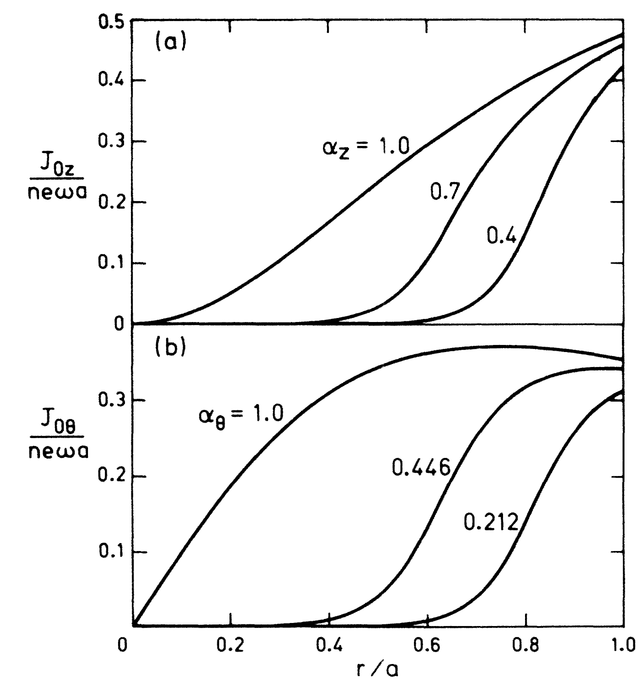

FIG. 3. Radial distribution of normalized steady current densities for full penetration ($\alpha = 1.0$) and partial penetration ($\alpha < 1.0$). Here $\kappa = 1.35$ and $\lambda = 10.0$.

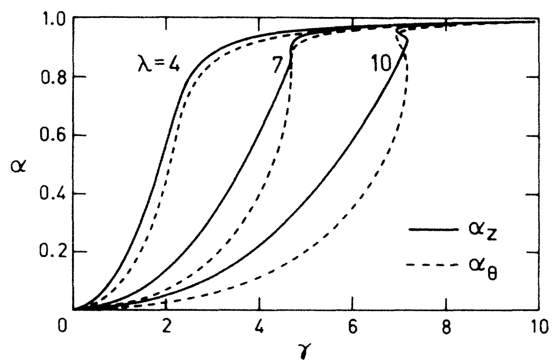

FIG. 4. The dependence of the steady normalized toroidal and polodial currents, α_z and α_θ, on increasing rf field strength (γ increasing) for various values of the resistivity ($\eta \propto \lambda^{-2}$).

dominance. Notice that if $\gamma > \lambda$ almost all of the maximum possible current can be driven. As observed by Hugrass[11] for the case $\kappa = 0$ (straight conductors) the steady currents are sensitive and sometimes nonunique functions of the driving rf fields. The actual current for a given value of γ is determined by the startup conditions.

We have shown that both poloidal and toroidal steady currents can be driven by rf currents in sets of external helical coils. The optimum conditions for this mechanism of current drive are $\gamma > \lambda$ and $\kappa = 1.35$. It should be noted for tokamak applications that the sense of the pitch of the helical coils will determine the direction of the poloidal driven current: An appropriate choice will result in an enhanced q value on axis.

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[1]R. A. Demirkhanov, A. G. Kirov, L. F. Ruchko, and A. V. Sukachev, in *Proceedings of the Tenth European Conference on Controlled Fusion and Plasma Physics, Moscow, 14–19 September 1981* (European Physical Society, Petit-Lancy, Switzerland, 1981), paper E7.

[2]T. Kikunaga, E. Hotta, and I. Hayashi, Jpn. J. Appl. Phys. **21**, 1657 (1982).

[3]E. Hotta, M. Suzuki, H. Ohta, and I. Hayashi, Jpn. J. Appl. Phys. **24**, 110 (1985).

[4]J. D. Evans, K. F. Lai, and R. J. Taylor, Bull. Am. Phys. Soc. **30**, 1623 (1985).

[5]M. Dutch and A. L. McCarthy, to be published.

[6]W. N. Thonemann, W. T. Cowhig, and P. A. Davenport, Nature (London) **169**, 34 (1952).

[7]W. N. Hugrass, J. Plasma Phys. **28**, 369 (1982).

[8]P. M. Bellan, Phys. Rev. Lett. **54**, 1381 (1985).

[9]I. R. Jones, in Proceedings of the International Conference on Plasma Physics, Lausanne, Switzerland, 27 June–3 July 1984 (to be published), Invited Papers, Vol. 1, p. 473.

[10]C.-M. Ryu, Ph.D. dissertation, Princeton University, 1983 (unpublished).

[11]W. N. Hugrass, Aust. J. Phys. **38**, 157 (1985).