Tensor Polarization of the Deuteron in Elastic $e^- d$ Scattering

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It is argued that neither spin observables in electron-deuteron scattering nor form factors can serve as an unambiguous testing ground of predictions of perturbative QCD in the momentum-transfer region of a few GeV/c. The calculations based on nucleon and Δ-isobar degrees of freedom give results similar to those of perturbative QCD. The influence of Δ isobars on the short-range behavior of the nucleon components of the deuteron wave function is stressed.

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Quantum chromodynamics (QCD) is considered to be the fundamental theory for strong interactions and it has often been asked if there are clear manifestations of the explicit quark and gluon degrees of freedom in nuclei. The answer is not simple. It is found,[1] for example, that the asymptotic falloff of the deuteron form factor $A^{1/2}(Q)$ at large values of Q may be deduced, from perturbative QCD, to be Q^{-10}. It was suggested[1,2] that this asymptotic region may already be in the nuclear-physics domain $Q \approx 5$–$10\ \mathrm{fm}^{-1}$. Unfortunately, an effective theory of strong interactions based on nucleons and mesons leads to a similar falloff at large Q if appropriate momentum dependence at each of the vertices is assumed.[2]

Recently it has also been suggested[2] that a distinctive signature may be found in a measurement of the spin variables in elastic electron-deutron scattering. It was shown,[2] for example, that the ratio of vector to tensor polarization (t_{21}), in the Q region 5–10 fm^{-1}, was negative when calculated in the traditional approach with several different models of the deuteron wave function, and was positive when calculated within perturbative QCD. The QCD value of this ratio comes from the unique prediction of perturbative QCD that

$$x = \frac{2}{3}\eta \frac{G_q(Q)}{G_c(Q)} \underset{Q^2 \to \infty}{\longrightarrow} 1, \tag{1}$$

where $G_q(Q)$ and $G_c(Q)$ are the deuteron quadrupole and charge form factors, respectively, and $\eta = Q^2/4M_d$ with M_d the deuteron mass. In perturbative QCD the tensor polarization parameter t_{20} in the limit $x \to 1$ has a value of $-\sqrt{2}$ for a particular choice of the kinematical conditions (see discussion later), while the traditional approach gives values which are positive in the Q range of 5–10 fm^{-1}. There is then apparently a striking contrast between the results from perturbative QCD and the nucleons-only wave functions of the deuteron. It has been argued that this may well serve as the distinctive feature that distinguishes the QCD region from that where theories with nucleons and mesons are applicable.

To study this second region in a consistent way we have developed a nonrelativistic model of coupled nucleons and Δ isobars (lowest Δ isobar at 1232 MeV) within the one-boson-exchange potential approach. This system may be interpreted as a lowest-order (or effective-channel) approximation to the more general system of coupled nucleons, Δ's, and all their excited states derived within QCD from a six-quark wave function.[3] In our model the interaction between nucleons and between isobars, and transition potentials between nucleons and isobars, are expressed in terms of a one-boson-exchange potential that is an extension of the Bonn potential,[4] where the isobar degrees of freedom are not retained explicitly. The parameters of the potential, i.e., coupling constants, cutoff parameters, and mass of the isoscalar-scalar particle (σ), are varied slightly from those in the Bonn potential to accommodate the isobar degrees of freedom. The relationship between nucleon and isobar coupling constants is taken from the quark model. With this potential we solved the coupled Schrödinger equations to obtain a reasonable description of the nucleon-nucleon scattering phase shifts up to 300 MeV and simultaneously the ground-state properties of the deuteron. The details of the calculations will be published separately,[5] but a few comments about our model should be made here. As in the Bonn model[4] with nucleons only or in other models with isobar degrees of freedom included,[6] the predictions of our model are not unique because of uncertainties in parameters of the potential. The cutoff parameters, for example, which determine the short-range part of the potential, are not severely restricted by the experimental phase shifts and the deuteron properties. As a result several sets of potential parameters may give equally good descriptions of the available experimental nucleon-nucleon data. Further restrictions on parameters of the model can come only from testing the model in few-body systems.

Here we consider two models for the deuteron wave function with probabilities 1.27% and 0.33% of the Δ

TABLE I. The nucleon (NN) and isobar ($\Delta\Delta$) component percentage of the deuteron wave function.

	NN (%)		$\Delta\Delta$ (%)						
	$^3D_1^N$	$^3S_1^N$ 0–1 fm	$^3S_1^\Delta$	$^3D_1^\Delta$	$^7D_1^\Delta$	$^7G_1^\Delta$	Sum	Sum 0–1 fm	Nodes in $^3S_1^\Delta$ and $^3D_1^\Delta$
B1	5.48	7.93	1.06	0.02	0.18	0.01	1.27	1.14	Yes
B2	5.85	6.99	0.04	0.02	0.25	0.02	0.33	0.16	No
RSC	6.47	3.99	...	...	...	...	...	...	...

component (called B1 and B2, respectively). In Table I we specify probabilities of nucleon and isobar components explicitly. For comparison purposes we list in Table I the relevant characteristics of the Reid soft-core (RSC) deuteron wave function.[7] The percentage of D^N state in all three models is similar. The isobar components of the wave function have quite different characteristics in the two models: (1) The total probability of isobar component is almost four times larger in model B1 than B2; (2) the dominant state in model B1 is $^3S_1^\Delta$ while in model B2 the dominant state is $^7D_1^\Delta$; (3) almost the whole strength of the Δ components in model B1 is accumulated at small distances ($r < 1$ fm) while it is equally distributed between regions $r < 1$ fm and $r > 1$ fm in model B2; and (4) the $^3S_1^\Delta$ and $^3D_1^\Delta$ components in model B1 have nodes at small distances ($r \simeq 0.5$ fm). The presence of these nodes is related to the coupling strength among various coupled states. The particular behavior of the isobar components of the wave function has a large impact on the short-range behavior of the nucleon components of the wave function—mainly on the S state. While probabilities of the D^N state at small distances ($r \leqslant 1$ fm) are almost the same for all three models ($\sim 0.7\%$), the probabilities of the nucleon S state in this region is largest in model B1, two times larger than in the RSC model. These differences have a large implication for large-Q behavior of the form factors and t_{20}. We have checked for example that the differences in form factors between the predictions of the three models in nucleon-only approximation are due entirely to the differences in small-range behavior of the deuteron wave function ($r \leqslant 1$ fm). Almost all studies[6] of the role of isobars in the deuteron reported so far have considered isobar wave functions similar to our B2 model (although frequently with higher probability).

The charge, the quadrupole, and the magnetic form factors of the deuteron have been calculated with the wave functions discussed above and the experimental data are well described.[5] The isobar contributions to the form factors were evaluated as in Ref. 6 and the corrections due to the meson-exchange correction (MEC) (π-pair and $\rho\gamma\pi$) have been included according to the model developed by Gari and Hyuga.[8] The dipole form was used for isoscalar nucleon electric and magnetic form factors (neutron electric form factor was neglected). For Δ's we used the same form factors as for nucleons.

The tensor polarization t_{20} of the recoiling deuteron in elastic $e^- d$ scattering can be written

$$t_{20} = -\sqrt{2}\frac{x(x+2)+y/2}{1+2(x^2+y)}, \tag{2}$$

where $y(Q) = \frac{2}{3}\eta[G_m(Q)/G_c(Q)]^2 f(\theta)$ with $f(\theta) = \frac{1}{2} + (1+\eta)\tan^2(\theta/2)$. Here $G_m(Q)$ is the magnetic form factor of the deuteron and θ is a scattering angle. In perturbative QCD G_m falls like Q^{-12} and F_c falls like Q^{-10}. With an appropriate choice of θ, we have $y \cong 0$ and, in the limit $x \to 1$, $t_{20} \to -\sqrt{2}$.

In Fig. 1 we show t_{20} calculated for $\theta = 70°$ within the models discussed above. The three curves for each model (two for RSC) represent the three different approximations in the evaluation of the form factors: (i) nucleon-only components of the deuteron wave function are taken into account (N), (ii) full ($N+\Delta$) wave functions are considered, and (iii) full wave functions + MEC corrections are used.

The predictions of all the models at small Q are very similar and all fit the experimental data quite well. The role of isobars and MEC at small Q is negligible. At larger Q (> 3 fm^{-1}) the predictions of the models are different already in approximation (i). In the RSC model t_{20} oscillates as a function of Q and in models B1 and B2 the t_{20} curves are smooth at large Q and assume different asymptotic values in the two models.

In approximation (ii) the effect of isobars is large in model B1 and is small in model B2 for Q in the range 4–20 fm^{-1}. We want to stress here that the large effect of the isobars in model B1 is determined mainly by the characteristics (2)–(4) listed earlier and not by the total probability of the Δ components. To check this we made test calculations within model B2 with renormalized Δ wave functions (1.27% probability as in model B1) and we obtained a prediction for t_{20} very similar to that of the original model B2 [Fig. 1(b)] and not to that of model B1 [Fig. 1(c)].

The MEC corrections introduce substantial modifications to the calculated t_{20}. They smooth the curve in the RSC model giving a small negative value of t_{20} at

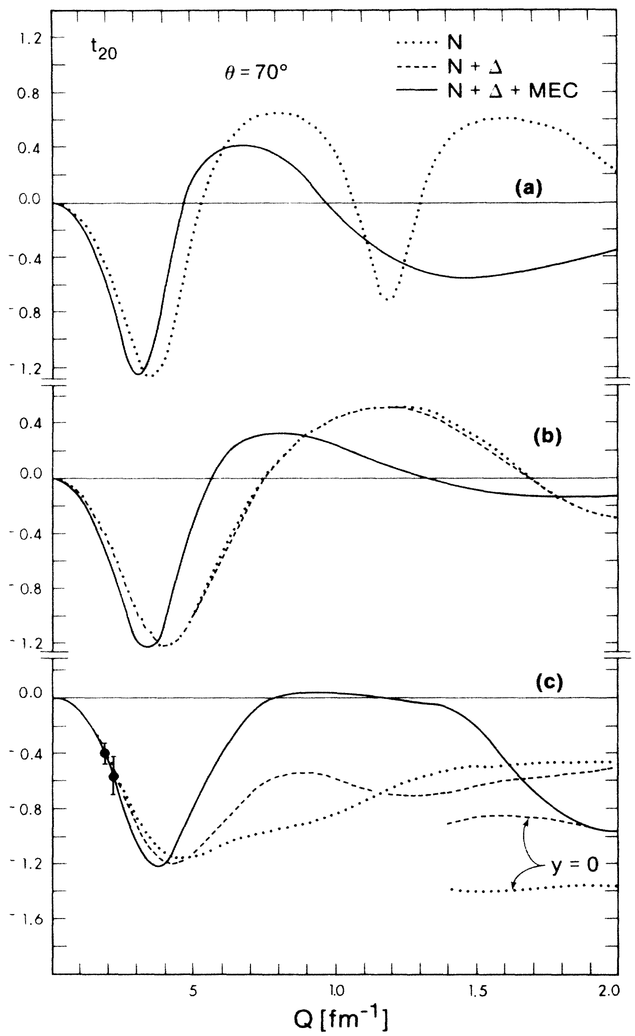

FIG. 1. The tensor polarization t_{20} in $e^- d$ elastic scattering for different models of the deuteron wave function: (a) RSC, (b) B2, and (c) B1 (based on one-boson-exchange potentials). The three approximations are N, nucleon-only component of the wave function, $N+\Delta$, full wave function (nucleons + isobar), and $N+\Delta+$MEC, meson exchange correction added. All curves were calculated according to Eq. (2) except two additional curves in (c) calculated with approximation $y=0$. See the text for details. (The experimental data are from Schulze *et al.*, Ref. 9.)

large Q. Similarly a small negative value is predicted in model B2 for large Q. In model B1 the predicted asymptotic value is also negative but relatively large (-0.9).

In Fig. 1(c) we have plotted, in addition to the three curves representing approximations (i)-(iii), predictions of model B1 for special kinematic conditions $y=0$. One can see that our nucleon-only approximation gives for $Q > 15$ fm^{-1} values of t_{20} very close to $-\sqrt{2}$, i.e., the perturbative QCD prediction. The value of t_{20} for our nucleon-plus-isobar approximation seems to approach this limit at higher Q.

In conclusion, we have presented model calculations with coupled nucleon and isobar degrees of freedom. The results shown in Fig. 1 indicate clearly that even within this model there can be considerable variation in the magnitude of t_{20} and even more dramatic differences from the results of RSC potentials that consider only nucleon degrees of freedom. There is no doubt that we would like to establish a connection with the perturbative QCD calculations,[2] but it is not obvious at what value of Q they will be valid. But within the simple models considered here the magnitude of t_{20} at large values of Q is negative and comparable to the results of perturbative QCD.

Before closing we would like to make a few comments about the present model calculations.

(a) The behavior of t_{20} at large Q depends largely on the deuteron wave function for $r \leq 1.0$ fm. There are numerous attempts[10] to cut off the nucleon components of the wave function at some radial distance and to introduce a six-quark wave function. The present model has wave functions that arise naturally from the strength of the coupling within a conventional description of baryons and mesons.

(b) Our calculations are nonrelativistic in nature and may not be quantitative at high momentum transfer. The relativistic corrections[11] modify form factors at large Q. Our preliminary calculations with a relativistic deuteron wave function (nucleon components only) give a picture qualitatively similar to that of the RSC model.

(c) The deuteron form factors depend sensitively on the proton and the neutron form factors at large Q where they are not known very well. However, the effect of the nucleon form factors cancels out to a large extent because only ratios of form factors enter the t_{20} formula.

(d) The results depend strongly on the contribution due to MEC. At larger Q, the MEC is the dominant part of the form factors.

(e) The experimental determination of t_{20} is in progress[12] and it is hoped that it will be known for $Q \leq 5$ fm. This should help to determine the short-distance behavior of the deuteron wave function because at $Q = 5$ fm^{-1} the predictions of the models discussed are quite different.

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