Possible Explanation of the Solar-Neutrino Puzzle

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Mikheyev and Smirnov have shown that electron neutrinos above a certain minimum energy E_m may all be converted into μ neutrinos on their way out through the sun. We assume here that this is the reason why Davis and collaborators, in their experiments, find many fewer solar neutrinos than predicted. The minimum energy E_m is found to be about 6 MeV, the mass m of the μ neutrino must be greater than that of the electron neutrino, $m_2^2 - m_1^2 = 6\times 10^{-5}$ eV^2, and there is a very minor restriction on the neutrino mixing angle.

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Mikheyev and Smirnov (MS), in a very important paper,[1] have discovered a mechanism by which a large fraction of the neutrinos ν_e emitted in the sun may be converted into ν_μ when traversing the sun, and thereby be rendered unobservable. We shall first derive the MS results in a different but equivalent way, and then draw conclusions from it. Like MS, we shall consider only two neutrino flavors, ν_e and ν_μ; the third flavor, ν_τ, will be discussed at the end.

It is generally believed that the neutrinos ν_e and ν_μ are not the ones that propagate in free space, ν_1 and ν_2. Conventionally, we write

$$|\nu_e\rangle = |\nu_1\rangle\cos\theta + |\nu_2\rangle\sin\theta, \quad |\nu_\mu\rangle = -|\nu_1\rangle\sin\theta + |\nu_2\rangle\cos\theta, \tag{1}$$

and assume $\theta < 45°$. The masses are m_1 and m_2. With a transformation to the ν_e, ν_μ representation, the square of the mass is then given by the matrix

$$\mathcal{M} = \tfrac{1}{2}(m_1^2 + m_2^2)\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} + \tfrac{1}{2}(m_2^2 - m_1^2)\begin{pmatrix}-\cos 2\theta & \sin 2\theta\\ \sin 2\theta & \cos 2\theta\end{pmatrix}. \tag{2}$$

Wolfenstein[2] has pointed out that in matter, the masses are changed as a result of the weak-current interaction. The neutral weak current acts equally on ν_e and ν_μ and is therefore unimportant for our purpose. But the charged current will exchange ν_e with the electrons in the matter, while it has no effect on ν_μ. This exchange gives an extra term in the Hamiltonian,[2]

$$H_{\rm int} = (G/\sqrt{2})\bar{\nu}_e\gamma_\lambda(1+\gamma_5)\nu_e[\bar{e}\gamma_\lambda(1+\gamma_5)e], \tag{3}$$

where G is the Fermi constant of weak interactions. For electrons at rest, only γ_4 $(=1)$ contributes, and for neutrinos $1+\gamma_5 = 2$; so (3) is equivalent to a potential energy for ν_e

$$V = G\sqrt{2}N_e, \tag{4}$$

where N_e is the number of electrons per unit volume.

The momentum k of an electron neutrino is then related to its energy E by

$$k^2 + m^2 = (E - V)^2 = E^2 - 2EV, \tag{5}$$

with neglect of the small V^2. Thus, V is equivalent to an addition to m^2 of

$$m_i^2 = 2EV = 2\sqrt{2}(GY_e/m_n)\rho E = A, \tag{6}$$

where m_n is the mass of the nucleon and Y_e the number of electrons per nucleon in the matter, generally $Y_e = \frac{1}{2}$.

When m_i^2 is added to $\mathcal{M}$ the mass matrix (2) becomes

$$\mathcal{M} = \tfrac{1}{2}(m_1^2 + m_2^2 + A)\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} + \frac{1}{2}\begin{pmatrix}A - \Delta\cos 2\theta & \Delta\sin 2\theta\\ \Delta\sin 2\theta & -A + \Delta\cos 2\theta\end{pmatrix}, \tag{7}$$

where

$$\Delta = m_2^2 - m_1^2. \tag{7a}$$

The eigenvalues are

$$m_\nu^2 = \tfrac{1}{2}(m_1^2 + m_2^2 + A) \pm \tfrac{1}{2}[(\Delta\cos 2\theta - A)^2 + \Delta^2\sin^2 2\theta]^{1/2}. \tag{8}$$

The splitting between the two mass eigenvalues is given by the square root.

The splitting has a minimum as a function of A provided that

$$\Delta = m_2^2 - m_1^2 > 0 \tag{9}$$

[$A > 0$ according to (6), and $\cos 2\theta > 0$ by assumption]. The minimum occurs when

$$A = \Delta \cos 2\theta. \tag{10}$$

Figure 1 shows the two eigenvalues (8) of m_ν^2 as a function of A. At low A, i.e., small matter density ρ, the electron neutrino has the smaller mass, but when A reaches the value (10), the two curves would cross if it were not for the coupling term $\Delta \sin 2\theta$. The near-crossing point (10) is the resonance of MS. At larger A, beyond the crossing point, the electron neutrino has larger mass than the μ neutrino.

For these statements to be true, it is essential that the ν_e-e interaction has the positive sign, as indicated in (3) and (4). This sign was given by Wolfenstein in his first paper,[2] but then unfortunately was reversed in his second paper,[3] and the incorrect sign was taken over by MS.[1] They therefore claimed that the crossing of the curves in Fig. 1 (resonance) would occur only if $\Delta < 0$, i.e., ν_e heavier than ν_μ. I am grateful to Paul Langacker for pointing out the correct sign to me. The main features of the theory, however, are independent of the sign, and it is the great contribution of MS to have discovered the significance of the curve crossing (resonance).

Assume now that an electron neutrino is produced in the sun at sufficiently high density that $A > \Delta \cos 2\theta$. Then its m^2 will clearly be given by the + sign in (8). As the neutrino moves outward, A will decrease, Eq. (6), and it will finally hit the resonance (10). At that point, its mass will continue to follow the upper curve in Fig. 1, and it will therefore *emerge from the sun as a μ neutrino* (which cannot be detected). This is the result obtained by MS.

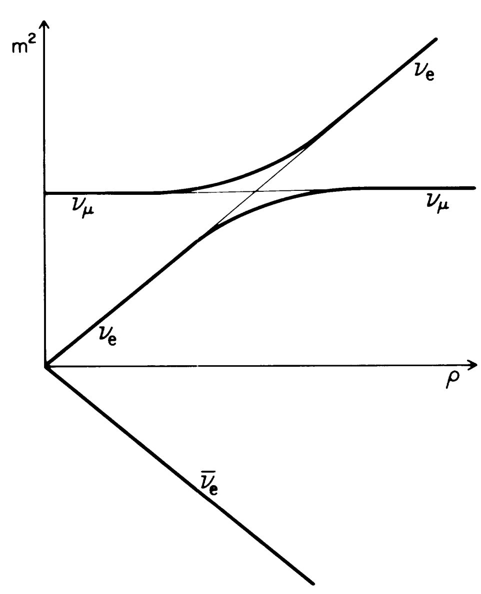

FIG. 1. The masses of two flavors of neutrinos as a function of density. The curves nearly cross at one point. The electron-antineutrino mass $\bar{\nu}_e$ is also shown.

What happens in the resonance is that, for the upper curve in Fig. 1, the state vector which was originally almost in the direction of $|\nu_e\rangle$ turns slowly to the direction $|\nu_\mu\rangle$. Evaluating (6) for $Y_e = \frac{1}{2}$ gives

$$A = 0.76 \times 10^{-7} \rho E \tag{11}$$

if ρ is in grams per cubic centimeter, E in megaelectronvolts, and A in electronvolts squared.

So far the MS theory. Now I propose to take this theory seriously; i.e., we assume that this conversion of ν_e into ν_μ is indeed the cause of the depletion of observable neutrinos from the sun. For any given neutrino energy E, the resonance (10) occurs at a definite density ρ_E. From (10),

$$E\rho_E = 1.3 \times 10^7 \Delta m^2 \cos 2\theta = \Lambda. \tag{12}$$

There is a critical energy $E_c = \Lambda/\rho_c$, where ρ_c is the density at the center of the sun (a definition to be modified later). All neutrinos of energy $E > E_c$ have to go through the resonance; they will emerge as ν_μ and hence be undetectable. The less energetic neutrinos, $E < E_c$, will not go through the resonance and will emerge unscathed as ν_e.

This means that Davis and his collaborators will observe only the solar neutrinos of energy below E_c, but will observe these at full strength. On this assumption, we shall now determine E_c from experiment, using the data from Bahcall *et al.*[4] Table I of that paper gives the composition of the solar neutrino units (SNU) of neutrinos detectable by ^{37}Cl as follows: (i) from ^{8}B, 4.3 SNU; (ii) from all other nuclear species (*pep*, ^{7}Be, ^{13}N, and ^{15}O), 1.6 ± 0.2 SNU. All these other neutrinos have maximum energies of 2.8 MeV or less, which will be found to lie below E_c, so that they will be fully detectable. The ^{8}B neutrinos have a continuous spectrum extending to 14.0 MeV.

The number observed by Davis *et al.* is[4] 2.1 ± 0.3 SNU. Subtracting the expected number from other species, we get for the "observed" neutrinos from ^{8}B

$$S(^8\text{B}, \text{obs}) = 0.5 \pm 0.5. \tag{13}$$

Therefore only a fraction of the ^{8}B neutrinos are observed, viz.,

$$F(^8\text{B}) = (12 \pm 12)\%. \tag{13a}$$

According to our theory then, 12% of the neutrinos emitted by ^{8}B should be below the critical neutrino energy E_c for the sun. This permits a determination of E_c.

We assume that all the ^{8}B decays lead to the same state of ^{8}Be, in an allowed transition. Then the spectrum of neutrinos should be $x^2(1-x)^2\,dx$, where $x=\epsilon_\nu/Q$, and $Q=14.0$ MeV is the energy release in the β decay. We assume that the neutrino detection probability is simply proportional to x^2 provided that $x>x_1=\epsilon_{th}/Q$, where ϵ_{th} is the detection threshold which is 0.82 MeV for the ^{37}Cl detector. Then, if we set $E_c/Q=x_2$,

$$\int_{x_1}^{x_2} x^4(1-x)^2\,dx = F(^8\mathrm{B})\int_{x_1}^{1} x^4(1-x)^2\,dx = 9.4\times10^{-3}F(^8\mathrm{B}). \tag{14}$$

This determines $x_2=0.42$, or

$$E_c = 5.9^{+1.1}_{-3}\ \mathrm{MeV}. \tag{15}$$

This is safely above the energy of all neutrinos other than those from ^{8}B.

Again taking the theory seriously, (12) permits the determination of $\Delta=m_2^2-m_1^2$. For ρ_c, I take the solar density at a radius which includes half the ^{8}B reactions,

$$\rho_c = 130\ \mathrm{g/cm^3}$$

according to the standard solar model of Bahcall *et al.* Then

$$\Delta\cos2\theta = +5.9\times10^{-5}\ \mathrm{eV^2}. \tag{16}$$

This is small, and in the range predicted by MS. If we assume $m_1=0$ and $\cos2\theta=1$, then

$$m_2 = 0.008\ \mathrm{eV}. \tag{16a}$$

With Δ as small as (16), it is likely to be extremely difficult to confirm this result by laboratory experiments. Thus the astrophysical evidence, from the observed deficiency of solar neutrinos, seems to be the best, perhaps the only, way to determine[5] the elusive Δ.

With use of the conclusion (13a), it is now possible to predict the result of future experiments using detectors other than ^{37}Cl. We merely need to take the prediction by Bahcall *et al.* for the SNU from other species, and add 12% of the SNU from ^{8}B. For instance, for the ^{71}Ga detector,[6]

SNU from other species = 106

12% of SNU from ^{8}B = 1.6

Total = 108 SNU. (17)

For this detector, the conversion of ν_e into ν_μ decreases the expected SNU by only 10%. Observation of solar neutrinos by the Ga detector, therefore, would be the best confirmation (or disproof) of theory presented here.[7]

We have tactily assumed that the mass of the neutrino follows the upper curve in Fig. 1. As MS point out, this is true only if the change of density near the crossing point is adiabatic. According to (8), the width of the resonance is $\Gamma=2\Delta\sin2\theta$, and so in the resonance A goes from $\Delta\cos2\theta-\frac{1}{2}\Gamma$ to $\Delta\cos2\theta+\frac{1}{2}\Gamma$. Since A is proportional to ρ, this corresponds to a relative change of density of

$$\delta\rho/\rho = \Gamma/A\cos2\theta = 2\tan2\theta, \tag{18}$$

and this happens in a distance

$$\delta r = 2\left[-\frac{1}{\rho}\frac{d\rho}{dr}\right]^{-1}\tan2\theta, \tag{18a}$$

where the bracketed quantity is taken from the density distribution in the sun. To make the transition adiabatic, δr must be large compared to the neutrino oscillation distance at the resonance,

$$L = \frac{2\pi}{\delta k} = \frac{2\pi\times2E}{\Delta\sin2\theta} = \frac{4\pi\times1.3\times10^7}{\rho\tan2\theta}\ \mathrm{cm}, \tag{19}$$

where (5), (10), and (11) have been used. The adiabatic condition $\delta r \gg L$ then requires

$$\tan^2 2\theta > 0.8\times10^8\left(-\frac{1}{\rho^2}\frac{d\rho}{dr}\right) = 0.8\times10^8\frac{d}{dr}\left(\frac{1}{\rho}\right). \tag{20}$$

In the relevant region of the sun, i.e., where neutrinos between 5 and 14 MeV go through the resonance, the density is between 130 and 50. In the standard model of the sun,[8] in this density region

$$\frac{d}{dr}\left(\frac{1}{\rho}\right) = (0.8\text{–}2)\times10^{-12}\ \frac{\mathrm{cm^2}}{\mathrm{g}}, \tag{20a}$$

the larger number being the relevant one. Then (20a) gives $\tan^2 2\theta > 1.6\times10^{-4}$,

$$\theta > 0.0065\ \mathrm{rad} = 0.4^\circ. \tag{21}$$

Thus even quite small mixing angles are compatible with the present theory.

MS speculate about the effect of the neutrino resonance in supernova stars. If our theory of the solar-neutrino puzzle is correct, we can make definite predictions about the neutrinos in a supernova. These neutrinos range in energy from 5 MeV up; hence, according to (12), their resonance density in matter will range downward from the sun's central density, 130. Therefore, there is no chance for resonance conversion of ν_e into ν_μ in the dense core of a supernova where $\rho > 10^{10}$ g/cm^2, just as originally predicted by Wolfenstein.[3]

[If the mass of ν_τ is 2 eV, as we shall estimate below, the crossing between ν_τ and ν_e is at about 10^7 g/cm^3, still much below the supernova core density.]

However, the enormous number of neutrinos escaping from the collapsing core will all pass through a density region around 100 on their way out. Here the ν_e will be converted into ν_μ. Conversely, ν_μ emitted from the core will be converted into ν_e. So electron neutrinos will be observable (if we ever catch the neutrinos from a supernova), but they will not have started out as ν_e.

Antineutrinos $\bar{\nu}_e$ have an interaction of the opposite sign with matter. Thus they will *not* have a resonance with $\bar{\nu}_\mu$ but will escape from the star unchanged.

Gell-Mann, Ramond, and Slansky,[9] and also Yanagida[10] have proposed a "seesaw model" in which the mass of a neutrino of flavor i is related to that of the quark of the same flavor by

$$m(\nu_i) = m^2(q_i)/M, \tag{22}$$

where M is a superheavy mass. For definiteness, I replace q_i by the charged lepton of the same flavor, l_i; then

$$\frac{m(\nu_\tau)}{m(\nu_\mu)} = \frac{m^2(\tau)}{m^2(\mu)} \simeq 300 \tag{23}$$

and

$$m(\nu_\tau) \simeq 2.5 \text{ eV}. \tag{24}$$

Such a neutrino could have useful cosmological consequences. If I chose, instead of the τ lepton, some geometric mean between bottom and top quark, let us say $m(q_\tau) = 12$ GeV, then I would get $m'(\nu_\tau) = 100$ eV, giving cosmologists a choice between 2 and 100 eV. The mass of the electron neutrino, using (22), would be

$$m(\nu_e) = 2\times 10^{-7} \text{ eV}, \tag{25}$$

clearly unmeasurable. The superheavy mass would be

$$M \simeq 10^{18} \text{ eV} = 10^9 \text{ GeV}, \tag{26}$$

less than the usually assumed $M \simeq 10^{14}$ GeV.

The oscillation distance between ν_μ and ν_e in vacuum is

$$L_{e\mu} = \frac{\pi}{\delta k} = \frac{2\pi E}{m_2^2 - m_1^2} \simeq 4000 \text{ km} \tag{27}$$

(for $E = 200$ MeV), clearly impractical for a laboratory experiment. But if (24) is the correct mass of the τ neutrino, and if we take $E = 3$ GeV, then

$$L_{\mu\tau} \simeq L_{e\tau} \simeq 600 \text{ m} \tag{27a}$$

which may be a feasible distance for a laboratory experiment. However, the fraction of ν_μ which would convert into ν_τ at this (most favorable) distance is only $\sin^2 2\theta_{\mu\tau}$ which may perhaps be 1%. If ν_τ is to be detectable, its energy must be greater than the mass of the τ lepton (1.8 GeV). But if we use energies as large as this, there will be many τ and ν_τ produced in the first place, and it will be exceedingly difficult to find out if 1% of the ν_μ have been converted into ν_τ.

Coming back to the neutrinos escaping from a supernova and thus going from very high to low ρ, there is first a crossing of the masses of ν_e and ν_τ at a density around 10^7 g/cm^3, and then a crossing of ν_e and ν_μ at around $\rho = 10^2$. Thus the history of escaping neutrinos will be as follows:

$$\nu_e \to \nu_\tau; \quad \nu_\tau \to \nu_e \to \nu_\mu; \quad \nu_\mu \to \nu_e; \qquad \bar{\nu}_e \text{ remains } \bar{\nu}_e. \tag{28}$$

If supernova neutrinos are measured on Earth, ν_e and $\bar{\nu}_e$ can be measured which indicate the ν_μ and $\bar{\nu}_e$ originally produced in the supernova core.

I am much indebted to Ray Davis for drawing my attention to the paper by Mikyeyev and Smirnov. I also thank Paul Langacker for pointing out a mistake in sign in the first version of my paper (as well as that of MS), and drawing my attention to Refs. 9 and 10, and Michael Turner for checking my arithmetic.

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[5]S. P. Rosen and J. M. Gelb, unpublished, have pointed out a second solution in which the conversion of ν_e to ν_μ takes place near the surface of the sun. Δ is smaller, 10^{-7} to 10^{-4} eV^2, and θ is directly related to Δ. The Ga detector is predicted to give less SNU than in (17), so that the Ga experiment can decide between the two solutions.

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