New Precision Determination of the π^- Mass from Pionic X Rays

B. Jeckelmann and T. Nakada
Institut de Physique de l'Université de Fribourg, CH-1700 Fribourg, Switzerland

W. Beer, G. de Chambrier, O. Elsenhans, K. L. Giovanetti, P. F. A. Goudsmit, H. J. Leisi, and A. Rüetschi
Institute for Intermediate Energy Physics, Eidgenössische Technische Hochschule Zürich, Schweizerisches Institut für Nuklearforschung, CH-5234 Villigen, Switzerland

and

O. Piller and W. Schwitz
Eidgenössisches Amt für Messwesen, CH-3084 Wabern, Switzerland
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The wavelength of the pionic $4f$-$3d$ transition in ^{24}Mg has been measured with a curved-crystal spectrometer. The new value of the pion-electron mass ratio is deduced to be $m_{\pi^-}/m_{e^-} = 273.126\,77(71)$. The consequence of this value for the rest mass of the muon neutrino is discussed.

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The work described in this paper was carried out with the aim of determining the pion-electron mass ratio with the highest precision available. Such an accuracy is required for present efforts in the determination of the mass of the muon neutrino from measurements of p_{μ^+} in the decay of stopped pions[1] and for all precision experiments with pionic x rays, as for example the search for long-range van der Waals–type forces between hadrons.[2]

The pion-electron mass ratio was determined from a wavelength measurement of the pionic $4f$-$3d$ transition (25.9 keV) in ^{24}Mg. This transition was selected for the following reasons: (1) The transition energy is ideal for a crystal-spectrometer measurement. (2) The theoretical corrections to the point-nucleus Klein-Gordon energy of the transition, such as the strong-interaction shift, the vacuum polarization, and the screening of a known electron configuration, are small and the uncertainties in the calculation are negligible compared to the experimental errors. (3) The number of K electrons present during the pionic transition can be determined experimentally from the measured line shape of the pionic-x-ray peak by virtue of the extreme energy resolution of the crystal spectrometer in the 20-keV region. (4) A calibration line lying very close to the chosen pionic x ray ($\Delta E = 0.3$ keV) is available, namely the 25.6-keV γ transition in the decay of ^{161}Tb.

The measurement was performed with the Swiss Institute for Nuclear Research crystal spectrometer especially designed for the pionic-x-ray work. The instrument and the experimental details are described elsewhere.[3–5]

In a first step the calibration line (25.6-keV γ line, ^{161}Tb) was related to a standard of the National Bureau of Standards and its wavelength was determined with a precision of 1.3 ppm. The result of this part of the experiment is presented and discussed in a recent publication.[4]

Secondly, the line shape of the pionic $4f$-$3d$ transition in ^{24}Mg was measured with a 2.1-mm-thick (110) quartz crystal in third order of diffraction ($\theta_{\text{Bragg}} = 17°$). The target consisted of metallic $^{\text{nat}}$Mg. The response function of the spectrometer was measured with the 25.6-keV Tb line.

In a third step the wavelength of the pionic $4f$-$3d$ transition in ^{24}Mg was determined by measurement of the Bragg reflections of the pionic line and the 25.6-keV calibration line for both positive and negative angles. A 1.7-mm-thick quartz crystal reflecting from the (310) planes and the 2.1-mm-thick (110) crystal in second order of diffraction were used. The target consisted of pressed metallic ^{24}Mg powder (purity = 99.7%).

The distribution of the K electrons during the observed pionic transition was obtained from the data of step 2 in the following way: For an ideal resolution and zero natural width we would see three different x-ray lines separated by ~ 0.5 eV which correspond to the cases of having two, one, or zero K electrons present. In reality, as a result of the limited resolution of the instrument and the finite linewidth the pionic line profile looks as shown in Fig. 1. The width of this peak is $(13 \pm 3)\%$ larger than the width of the spectrometer response function folded with the natural linewidth of the transition, giving a clear evidence of the underlying structure.

The analysis of the line shape of Fig. 1 allows two al-

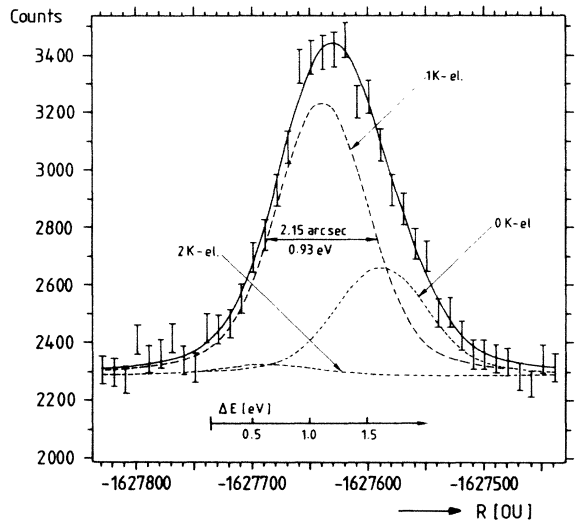

FIG. 1. Bragg reflection of the $4f$-$3d$ transition in pionic ^{24}Mg measured with a (110) quartz crystal in third order of diffraction. R is the reading of the angle-measuring system in optical units. The fitted function is shown by the solid line; it is the sum of three individual peaks corresponding to the cases of two, one, and zero K electrons present during the pionic transition. The line shape of the peaks is obtained by folding the response function of the spectrometer with the natural linewidth of the transition (40% of the instrumental width).

ternative interpretations. The main peak can represent the case of having either one or two K electrons present during the pionic transition. In order to eliminate this ambiguity a separate measurement was performed[6] on the relative intensities of the $4f$-$3d$ and $3d$-$2p$ transitions in pionic Mg. Whereas the intensity of the first transition is dependent—through the Auger process—on the same K-electron population, the second transition is almost purely radiative. This intensity measurement resulted in a value of 0.44 ± 0.30 K electron present during the transition. This result clearly rules out the second of the above two possibilities (corresponding to roughly 1.70 ± 0.15 K electrons) whereas it is still in agreement within 1 standard deviation with the first one (see below). Knowing, thus, that the strongest component corresponds to the one-K-electron case we fitted the measured reflection, as shown in Fig. 1, with the line shapes of three different peaks, the distance between them being fixed by the calculated values. A constant background was assumed. From the fit we derive a probability of $(3 \pm 3)\%$ for having two K electrons, $(70 \pm 6)\%$ for one K electron, and $(27 \pm 8)\%$ for an empty K shell during the pionic transition.

Combining this result with the result of the wavelength measurement (step 3) we find the wavelength of the pionic $4f$-$3d$ transition with one K electron to be

$$\lambda_{\text{expt}}^{(1K)} = 47.838\,23(11)\,\text{pm}. \quad (1)$$

Table I gives the details of the result. The different sources of error and their estimates are listed in Table II. Figure 2 shows the angular spectrum of the pionic x ray measured with the (310) quartz crystal.

As a result of perfect agreement between the results with the two different crystals (see Table I) we conclude that multiple diffraction effects were absent in the present experiment. The possibility that a weak, unnoticed γ line lying under the x-ray peak falsified the measurement was investigated as described in Ref. 5 and the work of Giovanetti and Sergeev.[7] An upper limit for the deviation of the x-ray peaks due to the

TABLE I. Experimental wavelength of the pionic $4f$-$3d$ transition in ^{24}Mg with one K electron present. The calibration wavelength used is λ (^{161}Tb, 26 keV) = 48.334 488(64) pm (Ref. 4). The different contributions to the errors are given in Table II.

	$\lambda_{Mg}/\lambda_{Tb}$	λ_{Mg} (pm)
(110) crystal	0.989 732 3(19)	
(310) crystal	0.989 733 3(16)	
Weighted average	0.989 732 9(12)	47.838 23(11)

TABLE II. Contributions to the wavelength uncertainty (one-standard-deviation errors).

Source of uncertainty	Error (ppm)	
	$\lambda_{Mg}/\lambda_{Tb}$	
	(110) crystal	(310) crystal
Statistics, Mg line	1.41	1.02
Statistics, calibration line	0.40	0.37
Source high effect	0.03	0.03
Nonlinearity of the spectrometer	0.02	0.02
Background inclination, Mg spectra	1.30	0.90
Inhomogeneous π-stopping distribution in the target	0.13	0.60
Temperature fluctuations	0.10	0.30
Combined uncertainty	1.97	1.57
	λ_{Mg}	
Error from above for the mean value of $\lambda_{Mg}/\lambda_{Tb}$	1.24	
Line-shape analysis of the Mg reflection (Fig. 1)	1.40	
Wavelength error of the calibration line	1.33	
Combined uncertainty (quadratic sum)	2.30	

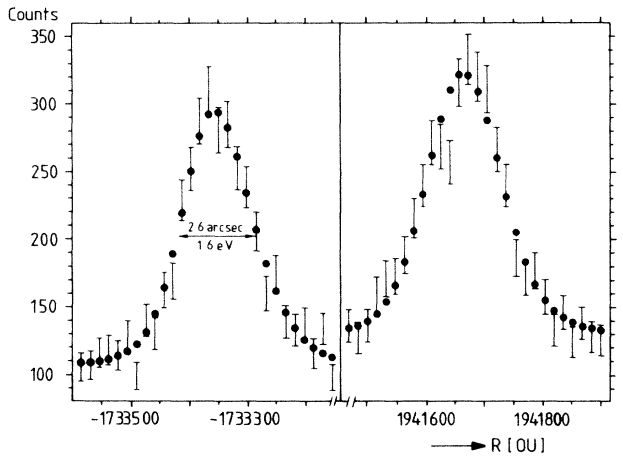

FIG. 2. Reflection pair of the $4f$-$3d$ transition in pionic ^{24}Mg measured with a (310) quartz crystal. The fitted function is indicated by dots. R is the reading of the angle-measuring system in optical units.

possible presence of hidden γ lines was taken into account in the determination of the experimental error (see Table II).

The measured wavelength (1) is related to visible-light reference standards through the relation to a National Bureau of Standards γ-ray reference line.[4,8] In general, visible transitions are expressed in terms of the Rydberg constant R_∞ which is known to a high precision [$R_\infty = 109\,737.314\,76(32)$ cm^{-1}].[9] It is thus physically significant to base the present measurement on the Rydberg wavelength scale. In this framework the point-nucleus Klein-Gordon wavelength $\lambda_0^{i\text{-}f}$ of the pionic $4f$-$3d$ transition has to be expressed as follows:

$$\lambda_0^{i\text{-}f} = \lambda_{\text{expt}}^{(1K)} + \Delta\lambda = \frac{1}{R_\infty Z^2}\,\frac{m_{e^-}}{m_{\pi^-}}\left(1+\frac{m_{\pi}}{M}\right)L^{i\text{-}f}, \quad (2)$$

TABLE III. Corrections to the point-nucleus transition energy (E_0) of the $4f$-$3d$ transition in pionic ^{24}Mg. $E_{\text{expt}}^{(1K)} = E_0 + \Delta E$ ($E_{\text{expt}}^{(1K)}$ is the measured pionic $4f$-$3d$ transition energy with one K electron present).

Correction	ΔE (eV)
Vacuum polarization $\alpha(Z\alpha)$ for a finite nuclear charge distribution	50.070(2)
Vacuum polarization $\alpha^2(Z\alpha)$	0.365(1)
Vacuum polarization $\alpha(Z\alpha)^{3,5}$	−0.039(4)
Vertex correction	0.004
Strong interaction	0.040(4)
Relativistic recoil	0.053
Nuclear recoil	−0.008(8)
Screening of one K electron	−0.501(5)
Screening of L_{I} electrons	−0.030(30)
Total	49.954(32)

where M is the mass of the nucleus and $L^{i\text{-}f}$ a known function of n, l, and $(Z\alpha)$.[2,3]

All relevant correction terms ($\Delta\lambda$) which are necessary for the evaluation of the point-nucleus wavelength $\lambda_0^{i\text{-}f}$ from the measured wavelength $\lambda_{\text{expt}}^{(1K)}$ are given in Table III. The first-order vacuum-polarization shift and the strong-interaction shift were calculated by a numerical solution of the Klein-Gordon equation with standard potentials (computer code BIPA[10]). The higher-order vacuum-polarization, vertex, and relativistic-recoil effects were calculated in first-order perturbation theory. The energy shift due to the screening effect of different electron configurations for the pionic $4f$-$3d$ transition in magnesium was calculated by Fricke and Rashid[11] on the basis of a self-consistent Dirac-Fock procedure,[12] the accuracy being better than 1%.[11] From the measured population of the electronic K shell we conclude that probably only one L_{I} electron is left in the L shell during the pionic $4f$-$3d$ transition (for details see Jeckelmann *et al.*[13]). By conservatively taking 0.5 ± 0.5 of the screening effect of the two L_{I} electrons into account we find the value given in Table III.

The sum of all the corrections listed in Table III in wavelength units gives

$$\Delta\lambda = 0.092\,384(59) \text{ pm}. \quad (3)$$

With the results of (1) and (3) and with use of Eq. (2) we get, finally, for the pion-electron mass ratio

$$m_{\pi^-}/m_{e^-} = 273.126\,77(71). \quad (4)$$

With $m_e = 0.511\,003\,4(14)$ MeV/c^2 we obtain for the pion mass

$$m_{\pi^-} = 139.568\,71(53) \text{ MeV}/c^2. \quad (5)$$

As a result of the uncertainty of the constant hc, the error of m_π is increased by this procedure from the error of 2.6 ppm in the ratio m_π/m_e to 3.8 ppm.

In Table IV our mass value is compared with the best previous measurements. The values of

TABLE IV. Comparison of the π^- mass with previous publications.

	Mass (MeV/c^2)	Error (ppm)	Ref.
(a)	139.568 71(53)(36)[a]	3.8/2.6[a]	This work
(b)	139.567 50(90)	6.4	14
(c)	139.566 70(240)	17	15,16
(d)	139.568 60(200)	14	17
(e)	139.570 40(110)[b]	7.9	1

[a]The error from the uncertainty of the wavelength-energy conversion and the purely experimental error are both given. For comparison with the crystal-spectrometer measurements (b) and (c) the experimental error should be used.

[b]Assuming an upper limit of 40 eV/c^2 for the muon-neutrino mass.

Marushenko *et al.*[16] (c) and Carter *et al.*[17] (d) are in excellent agreement with our work; the more accurate mass of Lu *et al.*[14] (b), measured with a curved-crystal spectrometer, differs by 1.2 standard deviations.

With the new pion-mass value we reevaluate the square of the mass of the muon neutrino using the recently published muon momentum in the $\pi^+ \rightarrow \mu^+ \nu_\mu$ decay at rest[1] and assuming the π^+ and π^- masses to be equal. The result is

$$m_{\nu_\mu}^2 = (-0.097 \pm 0.072)\ \mathrm{MeV}^2/c^4. \quad (6)$$

The error originates dominantly from the p_μ measurement. The previous value is $m_{\nu_\mu}^2 = -0.163(80)$ MeV^2/c^4.[1] For the upper limit of the muon-neutrino mass we deduce from (6)

$$m_{\nu_\mu} < 0.27\ \mathrm{MeV}/c^2\ (90\%\ \text{confidence level}). \quad (7)$$

The authors of Ref. 1 interpret their result alternatively as a π^+ mass determination by assuming an upper limit of 40 eV/c^2 for the muon-neutrino mass (from cosmological arguments). This leads to the result given in Table IV (e) which differs [as can be seen directly in Eq. (6)] by 1.4 standard deviations from our value.

In the present work we have measured the π^- mass with a precision improved by a factor of 2.5 as compared to the best previous experiment. With this new result the uncomfortably large negative value for the square of the muon-neutrino mass is reduced. Any further improvement of the ν_μ mass limit requires an improved p_μ measurement. The new precision in the π^- mass is urgently needed for a current analysis of the π-nucleus optical potential with recent crystal-spectrometer data[5] and for a search of strong van der Waals interactions.[2]

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