Scale-Invariant Hypercolor Model and a Dilaton

Koichi Yamawaki, Masako Bando,[(a)] and Ken-iti Matumoto[(b)]
Department of Physics, Nagoya University, Nagoya 464, Japan
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We propose a scale-invariant hypercolor model with a nontrivial ultraviolet fixed point having large anomalous dimension, which resolves the notorious flavor-changing neutral-current problem in hypercolor models, and at the same time predicts a $J^{PC}=0^{++}$ Nambu-Goldstone boson (dilaton) associated with the spontaneous breakdown of the scale invariance.

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Hypercolor (HC)[1] is a very attractive idea to account for the spontaneous breakdown of the electroweak gauge symmetry in a natural way. However, if one wishes to obtain a realistic mass spectrum of quarks and leptons based on this idea, one must necessarily introduce transitions between the quarks or leptons and the hyperfermions. A typical way to do it without elementary scalars is extended hypercolor (EHC)[2] which connects the quarks and leptons with the hyperfermions via exchange of the extended HC (sideways) gauge bosons. However, the idea unfortunately leads to a phenomenological disaster, namely, excessive flavor-changing neutral currents (FCNC's).[3] The same is also applied to the hypercolored preon model[4] in which the hyperfermions are composites on the same footing as the quarks and leptons, the preonic dynamics (subcolor) being responsible for such transitions among the composites.

In this Letter, we propose an asymptotically *non*free scale-invariant HC model, which will be found to have a nontrivial ultraviolet fixed point. The appearance of large anomalous dimension at the fixed point in this model automatically resolves the FCNC syndrome,[5] which has long been a central problem of any sensible HC model. In addition, a novel feature of this HC model is a prediction of the dilaton (referred to as "hyperdilaton" hereafter), a new type of Nambu-Goldstone (NG) boson associated with the spontaneous breakdown of the scale invariance. This hyperdilaton, having $J^{PC}=0^{++}$, should be an outstanding low-energy signature of the HC of this kind, in sharp contrast to the asymptotically free HC models whose low-energy signature is the existence of light 0^{-+} pseudo-NG bosons (hyperpions) associated with the spontaneous breakdown of the chiral symmetry.

We start with a HC model of general type which may be regarded as a low-energy effective theory of the EHC or the hypercolored preon model. Below the mass scale, Λ_S, of the EHC (sideways) or preonic dynamics (subcolor), the Lagrangean may be written as

$$\mathscr{L}=\mathscr{L}_{\rm HC}+\mathscr{L}', \quad (1)$$

$$\mathscr{L}'=G_1\bar{F}F\bar{F}F+G_2\bar{F}F\bar{f}f+G_3\bar{f}f\bar{f}f, \quad (2)$$

where $\mathscr{L}_{\rm HC}$ is the HC Lagrangean consisting of hyperfermions and hypergluons, F and f stand for the hyperfermions and the quarks or leptons, respectively, and G_1, G_2, and G_3 are the coupling constants of the chirally invariant four-fermion interactions of order $O(1/\Lambda_S^2)$.

Let us consider the $\mathscr{L}_{\rm HC}$ part first. The fermion contents of this model are arranged in such a way that the HC is asymptotically *non*free,[6] in which case the dynamical feature of the model may be characterized by the truncation to an effective Abelian gauge theory, massless QED, which is also asymptotically nonfree. We are mainly interested in the spontaneous breakdown of the chiral symmetry and the scale symmetry and thus are concerned with the hyperfermion self-energy $\Sigma(p)$. In the case at hand, the Schwinger-Dyson (SD) equations for $\Sigma(p)$ may be simulated by those of the QED with vanishing fermion mass in the ladder approximation.

In Euclidean space, the SD equation of the ladder QED is written in the Landau gauge as

$$\Sigma(p)=m_0+\frac{3g^2}{(2\pi)^4}\int d^4q\,\frac{1}{(p-q)^2}\,\frac{\Sigma(q)}{q^2+\Sigma(q)^2}, \quad (3)$$

where m_0 and g are the fermion bare mass and the gauge coupling constant, respectively. Equation (3) has been extensively studied by many authors[7-9]: Solutions to the homogeneous SD equation in the weak-coupling phase[7] ($\alpha\equiv g^2/4\pi<\alpha_c=\pi/3$) correspond to explicit chiral-symmetry breaking,[8,9] whereas in the strong-coupling phase ($\alpha>\alpha_c$) there exist solutions of the spontaneous breakdown of the chiral symmetry,[8,9] with $\Sigma(0)$ being nonzero for $m_0=0$. The cutoff (Λ) dependence of α of these solutions in the strong-coupling region was obtained in Ref. 9, yielding a negative β function, $\beta(\alpha)\equiv\Lambda\,\partial\alpha(\Lambda)/\partial\Lambda<0$ for $\alpha>\alpha_c$; namely, the critical point $\alpha=\alpha_c$ should be regarded as a nontrivial ultraviolet fixed point. This interpretation has recently been made clear[10] in connection with studies of the lattice QED.[11]

Actually the β function of the ladder QED takes the form[10,12]

$$\beta(\alpha)=0 \quad (\alpha<\alpha_c) \text{ (Ref. 13)}, \tag{4}$$

$$\beta(\alpha)=-\tfrac{2}{3}(\alpha/\alpha_c-1)^{3/2} \quad (\alpha>\alpha_c), \tag{5}$$

which is depicted in Fig. 1(a), indicating clear evidence for a nontrivial ultraviolet fixed point at $\alpha=\alpha_c=\pi/3$.

Accordingly, the anomalous dimension, $\gamma(\alpha) \equiv 3-d_{\bar{\psi}\psi}$, of the fermion bilinear operator $\bar{\psi}\psi$ with dimension $d_{\bar{\psi}\psi}$ is given by

$$\gamma(\alpha)=1-(1-\alpha/\alpha_c)^{1/2} \quad (\alpha<\alpha_c), \tag{6}$$

$$\gamma(\alpha)=1 \quad (\alpha>\alpha_c), \tag{7}$$

which is shown in Fig. 1(b); the asymptotic behavior of the solution to the SD equation in the renormalized form thus reads[14,9]

$$\Sigma(p)\sim m(p/\mu)^{-1+(1-\alpha/\alpha_c)^{1/2}} \quad (\alpha<\alpha_c), \tag{8}$$

$$\Sigma(p)\sim\{m+\mu\exp[-2\pi/(\alpha/\alpha_c-1)^{1/2}]\} \times(p/\mu)^{-1} \quad (\alpha>\alpha_c), \tag{9}$$

where

$$\mu\exp[-\pi/(\alpha/\alpha_c-1)^{1/2}]\equiv\Lambda_{\text{QED}}$$

is a scale parameter of QED in the strong-coupling phase[15] and hence $\Sigma(p)\sim\Lambda_{\text{QED}}^2/p$ in the chiral-symmetry limit ($m=0$). Equations (8) and (9) satisfy the renormalization-group equation,

$$\left[\mu\frac{\partial}{\partial\mu}+\beta(\alpha)\frac{\partial}{\partial\alpha}-\gamma(\alpha)m\frac{\partial}{\partial m}\right]\Sigma(p)=0,$$

with $\beta(\alpha)$ and $\gamma(\alpha)$ given by (4)-(7).

Now, in the asymptotically nonfree HC model, if we adopt the ladder approximation, the SD equation takes the same form as the above up to numerical factors ($m=0$ in this case). Thus we suggest that the above result in the ladder QED remains essentially the same especially in the strong-coupling phase in our HC model, the β function being of the form shown in Fig. 1(a). The strong-coupling phase $\alpha^{(\text{HC})}>\alpha_c^{(\text{HC})}$ which we are interested in may be identified with a confining phase similar to that of an asymptotically free model

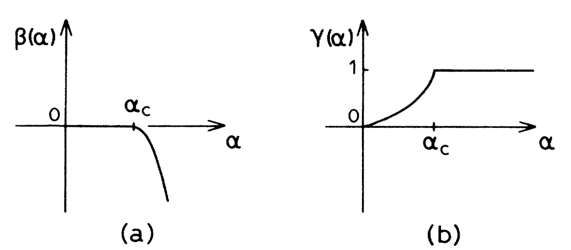

FIG. 1. (a) β function of the QED in the ladder approximation. (b) Anomalous dimension of $\bar{\psi}\psi$ of the QED in the ladder approximation.

except that $\alpha^{(\text{HC})}$ approaches a nonvanishing value $\alpha_c^{(\text{HC})}$ at the ultraviolet limit.[5] As to anomalous dimension, one may expect from (7) that $\gamma(\alpha^{(\text{HC})})$ takes a special value at the ultraviolet fixed point $\alpha^{(\text{HC})}=\alpha_c^{(\text{HC})}$ as

$$\gamma^*\equiv\gamma(\alpha^{(\text{HC})}=\alpha_c^{(\text{HC})})=1, \tag{10}$$

although the precise form of $\gamma(\alpha^{(\text{HC})})$ for $\alpha^{(\text{HC})}>\alpha_c^{(\text{HC})}$ may be modified in the calculations beyond the ladder approximation. Then the asymptotic behavior of the hyperfermion self-energy $\Sigma(p)$ is read off from (9);

$$\Sigma(p)\sim\Lambda_{\text{HC}}^2/p, \tag{11}$$

where Λ_{HC} (~ 1 TeV), like Λ_{QED}[15] in the ladder QED, stands for the scale parameter of the asymptotically nonfree HC in the strong-coupling phase. Equation (11) should be compared with the asymptotically free HC in which $\Sigma(p)\sim\Lambda_{\text{HC}}^3/p^2$ up to logarithms.[2,16]

Next we proceed to discuss the four-fermion interactions $\mathscr{L}'$ in (1) and (2). The terms $G_2\bar{F}F\bar{f}f$ and $G_3\bar{f}f\bar{f}f$ yield quark or lepton masses, $m_{q/l}$, and FCNC's, respectively, with the couplings G_2 and G_3 being of the same order, $O(1/\Lambda_S^2)$, which has been a cause of the fatal disease of HC models. Having Eqs. (10) and (11) in our HC model, however, we obtain[5]

$$m_{q/l}\sim G_2\Lambda_{\text{HC}}^2\Lambda_S\sim O(\Lambda_{\text{HC}}^2/\Lambda_S), \tag{12}$$

which is compared with those of the asymptotically free HC,[3] $m_{q/l}\sim O(\Lambda_{\text{HC}}^3/\Lambda_S^2)$. For $\Lambda_{\text{HC}}\sim 1$ TeV, (12) requires Λ_S to be $\sim 10^4$ TeV in order to produce typical quark or lepton masses of order $\sim 10^2$ MeV, thus yielding FCNC's on the order of $G_3 \sim O(1/\Lambda_S^2)\sim 10^{-8}$ TeV^{-2}, which is perfectly consistent with the present experimental limit $\sim 10^{-6}$ TeV^{-2}. Moreover, if there exist transitions between hyperquarks Q and hyperleptons L such that $\mathscr{L}_{QL}\sim(1/\Lambda_{QL}^2)\bar{Q}L\bar{L}Q$, with Λ_{QL} being either the scale of the Pati-Salam-type gauge interaction[3] or the preonic interaction ($\Lambda_{QL}=\Lambda_S$),[17] then even the lightest pseudo-NG bosons often called P^0 and P^3,[3] acquire mass of order $O(\Lambda_{\text{HC}})$ as[5]

$$m_{P^0,P^3}^2\sim\frac{1}{F_\pi^2}\left[\frac{1}{\Lambda_{QL}^2}\Lambda_{QL}^2\Lambda_{\text{HC}}^4\right]\sim O(\Lambda_{\text{HC}}^2), \tag{13}$$

where $F_\pi\sim O(\Lambda_{\text{HC}})$ is the decay constant of the hyperpions,[3] and (10) and (11) have been used. Thus there are *no light* O^{-+} pseudo-NG bosons in our HC model.

Most remarkably, the term $G_1\bar{F}F\bar{F}F$ in (2)[18] plays a vital role in the asymptotically nonfree HC model; if we consider the Lagrangean (1) with (2) in the ladder approximation, then $G_1\bar{F}F\bar{F}F$ is essential to recovering the scale invariance at the critical point (ultraviolet fixed point), as has been recently demonstrated by

Bardeen, Leung, and Love[12] in the ladder QED, whereas $G_2\bar{F}F\bar{f}f$ and $G_3\bar{f}f\bar{f}f$ are irrelevant to the SD equation.

Bardeen, Leung, and Love[12] actually considered the QED Lagrangean plus the chiral-invariant four-fermion term

$$\mathscr{L}_{4\text{-F}} = (G_0/2)[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\psi)^2],$$

the SD equation in the ladder approximation being depicted in Fig. 2. The analysis is completely the same as the previous one $(G_0=0)$[8,9] except that the bare mass m_0 is replaced by an "effective bare mass" $\bar{m}_0 = m_0 - G_0\langle\bar{\psi}\psi\rangle$, where $\langle\bar{\psi}\psi\rangle$ is determined in a self-consistent way. The result remains essentially the same as before[8,9] as to the chiral symmetry; there exists a critical point $\alpha = \alpha_c = \pi/3$ (again the ultraviolet fixed point), above which $(\alpha > \alpha_c)$ the spontaneous breakdown of the chiral symmetry takes place and the corresponding NG-boson pole $(J^{PC}=0^{-+})$ appears in in the chiral-symmetry limit $m \equiv m_0\Lambda/\mu \to 0$. On the other hand, the situation is drastically changed as to the scale invariance[12]: Although the scale invariance is also to be spontaneously broken in the strong-coupling phase $(\alpha > \alpha_c)$ together with the chiral symmetry, there is no dilaton pole $(J^{PC}=0^{++})$ in pure QED $(G_0=0)$. However, the inclusion of the four-fermion term $(G_0 \neq 0)$ makes the theory scale invariant at the critical point, $\alpha \to \alpha_c + 0$ *and* $G \equiv G_0\Lambda^2/\pi^2 \to G_c = 1$ $(\Lambda \to \infty)$, at which point the dilaton pole does in fact exist in the $J^{PC}=0^{++}$ channel of the fermion-antifermion scattering amplitude in the chiral-symmetry limit $m=0$.

Now, the above result can be readily translated into our asymptotically nonfree HC model in the ladder approximation, the SD equation taking the same form up to numerical factors. The theory (1) with (2) thus will become scale invariant at an ultraviolet fixed point, $\alpha^{(\text{HC})} \to \alpha_c^{(\text{HC})}$ *and* $\tilde{G}_1 \equiv G_1\Lambda_S^2/\pi^2 \to \tilde{G}_{1c}$, the exact values of $\alpha_c^{(\text{HC})}$ and $\tilde{G}_{1c}$ being irrelevant to our discussions here. Accordingly, the dilaton pole (hyperdilaton) will appear at this fixed point in the $J^{PC}=0^{++}$ channel of the hyperfermion-antihyperfermion scattering amplitude.

Where should we find the hyperdilaton, then? It cannot be exactly massless, since the scale invariance must be slightly violated by the nonvanishing β function off the critical point.[19] Its (mass)^2 may be estimated via Dashen's formula as $\sim\beta\Lambda_{\text{HC}}^2 << \Lambda_{\text{HC}}^2$,

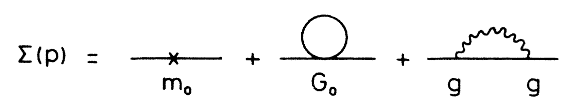

FIG. 2. The Schwinger-Dyson equation in the ladder QED with four-fermion interactions. Internal lines are full propagators.

which may be much lighter than the hyperpions whose masses are very large in our scale-invariant HC model as have been mentioned earlier.[20] The interaction of the hyperdilaton is expected to be quite similar to that of the neutral Higgs boson, both of which possess $J^{PC}=0^{++}$ in contrast to the hyperpions carrying $J^{PC}=0^{-+}$. Thus, we would suggest, for example, that $\xi(2.2)$ observed in the Mark III experiment may be identified with the hyperdilaton, which is consistent with the 0^{++} assignment of $\xi(2.2)$ and the branching ratio as well.[21] Details will be reported elsewhere.

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[(a)]Department of Physics, Kyoto University, Kyoto 606, Japan.

[(b)]Department of Physics, Toyama University, Toyama 930, Japan.

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[6]In the hypercolored preon model, the HC which is asymptotically free at the preon level readily becomes asymptotically nonfree at the composite level due to the abundance of the composite spectrum carrying the HC charge. See Yamawaki and Yokota, Refs. 4 and 5. As to the EHC scenario, see B. Holdom, Phys. Rev. D **23**, 1637 (1981).

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[13]Vanishing β function for $\alpha < \alpha_c$ simply reflects the characteristics of the ladder approximation in which the charge is not renormalized perturbatively. Actually QED is asymptotically nonfree ($\beta > 0$ for $\alpha \sim 0$) beyond the ladder approximation.

[14]T. Kugo, private communication.

[15]Actually, the dimensional transmutation takes place in the strong-coupling phase, i.e., $\Lambda_{\rm QED} \equiv \mu \exp[-\pi/(\alpha/\alpha_c - 1)^{1/2}]$ is a renormalization-group invariant similar to the scale parameter $\Lambda_{\rm QCD}$ in QCD.

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[18]Of course it may contain $\mathscr{L}_{QL}$ mentioned above. Also note that the hyperfermion flavors are implicit in this Letter.

[19]The true hyperdilaton may be a mixture of a hyperfermion-antihyperfermion bound state and hypergluonium.

[20]$\mathscr{L}_{QL}$ does not give mass to the hyperdilaton similarly to $\Pi^{\pm,0}$ which are absorbed into $W^{\pm}$ and Z^0 (Ref. 3).

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