Proposed Experimental Realization of Anderson Localization in Random and Incommensurate Artificially Layered Systems

S. Das Sarma, Akiko Kobayashi, and R. E. Prange

Department of Physics and Astronomy and Institute for Physical Science and Technology, University of Maryland, College Park, Maryland 20742

(Received 1 July 1985)

We propose that random and incommensurate superlattices be fabricated to study the Anderson localization of their plasma oscillations, which can be measured by Raman scattering. Specific configurations are suggested which, by duality considerations, allow control of the wave functions relative to the mobility edge in the incommensurate case. Localization effects should be easily observable with presently achievable superlattice parameters.

PACS numbers: 73.60.Fw, 71.50.+t, 71.55.Jv, 72.30.+q

The new ability to position individual atomic layers one by one, and thus to create crystals or "superlattices" with prescribed properties, is a revolutionary advance which has inspired much recent work.[1] A natural way to exploit this power is to create superlattices which test important theories. It is all the more interesting if the theories in question are the simplest prototypes of basic mathematical and physical problems, and if they have never before been directly tested in the laboratory.

A rather obvious set of candidate theories are those which deal with "electron" propagation in random or incommensurate lattices. The central idea of this genre of problem is that of Anderson localization and of possible transitions (mobility edges) between localized and extended states.[2] The best studied case is that of one-dimensional tight-binding Hamiltonians. Then the results for the random lattice problem are especially simple, as the eigenstates are always localized. Incommensurate lattices in one dimension give a rich structure which includes the possibility of a transition between localized and extended states as a system parameter is varied.[3] No very direct observation of these properties, particularly in the incommensurate case, has ever been made, however.

The possibility of constructing an incommensurate or random superlattice has occurred to many workers.[4] However, observation of single-electron tunneling currents between superlattice layers has only recently been achieved.[5] Further experiments along this line have a bright future.

It is well known, however, that the problem of localization is not essentially electronic or quantum in nature, but rather turns on the wave nature of the entity involved. Thus, it is not necessary to think of individual electrons in an array of quantum wells, but one can instead consider collective motions of the electrons. The phonons can also be studied,[6] but their properties are weakly modulated by the artificial structures most often constructed.

It is our purpose here to point out that the plasma excitations in an artificially layered system serve admirably to illustrate the main results of localization theory. The plasmons in periodic superlattices are already well understood.[7] We thus imagine a sequence of layers, labeled by integers l. Each layer itself is translationally invariant in the transverse directions. Thus the transverse wave number q is invariant, and may be controlled by the experimenter. It is a good approximation for our purpose[8] to neglect the degrees of freedom (subband structure) of the individual electrons in each layer, and to treat them simply as a classical two-dimensional plasma embedded in three dimensions.

Radiation Raman scattered from the superlattice has intensity proportional to

$$I(q,k,\omega)=\sum_{l,l'}\mathrm{Im}\left[\frac{1}{\Pi^{-1}-\tilde{V}}\right]_{ll'}e^{ik(z_l-z_{l'})}. \tag{1}$$

Here k is the wave number perpendicular to the layer and (q,k,ω) is the wave-number–frequency difference of incident and outgoing photon. The L layers are centered at distance z_l. The dynamic polarizability[9] is

$$\Pi^{-1}=\delta_{ll'}\left[\frac{\omega^2+i\omega\Gamma-\beta_l q^2}{q^2N_l/m}\right]. \tag{2}$$

In this formula N_l is the electronic areal density of the lth layer, which is under the control of the experimenter and can be varied by a factor of more than 10. The effective mass m_l, in principle, can vary from layer to layer as well, but we shall regard it as a constant, m. The dispersion term $\beta_l=3\pi N_l/2m^2$ makes an uninteresting small modification, and we shall drop it from the formulas for clarity, although it has been included in the numerical work. The linewidth parameter Γ could be made to depend on layer as well, but we assume it is constant. In the ideal case, $\Gamma\to 0$ and $I\propto\delta(\Pi^{-1}-\tilde{V})$. For realistic systems, Γ can be fixed empirically, and is in agreement with theory.[10] Its ex-

istence is an important limitation on the resolution of the experiment.[10]

Adjacent layers are coupled via the Coulomb potential given by

$$\tilde{V}_{ll'} = (2\pi e^2/\kappa q) e^{-q|z_l - z_{l'}|}. \quad (3)$$

The potential thus decays with an exponent controlled in the experiment, rather than being long range. Thus one is able to control the strength of the hopping parameter in the tight-binding Anderson model derived below. If there is one layer only, the Raman intensity is independent of k, and peaks at the two-dimensional plasma frequency, ω_q, where

$$\omega_q^2 = 2\pi \bar{N} e^2 q/\kappa m. \quad (4)$$

Here κ is the dielectric constant and $\bar{N}$ is the electronic density of the layer. We shall, however, use Eq. (4) below, where $\bar{N}$ has the meaning of some nominal "average" density, introduced for dimensional convenience.

We now introduce a notation which brings out the relation of the Raman intensity to Anderson's tight-binding model. Let $T_l = \bar{N}/N_l = t_l^2$. The intention is to introduce incommensuration or disorder through the parameters T_l, that is, through the control of the density. One may also make z_l random which introduces "off-diagonal" disorder. We have studied this possibility and will report on it elsewhere. Here we take $z_l = al$ where a is the constant interlayer separation. We take a as the unit of length. We call $\omega_q^2 = E$, $V_{ll'} = -\omega_q^2 e^{-q|l-l'|}$ $(l \neq l')$, $V_{ll} = 0$, and $\eta_l = \omega \Gamma \bar{N}/N_l$. A constant part of the parameter T_l can be included in the "energy" E. Then

$$I(q,k,\omega) = \bar{N}\frac{q^2}{m} \sum_{l,l'} \mathrm{Im}\left[\frac{1}{E - \omega^2 T - V - i\eta}\right]_{ll'} e^{ik(l-l')}. \quad (5)$$

The wave function (density fluctuation) u_l satisfies

$$\omega^2 T_l u_l + \sum_{l'} V_{ll'} u_{l'} = E u_l. \quad (6)$$

The standard way of studying this tight-binding equation is to regard ω^2 as a parameter and E as an eigenvalue. It is experimentally more convenient to regard E as a parameter and ω^2 as the eigenvalue. Since $T_l > 0$ we may introduce $w_l = t_l u_l$. Then the orthonormal eigenstates w_l^ν satisfy

$$\sum_{l'} \omega_q^2 t_l^{-1} e^{-q|l-l'|} t_{l'}^{-1} w_{l'}^\nu = \omega_\nu^2 w_l^\nu. \quad (7)$$

Thus $I \propto \sum_\nu |u^\nu(k)|^2 \delta(\omega^2 - \omega_\nu^2)$, with $u^\nu(k) = \sum_l e^{ikl} \times w_l^\nu / t_l$. This displays the character of I as a local density of states, "local" in the momentum space representation.

To illustrate numerically the results expected, we parametrize T_l by

$$T_l = (\cosh q_0 - c_l)/\sinh q_0,$$

with $|c_l| \leq 1$ and $\sum_l c_l = 0$. We shall see below that for $q_0 << q$ the variation of T_l dominates favoring localization, while for $q_0 >> q$ extension is favored.

We have diagonalized Eq. (7) for $L = 21$ and chosen ω^2 to be at a central (tenth) eigenvalue.[11] In Fig. 1 we give the Raman intensity for four cases with c_l the cosine of a random angle. The most extreme cases are (a) $q_0 = 0.75$, $q = 7.5$, favoring localization, with I almost independent of k, and (b) the reverse case giving results essentially identical to the periodic lattice in this case of finite L. We have chosen also (c) $q_0 = 0.4$, $q = 2.0$ and (d) the reverse. This set is motivated by the fact that present superlattices have achieved a variation of a factor of 30 in density and 2 is the approximate upper limit of q reported by Sooryakumar *et al.*[12] However, we expect that more extreme values can be achieved in time. The localization length ξ labels each curve.

We next give results for the incommensurate superlattice. We take $c_l = f(\alpha l + \lambda)$, with $f(x) = f(x+1)$. The incommensuration parameter α is irrational. We take it to be the golden mean $(\sqrt{5}-1)/2$ in the numerical work. We let $f(x) = \cos x$ not only because it is a very well-studied case but because of duality considerations which we now discuss. If $V_{ll'}$ connected only neighboring layers there would be the important simplification of Aubry duality.[3] Aubry duality is a transformation (essentially a Fourier transform) which carries extended states into localized ones and vice versa, and which transforms the Hamiltonian into itself, but with new parameters. If the new parameters are identical to the old, the Hamiltonian is self-dual,

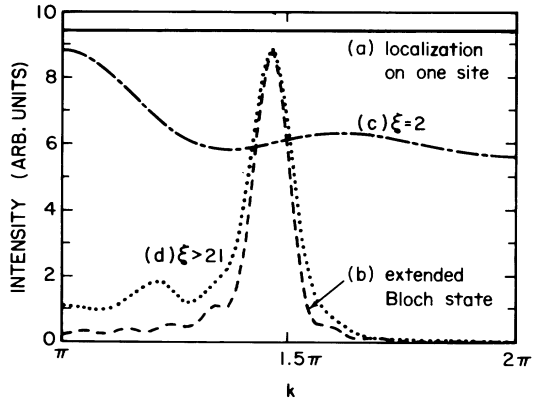

FIG. 1. Raman intensity as a function of the longitudinal wave-vector transfer for the random case. Four different situations (as described in the text) are shown with their respective localization lengths (ξ) labeled on the curves.

and the states are between extended and localized. This is clearly a very interesting critical situation, namely, it is the mobility edge.

We wish to consider[13] a Hamiltonian dual to (7). Aubry's transformation does not work. However, we can make the related transformation

$$\hat{w}_p = \hat{t}_p e^{-2\pi i p\lambda} \sum_l e^{2\pi i l(\alpha p+\mu)} t_l w_l,$$

where

$$\hat{t}_p^{-2} = \sum_l e^{2\pi i l(\alpha p+\lambda)} e^{-q|l|},$$

$$t_l^{-2} = \sum_p e^{-2\pi i p(\alpha l+\lambda)} e^{-q_0|p|}.$$

Then

$$\omega^2 \hat{w}_p = \omega_q^2 \sum_{p'} \hat{t}_p^{-1} e^{-q_0|p-p'|} \hat{t}_{p'}^{-1} \hat{w}_{p'}. \quad (8)$$

Thus for $q = q_0$, the Hamiltonian is self-dual. We conjecture that for $q > q_0$ the states of Eq. (7) are localized, while for $q_0 > q$ they are extended. Note that for large $q \simeq q_0$, the band of states is narrow. In the band $\omega^2 \simeq \omega_q^2$ and the usual Aubry duality is recovered.

Finally, we present the numerical results for the incommensurate case. Figure 2(a) gives the spectrum of (7) for $q_0 = 2.0$ as a function of q. Notice how the spectrum changes from the smooth structure characteristic of the periodic lattice, $q \ll q_0$, to the "Cantor set" structure for $q \simeq q_0$. In Fig. 2(b) we give the tenth eigenstate for $q = 0.4, 2.0$ in the extended and critical ranges ($q_0 = 2.0$ for both). For the localized case we show the eigenstate for $q_0 = 0.4$, $q = 2.0$.

The numerical results for the Raman intensity in the incommensurate case are shown in Fig. 3. Four cases are plotted. In addition to the three above for which the wave functions are shown, we show the results for a second critical case, $q = q_0 = 0.4$. As before, the extended case is quite close to that of the perfect lattice, the localized case is considerably smoother, and the critical case is intermediate.

To summarize, we suggest that experiments can be performed with present technology which will depend in a direct and understandable way on the Anderson localization or extension of the wave functions of plasma oscillations in superlattice systems. With perfect frequency resolution the spectrum and Fourier transform of the wave functions can be measured. There are, of course, limitations to the resolution which can be achieved. There will also be limitations to the number of layers which can be studied. This limitation will probably be not so much from the size of the superlattice which can be constructed, as from extinction effects on the photons. A better treatment of the free surfaces and resultant surface plasmons[14] will be made in a future publication. While it is not yet clear that a detailed study of the wave functions at

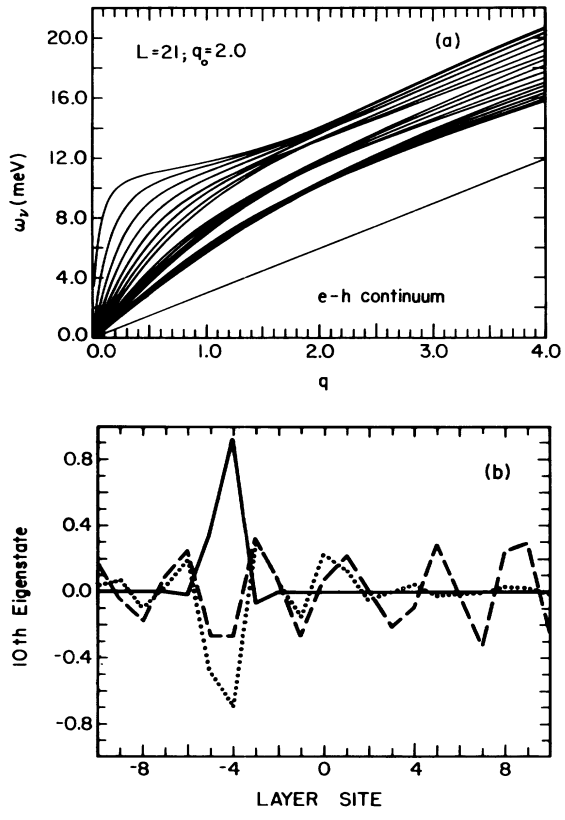

FIG. 2. (a) The plasmon spectrum as a function of the conserved transverse wave vector q for the incommensurate case with $q_0 = 2$ (the mobility edge is at $q = q_0 = 2.0$). (b) An extended eigenstate ($q = 0.4$, $q_0 = 2.0$), dashed line; a critical eigenstate ($q = 0.4$, $q_0 = 0.4$) at the mobility edge, dotted line; and a localized state ($q = 2.0$, $q_0 = 0.4$), solid line, for the incommensurate case.

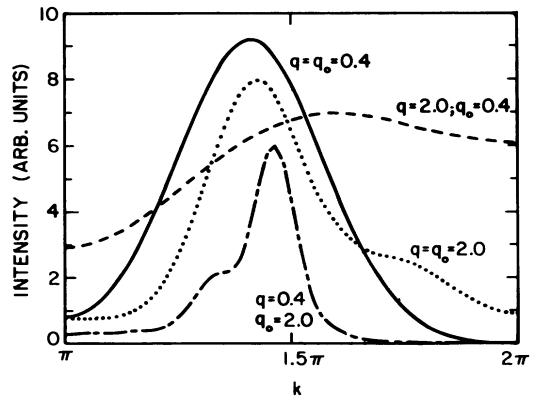

FIG. 3. Raman intensity as a function of the longitudinal wave-vector transfer for the incommensurate case. Two critical mobility-edge situations ($q = q_0 = 0.4$ and $q = q_0 = 2.0$), and one each of the extended ($q = 0.4$, $q_0 = 2.0$) and the localized ($q = 2.0$, $q_0 = 0.4$) situations, are shown.

the mobility edge will be possible by this method, it is highly probable that large effects directly attributable to localization can be observed. In many ways our proposed way of studying Anderson localization effects is better than the standard technique of transport measurements in disordered systems since (as we show in this Letter) the Raman intensity in our proposed experiments is a *direct measure* of "one-electron" local density of states, whereas conductivity measures two-electron properties.

This work is supported in part by the National Science Foundation through Grant No. NSF-DMR 82-13768 and by the U.S. Office of Naval Research through Contract No. N 00014-84-K0586.

---

[1]See, for example, Proceedings of the Fifth International Conference on Electronic Properties of Two-Dimensional Systems, Surf. Sci. **142** (1984), for a current status of the field.

[2]P. W. Anderson, Phys. Rev. **109**, 1492 (1958). For a current status, see P. A. Lee and T. V. Ramakrishnan, Rev. Mod. Phys. **57**, 287 (1985).

[3]S. Aubry and G. André, Ann. Isr. Phys. Soc. **3**, 133 (1980); B. Simon, Adv. Appl. Math. **3**, 463 (1982).

[4]A. C. Gossard, private communication; J. B. Sokoloff, Phys. Rev. B **22**, 5823 (1980).

[5]F. Capasso, K. Mohammed, A. Y. Cho, R. Hull, and A. L. Hutchison, Phys. Rev. Lett. **55**, 1156 (1985).

[6]A. K. Sood, J. Menéndez, M. Cardona, and K. Ploog, Phys. Rev. Lett. **54**, 2111, 2115 (1985).

[7]S. Das Sarma and J. J. Quinn, Phys. Rev. B **25**, 1603 (1982); D. Olego, A. Pinczuk, A. C. Gossard, and W. Wiegmann, Phys. Rev. B **26**, 7876 (1982); and references therein.

[8]S. Das Sarma, Phys. Rev B **29**, 2334 (1984).

[9]Our numerical results change little if the exact two-dimensional polarizability [F. Stern, Phys. Rev. Lett. **18**, 546 (1967)] is used.

[10]We have arbitrarily chosen $\Gamma = 0.2$ meV in this paper. This is an optimistic choice with the actual value more likely to be in the range $0.1\ \text{meV} \leq \Gamma \leq 1.0$ meV for good quality superlattices (S. Das Sarma, unpublished). Our calculated results with a more realistic Γ ($= 0.70$ meV) have the same features as presented in this paper.

[11]We have investigated other eigenvalues and L 's with the expected results. Finite-size studies will be interesting and can test the degree to which results near the mobility edge depend on ξ/L only. The end conditions are simple truncation of the tight-binding chain.

[12]R. Sooryakumar, A. Pinczuk, A. C. Gossard, and W. Wiegmann, Phys. Rev. B **31**, 2578 (1985).

[13]We have also worked with Eq. (6) and made it self-dual by a proper choice of N_l. These results will be presented elsewhere.

[14]G. F. Giuliani and J. J. Quinn, Phys. Rev. Lett. **51**, 919 (1983); J. K. Jain and P. B. Allen, Phys. Rev. Lett. **54**, 2437 (1985).