Splay Rigidity in the Diluted Central-Force Elastic Network

Percolation in a network of central-force springs has unusual properties. The first which was noted is that connectivity and rigidity thresholds are different[1]: For the two-dimensional triangular lattice, $p_c = 0.3473$ in the former case and[2] $p_{\mathrm{TR}} = 0.65 \pm 0.005$ in the latter. It was recently suggested by Wang and Harris[3] that yet another distinct threshold exists in that model, namely, the splay-rigidity threshold p_{SR} which would obey the relation $p_c < p_{\mathrm{SR}} < p_{\mathrm{TR}}$. Using series expansion techniques, Wang and Harris found $p_{\mathrm{SR}} = 0.61 \pm 0.02$.

We consider a triangular lattice with L triangular cells across each of two perpendicular directions, i.e., which contains $L \times L$ cells total. The top sites are attached to a rigid bus bar with infinite elastic moduli and the bottom sites to a separate identical bar. Pure unit torques of opposite directions are applied to the upper and lower bars. When the restoring force, as calculated numerically with use of the method of Ref. 2, is different from zero for the above external stresses, the sample is considered splay rigid. Every sample is also tested for compression rigidity.

Figure 1 plots the probability, as a function of the bond occupation probability p, that a sample is splay rigid but not compression rigid. Three system sizes $L = 6$, 10, and 16 are considered. The probability of being only splay rigid systematically decreases with increasing system size for all values of p. In the infinite-size limit, these results might extrapolate to a single delta function centered at $p \sim 0.65$ but not to the "inverted square well" function that one might have expected in light of the results of Ref. 3. There is no evidence for a new fixed point at $p_{\mathrm{SP}} \sim 0.61$.

Figure 2 displays histograms for the value of the Frank elastic constant calculated for the splay-rigid samples at $p = 0.63$. The arrows indicate the result that one would obtain for a single column of rhombi for each of the two system sizes considered. Larger values of the elastic constant come from samples whose splay rigidity is determined mainly by totally rigid clusters. The contribution of rhombi also seems to be decreasing with increasing system size.

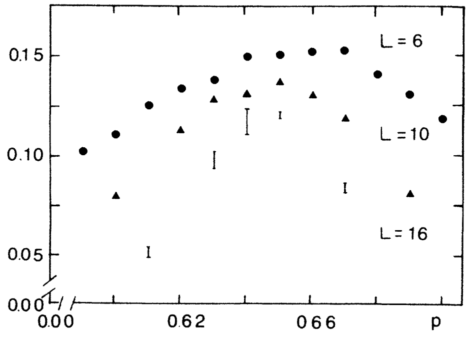

FIG. 1. Probability that a Monte Carlo–generated sample is splay rigid and not compression rigid as a function of the bond occupation probability p. More than N_s samples per value of p were considered. Circles, $L = 6$, $N_s = 8000$; triangles, $L = 10$, $N_s = 6000$; error bars, $L = 16$, $N_s = 1000$.

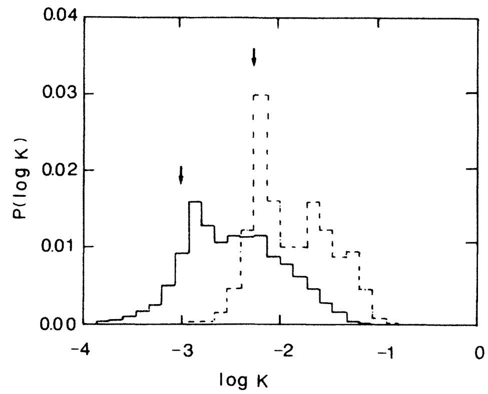

FIG. 2. Histogram $P(\log_{10}K)$ as a function of $\log_{10}K$, where the generalized elastic constant K equals $N^2/(2EL^2)$, with N being twice the torque on a single bus bar and E the corresponding elastic energy. Broken line for $L = 16$, $p = 0.63$, 24000 samples; solid line for $L = 10$, $p = 0.63$, 20000 samples. The surface under each histogram equals the corresponding probability defined in Fig. 1.

To conclude, numerical simulations show no evidence for a separate splay-rigidity threshold. One should keep in mind that the large correlation length associated with the p_{TR} threshold could be masking the effect in the range of p and L considered here but then similar problems should presumably have appeared in the series expansion.[3]

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R. R. Tremblay, A. R. Day, and A.-M. S. Tremblay
Département de Physique
et Centre de Recherche en Physique du Solide
Université de Sherbrooke
Sherbrooke, Québec J1K 2R1, Canada

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