Vortex Dynamics and Phase Transitions in a Two-Dimensional Array of Josephson Junctions

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The ac response of large two-dimensional arrays of proximity-effect Josephson junctions to an oscillating driving field has been studied as a function of temperature, applied transverse magnetic field, and frequency. For an integer number of flux quanta per unit cell, a peak in dissipation and a drop in superfluid density are observed near the superconducting transition of the array. These features as well as their frequency dependence provide clear evidence for the vortex-unbinding transition predicted by the Kosterlitz-Thouless theory.

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This Letter reports a study of vortex dynamics in two-dimensional (2D) square arrays of Josephson junctions exposed to a transverse magnetic field **B**. In the experiments described below, the complex ac response of vortex excitations to a driving oscillating field is inferred from measurements of the screening properties of the array performed with a two-coil mutual-inductance technique.[1] For an integer number of flux quanta per unit cell of the array we observe a peak in dissipation (as measured by the real part of the response) and a drop in superfluid density (as measured by the imaginary part of the response) in the vicinity of the superconducting transition of the array. The shapes of these structures, which are similar to those observed at the superfluid transition of 2D superfluid helium films[2] and of uniform 2D superconductors,[1] as well as their frequency dependence, provide clear evidence for the existence of a phase transition of the type predicted by the Kosterlitz-Thouless (KT) theory[3] and by its extensions[4,5] to finite frequencies.

In zero magnetic field a 2D array of identical Josephson junctions is isomorphic to an XY model with a temperature-dependent coupling energy $E_J(T) = \hbar i_c(T)/2e$, where $i_c(T)$ is the critical current of an isolated junction of the array in the absence of thermal fluctuations.[6] According to the KT theory,[3] below a critical temperature T_c such a system is populated by bound pairs of vortices with opposite circulation resulting from 2D fluctuations in the phase of the order parameter. The phase transition of the system at $T=T_c$ is attributed to the unbinding of vortex-antivortex pairs, a process creating free vortices which destroy the quasi long-range (or topological) order existing below T_c. Studies of the resistive transition and of the current-voltage characteristics of 2D arrays[7-11] were found to be consistent with the vortex-unbinding idea.

The physics of arrays of weak links exposed to a transverse **B** field is more complex and only partially understood at present. The interaction of the field-induced vortices with the pinning potential provided by the periodic structure of the array creates new and interesting phenomena which are most simply described by uniformly frustrated 2D lattice spin models,[12-15] the degree of frustration being controlled by the ratio $f=\phi/\phi_0$, where $\phi=Ba^2$ is the magnetic flux threading a unit cell of the array (a is the lattice spacing) and $\phi_0=hc/2e$ the superconducting flux quantum. The frustration parameter f determines the ground-state configuration of the vortex lattice and, in addition, has a profound effect on the nature of the phase transition at $T_c(f)$. As f changes, the array is driven through a sequence of pinned commensurate (C) vortex phases and "floating" incommensurate (I) vortex phases. As a consequence, critical currents and resistance show a complex periodic dependence on f,[9-11,16,17] similar to that observed earlier in other modulated superconducting structures.[18,19]

In order to explore the nature of the phase transition at $T_c(f)$, we have studied the dynamics of vortices in an array of proximity-effect Pb/Cu/Pb junctions using a modified version of the two-coil technique devised by Fiory and Hebard.[1,20] The experiments were performed on arrays consisting of $N\times N \approx 5\times 10^5$ square 4000-Å-thick Pb islands forming a square lattice with $a=8$ μm on a 2000-Å-thick Cu film, the length L of the Cu bridges connecting adjacent Pb islands being of the order of 1.7 μm. The zero-field dc resistance of the array shows, with decreasing temperature, the two distinct transitions observed in similar systems by other groups[7-9]: the sharp Bardeen-Cooper-Schrieffer transition of the individual Pb islands at $T_{cs}=7$ K and the subsequent transitions to the superconducting state at $T_c \approx 3.4$ K. For the ac measurements the array was positioned directly under a system of coaxial cylindrical coils consisting of an external driving coil and an inner astatic-pair receiving coil. An ac current of amplitude $I_{D\omega}=35$ μA rms and angular frequency ω (varying from $\sim 6\times 10^2$ s^{-1} up to $\sim 6\times 10^4$ s^{-1}) was applied to the driving coil and the signal voltage, δV_ω, at the receiving coil due merely to the screening currents flowing in the array was phase-sensitively detected.

Real and imaginary parts of δV_ω measured at 4.033 kHz as a function of f are shown in Fig. 1 for different temperatures. Prominent structures emerge in both the dissipative $[\mathrm{Re}(\delta V_\omega)]$ and inductive $[\mathrm{Im}(\delta V_\omega)]$ components of δV_ω in correspondence to C-vortex phases defined by integer $(f=p)$ and half-integer $(f=p/2)$ values of f. Structures at $f=p/3$ are also clearly resolved in most of the data shown in Fig. 1. A detailed interpretation of the array response as a function of driving frequency, frustration, and temperature requires a theory describing the dynamics of field-induced vortices and of thermally generated topological excitations (vortices, dislocations, domain walls) in a periodic force field. Such a theory is not available so far. However, some of the essential features of $\delta V_\omega(f,T)$ can be understood in terms of periodic vortex pinning and of the unique dynamic response of thermally excited vortices.

At low temperatures $(T \ll T_c)$, where the influence of topological excitations is negligible, structures in $\delta V_\omega(f)$ at rational $(f=p/q)$ values of f reflect the drastic change in pinning occurring at a CI transition. In a low-order (small q) C phase the mobility of the field-induced vortices is considerably reduced by the periodic pinning potential provided by the array, while the vortex lattice can slide freely in an I phase.[18] As a consequence, there is a marked reduction of dissipation in a low-order C phase, a process resulting in the periodic sequence of dips one observes in the $\mathrm{Re}[\delta V_\omega(f)]$ signals of Fig. 1(a) at low temperatures. On the other hand, since pinning is important in low-order C phases, considerable lag in response is expected for such vortex configurations. This is, of course, the mechanism responsible for the commensurate peaks occurring in the inductive component of $\delta V_\omega(f)$ shown in Fig. 1(b).

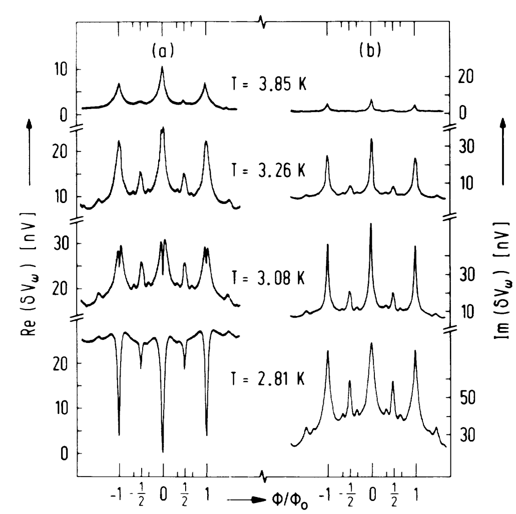

FIG. 1. (a) Real and (b) imaginary parts of the ac response at 4.033 kHz of a 2D array of proximity-effect Josephson junctions as a function of the frustration parameter $f=\phi/\phi_0$.

As the temperature rises and approaches T_c, the low-temperature commensurate dips in $\mathrm{Re}[\delta V_\omega(f)]$ progressively transform into peaks which finally vanish above T_c [Fig. 1(a)]. Simultaneously, a rapid degradation of the commensurate peaks in $\mathrm{Im}[\delta V_\omega(f)]$ is observed in the critical region [Fig. 1(b)]. The evolution of the commensurate structures in both $\mathrm{Re}[\delta V_\omega f)]$ and $\mathrm{Im}[\delta V_\omega(f)]$ suggests the presence of thermally activated defects[11] which, on account of their high mobility, generate additional dissipation and reduce the lag in response in a C phase. In the following we shall show that for integer f the dynamics of these defects has features allowing their unambiguous identification as the vortex excitations of the KT theory. To see this, we first consider the case $f=0$ which is comparatively simpler. As for uniform thin-film superconductors,[1] the complex sheet impedance, $Z_\square$, of the array can be written in the form $Z_\square(\omega,T)=i\omega L_{k\square}(T) \times\epsilon(\omega,T)$, where $L_{k\square}(T)=\hbar/2ei_c(T)$ is the bare sheet kinetic inductance of the array and $\epsilon(\omega,T)$ is the complex vortex dielectric constant which renormalizes the bare superfluid density $n^*(T)=m^*i_c(T)/2e\hbar$.[6] In a 2D Coulomb-gas analog, $\epsilon(\omega,T)$ is the sum of contributions from vortex-antivortex dipoles and free vortex charges.[4] In writing the above expression for $Z_\square$ we have assumed that normal currents do not appreciably contribute to the total current flowing in the array, a condition satisfied if $\omega L_{k\square} \ll r_n$, where r_n is the normal-state resistance of an individual junction. Then, in the weak-screening limit[20] appropriate to discussion of our experiments in the critical region,[21] the signal voltage $\delta V_\omega(T)$ turns out to be proportional to $Z_\square^{-1}(\omega,T)$ and can be written in the form[20]

$$\delta V_\omega(T)=Ci\omega I_{D\omega}i_c(T)/\epsilon(\omega,T), \qquad (1)$$

where C is a constant depending on the sample-coil geometrical configuration whose numerical value, $C\approx 0.74$ V s/A^2, was deduced by the study of the frequency dependence of the jump in $\mathrm{Re}[\delta V_\omega(T)]$ at T_{cs}. Since the monotonic temperature dependence of $i_c(T)$ does not significantly affect the response of the array in the vicinity of T_c, the behavior of $\delta V_\omega(T)$ in the transition region will be mostly determined by $\epsilon(\omega,T)$. This is quite clearly demonstrated by the $\delta V_\omega(T)$ signals for $f=0$ shown in Fig. 2, where the peak in dissipation and the drop in superfluid density reflect the fundamental features of $\epsilon^{-1}(\omega,T)$ predicted by dynamical extensions[4,5] of the KT theory.

Since the Hamiltonian of the system is periodic in f with period 1, the superconducting transition of the array is expected to be KT type also for $f=p\neq 0$. In this case the thermally activated defects are positive and

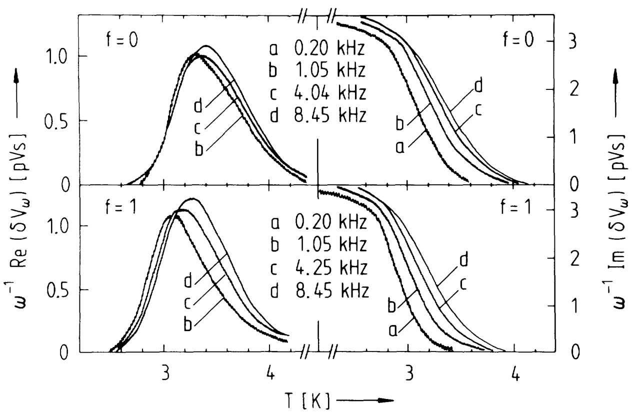

FIG. 2. Temperature dependence of the normalized complex ac response of the 2D array of Fig. 1 measured at different frequencies for $f=0$ and $f=1$. Signal voltages are normalized with respect to angular frequency.

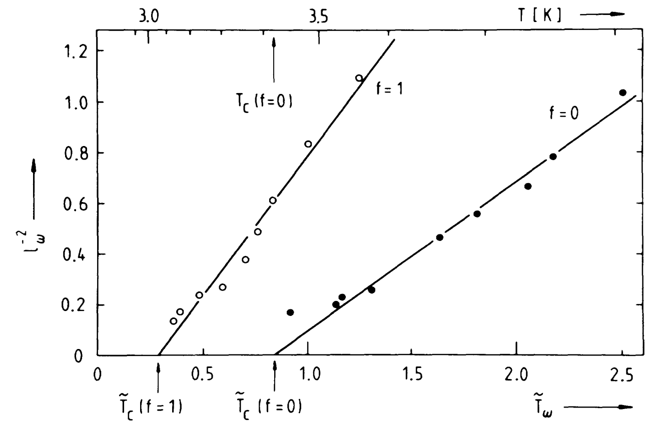

FIG. 3. Dependence of the scale parameter $l_\omega \equiv \ln(r_\omega/a)$ on the dimensionless vortex-unbinding temperature $\tilde{T}_\omega$. Solid lines are fits according to Eq. (2). The upper axis is a real temperature scale for $f=0$.

negative vacancies[11] associated, respectively, with $p-1$ and $p+1$ quantized vortices per unit cell. These vacancies of opposite sign can be viewed as free-moving vortex-antivortex excitations immersed in a pinned commensurate background of field-induced vortices with p flux quanta per unit cell. According to this picture, the response $\delta V_\omega(T)$ for $f=p\neq 0$ should be similar to that for $f=0$, a conjecture confirmed by the experimental results for $f=1$ shown in Fig. 2. When compared with the case $f=0$, however, the transition is seen to occur at a lower temperature. This is easily understood if one realizes that the finite size of the junctions makes i_c field dependent. More precisely, as demonstrated by the parabolic envelope characterizing most of the signals of Fig. 1, $i_c(T,f)$ is a quadratically decreasing function of f as long as $(A/a^2)f \ll 1$, where A is an effective area of the junctions. From the theoretical prediction[6,10] $i_c(T_c(p),p)/T_c(p) \approx 4ek_B/\pi\hbar \approx 27$ nA/K, it then follows that the KT transition is pushed to lower temperatures by increasing p. It is also clear that Eq. (1) still describes the response in the more general case $f=p$ provided one replaces $i_c(T)$ by $i_c(T,p)$.

Further evidence for a vortex-unbinding transition at $f=p$ is provided by a study of the frequency dependence of $\delta V_\omega(T,p)$. As shown in Fig. 2 by the data for $f=0$ and $f=1$, the peak in $\mathrm{Re}[\delta V_\omega(T)]$ shifts to higher temperatures and the falloff of $\mathrm{Im}[\delta V_\omega(T)]$ broadens with increasing ω. To discuss these results, we recall that the dynamical theories[4,5] introduce a vortex diffusion length $r_\omega \approx (14D/\omega)^{1/2}$ determining the separation of those vortex pairs which dominate the response. The vortex diffusivity D is easily deduced from the analysis of the flux-flow regime in arrays by Lobb, Abraham, and Tinkham[6] and is found to be given by $D=(c/\phi_0)^2 r_n a^2 k_B T$. At finite frequencies the crossover in response due to the unbinding of bound vortex pairs into free vortices will be seen at a temperature T_ω such that $r_\omega = \xi_+(\tilde{T}_\omega)$, where $\xi_+(\tilde{T}) \approx a\exp\{b[\tilde{T}-\tilde{T}_c(p)]^{-1/2}\}$ is the correlation length of the array, $\tilde{T}=2ek_BT/\hbar i_c(T,p)$ a dimensionless temperature parameter, and b a nonuniversal constant of order unity.[6] Following Ref. 1, we define T_ω by extrapolating to zero the steep portions of the $\mathrm{Im}(\delta V_\omega)$ vs T curves. We found that this procedure has the advantage of being largely independent of $I_{D\omega}$. Other definitions of T_ω, for instance that proposed in Ref. 5 which identifies T_ω as the temperature corresponding to the peak in $\mathrm{Re}[\delta V_\omega(T)]$, were found to be very sensitive to $I_{D\omega}$, thereby showing the importance of nonlinear effects at finite driving currents.[6-8] If we introduce a scale parameter $l_\omega \equiv \ln(r_\omega/a)$, the condition $r_\omega = \xi_+(\tilde{T}_\omega)$ can be written as

$$l_\omega^{-2} = b^{-2}[\tilde{T}_\omega - \tilde{T}_c(p)]. \qquad (2)$$

In Fig. 3 l_ω^{-2} is plotted vs $\tilde{T}_\omega$ for $f=0$ and $f=1$. The scale l_ω was calculated with $r_n = 2.2$ mΩ, a value inferred from the sheet resistance of the array at T_{cs}^-. In this connection, we notice that, for a given ω, l_ω in proximity-effect arrays is considerably smaller (in our experiments r_ω/a ranges from ~ 3 to ~ 13) than in uniform 2D superconductors,[1] a fact accounting for the much broader transitions observed in our system. The $i_c(T,p)$ curves needed to calculate $\tilde{T}_\omega$ were obtained by the fitting of low-temperature i_c measurements by the expression[22]

$$i_c(T,p) = i_0(p)[1-(T/T_{cs})]^2 \exp[-L/\xi_N(T)]$$

[using $\xi_N(T_{cs}) = 850$ Å for the Cu coherence length, $i_0(0) \approx 0.78$ A, and $i_0(1) \approx 0.26$ A] and by extrapolation of the theoretical curves in the critical region. Good fits to Eq. (2) are obtained for $b(f=0)=1.29$ and $b(f=1)=0.95$ and lead to $i_c(T_c(0),0)/T_c(0)$

$=49$ nA/K and $i_c(T_c(1),1)/T_c(1)=143$ nA/K which, in turn, give $T_c(0)=3.43$ K and $T_c(1)=3.08$ K. In comparing these i_c/T_c ratios with the approximate theoretical prediction of 27 nA/K, it should be remembered that this value, resting on the assumption of an unrenormalized $n^*(T,p)$, underestimates i_c/T_c. Actually, static screening, described by $\epsilon_c(p)=\epsilon(0, T_c(p))$, renormalizes $n^*(T,p)$ downwards, thereby leading to higher bare i_c/T_c ratios. For $f=0$ we obtain $\epsilon_c(0)=1.81$, a value in good agreement with an estimate for the XY model $[\epsilon_c(0)\approx 1.75]$ based on Monte Carlo calculations.[23] Because of additional dielectric screening provided by the commensurate vortex background,[11] renormalization effects should be more important for $f=1$ than for $f=0$ $[\epsilon_c(1)>\epsilon_c(0)]$. This conjecture is consistent with the experimental observation of a larger i_c/T_c ratio for $f=1$, although additional screening alone seems to be insufficient to account for the large ϵ_c value $[\epsilon_c(1)=5.3]$ found in this case.

Finally, we briefly comment on the case $f=\frac{1}{2}$. We found that the response $\delta V_\omega(T)$ looks quite similar to that for $f=p$, thereby suggesting a KT-type transition also for $f=p/2$. A study of the frequency dependence, however, did not confirm the exponential inverse square-root form for $\xi_+(\tilde{T})$. The precise nature of the phase transition at $T_c(p/2)$ (KT type or Ising type[12]) is therefore still unkown, a challenging problem which will deserve particular attention in our future work.

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