Experimental Observation of Bounce-Resonance Landau Damping in an Axisymmetric Mirror Plasma

M. Koepke,[a] R. F. Ellis, R. P. Majeski,[b] and M. J. McCarrick
Laboratory for Plasma and Fusion Energy Studies, University of Maryland, College Park, Maryland 20742
(Received 6 January 1986)

Suppression of the drift-cyclotron loss-cone instability (DCLC) in an axisymmetric mirror plasma has been observed for frequencies in the vicinity of the bounce frequency of electrostatically trapped thermal electrons. The location and width of the frequency region of suppression is in good agreement with calculations of bounce-resonance Landau damping.

PACS numbers: 52.35.–g, 52.55.Jd

It is well known that periodicities in particle orbits must be considered to describe accurately the response of plasma to fields fluctuating in time and space.[1,2] Energy can be transferred between trapped particles and a modulated electric[3] or magnetic[4] field if the modulation frequency ω is near a multiple of the bounce frequency, ω_b, of any of the particles. The energy exchange has a resonant character, with maxima when $\omega = n\omega_b$, where n is an integer. The case of an oscillating electric field interacting with charged particles is referred to as bounce-resonance Landau damping (or growth)[5–10] (BRLD) because the operating mechanism is the collisionless Landau interaction.[11]

While bounce-resonance damping has been discussed theoretically and invoked to explain plasma phenomena,[5,6,10] it has not been directly observed in the laboratory. In studying the drift-cyclotron loss-cone instability (DCLC) in an axisymmetric mirror machine, MIX 1, we have made the first laboratory observations of BRLD and we show that these are in good agreement with theoretical predictions.

DCLC is a microinstability of a magnetized collisionless mirror-confined plasma, which is driven by the radial density gradient and the loss-cone feature of the confined ions.[12,13] The unstable waves are characterized by a frequency which is at the ion cyclotron frequency $f_{ci} = eB/M_i 2\pi$ and its harmonics, a perpendicular wavelength $\lambda_\perp$ which is comparable to the ion gyroradius, a long parallel wavelength $\lambda_\parallel >> \lambda_\perp$, and a fluctuation amplitude which is very sensitive to the existence of a loss cone in the ion velocity distribution. The damping described here is attributed to the resonance of electrons bouncing between the ends of the mirror cell with $E_\parallel(t)$, the component of the DCLC wave electric field parallel to the magnetic field lines.

Our BRLD observations were possible because in MIX 1, DCLC is unstable for a substantial range of midplane magnetic fields and for three gas species, hydrogen, deuterium, and helium. In addition, DCLC is often unstable at many harmonics of the midplane ion cyclotron frequency. This has resulted in instability observations over the wide frequency range $f \approx 0.6$–4 MHz. Now, the electrons in MIX 1 are electrostatically confined by the mirror ambipolar potential ϕ ($e\phi \sim 5k_B T_e$, where T_e is the electron temperature) and the bounce frequency of a thermal velocity electron lies in that the same frequency range. What is observed is that in the vicinity of the fundamental bounce resonance, the DCLC mode is suppressed or stabilized.

This can be evidenced in a single shot, where a particular harmonic or harmonics are suppressed or stabilized, and is clearly shown in any extensive set of measurements of mode amplitude versus frequency, as are presented below. These data show the existence of a "damping or suppression window" and our identification of BRLD is based upon a comparison of the location and width of this suppression window with theoretical predictions of BRLD.

Turning to the theory, we obtain the relative BRLD rate from the linearized Vlasov-Poisson equations by integrating over the unperturbed periodic bounce orbits as described by McCune.[14] The dispersion relation is treated in the limit $\gamma/\omega_R << 1$, where $\omega = \omega_R + i\gamma$ is the complex instability frequency, and a Taylor expansion about $\omega = \omega_R$ is used. In this case,

$$\gamma = \sum_\alpha \gamma_\alpha = -\sum_\alpha \frac{\mathrm{Im}[\epsilon_\alpha(k,\omega_R)]}{\{\partial\,\mathrm{Re}[\epsilon(k,\omega)]/\partial\omega\}_{\omega-\omega_R}},$$

where ϵ is the plasma dielectric function and α denotes particle species, so that $\gamma_e \propto -\mathrm{Im}(\epsilon_e)$. This result is used to distinguish $\gamma_e(\omega_R)$ from $\gamma_i(\omega_R)$ in the experimentally relevant case where γ_i and $\partial\,\mathrm{Re}(\epsilon)/\partial\omega$ change negligibly over the range in ω_R for which γ_e changes dramatically. The perturbed potential is expressed as an expansion in a set of convenient basis functions,

$$\phi_1(\mathbf{x},t) = \sum_p \phi_p \exp(ik_p z + ik_\perp x_\perp - i\omega t),$$

where ϕ_p is the amplitude of the wave component with wave vector $\mathbf{k} = k_p\hat{\mathbf{z}} + k_\perp\hat{\mathbf{x}}_\perp$, $k_p = p\pi/L$, p is an integer, and L is the length of the mirror cell.

A Fourier transformation of Poisson's equation in z casts the dispersion relation in the form of a matrix eigenvalue equation for the coefficients ϕ_P. The solution of $\epsilon \cdot \boldsymbol{\phi}_1 = 0$ is obtained by setting $\det(\epsilon) = 0$, where the matrix elements of ϵ are $\epsilon^{qp} = \delta_{qp} - \epsilon_i^{qp} - \epsilon_e^{qp}$. To evaluate the electron damping alone, the eigenmodes of ϕ_1 are approximated as those for a homogeneous plasma in a square potential well, which are the same as the original basis functions. Thus, roots of the determinant are obtained from the set of uncoupled homogeneous equations $\epsilon^{qq} = 0$. From this result, $-\mathrm{Im}(\epsilon_e^{qq})$, which is proportional to γ_e, is obtained.

The one-dimensional electron dielectric function is Fourier transformed by representing the periodic position z' as a series in the orbit phase $\omega_b(t'-t) + \psi_0$, where ψ_0 corresponds to the particle's phase in its own orbit at time t, and rewriting the time integral with $\tau = t' - t$. A transformation from (z, v) space to (E, ψ_0) is made and the transform is then calculated, leading to

$$\epsilon_e^{qp} = -\frac{4\pi e^2}{m_e k^2} \int_0^\infty dE \frac{\partial f_{e0}(E)}{\partial E} \sum_{n,m} S_n(E,k_p) S_m^*(E,k_q)\, in \frac{\pi}{L} \int_0^\pi (d\psi_0/\pi)\, e^{-i(n-m)\psi_0} \int_{-\infty}^0 d\tau\, e^{i(n\omega_b - \omega)\tau}, \tag{1}$$

where

$$S_n(E,k_q) = \frac{1}{t_b} \int_{-t_b/2}^{t_b/2} dt' \exp[-in\omega_b(E)t' + ik_q z'], \tag{2}$$

$t_b = 2\pi/\omega_b$, and the particles are assumed to be distributed uniformly in initial orbit phase ψ_0.

The resonant denominator $(n\omega_b - \omega)$ that arises from the integral over τ in Eq. (1) is rewritten as $n(E - E_{\omega/n})[\partial\omega_b(E)/\partial E]_{E_{\omega/n}}$, where $\omega_b(E)$ has been expanded in a Taylor series about $E_{\omega/n}$, the energy defined by $\omega_b(E_{\omega/n}) = \omega/n$. Integrating over ψ_0 and E in Eq. (1) yields the electron bounce-resonance damping rate of the qth eigenmode,

$$\gamma_e^q \propto -\mathrm{Im}(\epsilon_e^{qq}) = \frac{4\pi^2 e^2}{m_e k^2} \sum_{n=-\infty}^{\infty} \frac{\pi}{L} \left[\frac{\partial f_{e0}(E)}{\partial E} \; \frac{S_n(E,k_q) S_n^*(E,k_q)}{\partial\omega_b(E)/\partial E} \right]_{E_{\omega/n}}. \tag{3}$$

From this point Eq. (3) can be applied to various potential shapes. We consider electrons which are electostatically trapped in a positive ambipolar potential ϕ which is modeled[14] as

$$\phi/m = 0, \quad |z| \leq z_0, \quad \phi/m = \tfrac{1}{2}\omega_{b0}^2(|z| - z_0)^2(1 + |z|/L_0)^2, \quad z_0 \leq |z| \leq L/2, \tag{4}$$

where m is electron mass, z is electron position from midplane, z_0 and L_0 are shape parameters, and ω_{b0} is a constant. The major role of the confining-potential model is to provide a functional for $\omega_b(E)$. For ϕ of Eq. (4) the approximate result is

$$\omega_b(x) = \omega_{b0}(1 + a/x)/(1 + b/x), \tag{5}$$

where $x = |v|(2k_B T_e/m_e)^{-1/2}$, $a = 2L(k_B T_e)^{1/2}/(\pi L_0 \phi_M^{1/2})$, $b = 4z_0\phi_M^{1/2}/(\pi L k_B T_e^{1/2})$, and ϕ_M is the maximum value of ϕ. Equation (5) is valid for a and b less than 0.3; ϕ is parabolic for $a = 0$, $b = 0$, and approximates a square well for $a = 0.3$, $b = 0.3$.

$S_n S_n^*$ in Eq. (3) is relatively insensitive to the shape of the confining potential and is approximated in the appropriate limit of the square well, for which $\omega_b(E) = (\pi/L)(2E/m)^{1/2}$,

$$S_n(E,k_p)S_n^*(E,k_q) = \frac{i^{q-p}}{4}\left[\mathrm{sinc}\frac{(n-p)\pi}{2}\, \mathrm{sinc}\frac{(n-q)\pi}{2} + \mathrm{sinc}\frac{(n+p)\pi}{2}\, \mathrm{sinc}\frac{(n+p)\pi}{2} + (-1)^n \left\{ \mathrm{sinc}\frac{(n-p)\pi}{2}\, \mathrm{sinc}\frac{(n+q)\pi}{2} + \mathrm{sinc}\frac{(n+p)\pi}{2}\, \mathrm{sinc}\frac{(n-q)\pi}{2} \right\} \right], \tag{6}$$

where $\mathrm{sinc}(\theta) = \sin\theta/\theta$.

We next use a Maxwellian distribution function in Eq. (3), with the measured T_e, and $n = p = q = 1$, consistent with the experimental observations. The magnitude of $\phi(z=0)$ is measured (see below) and the only unmeasured quantities are the shape parameters a and b which are obtained by a fit to the experimental data. Figure 1 shows the potential well shapes for the values $a = 0.15$ and $b = 0.12$.

Returning to the experiment, the MIX 1 mirror machine[15] was operated for two cases of mirror cell length, (a) $L = 92.5$ cm and (b) $L = 60$ cm, as illustrated in Fig. 2. The plasma is described by a central density of $(10^{12}\ \mathrm{cm}^{-3})\exp[-t/(40\ \mu\mathrm{sec})]$, a Gaussian radius of 4 cm, temperatures of 100 eV for ions and ~ 10 eV for electrons, and a midplane potential of $5k_B T_e$. The electron temperature was measured by Langmuir probes, the ion temperature by perpendicular and parallel ion energy analyzers, and the ambipolar

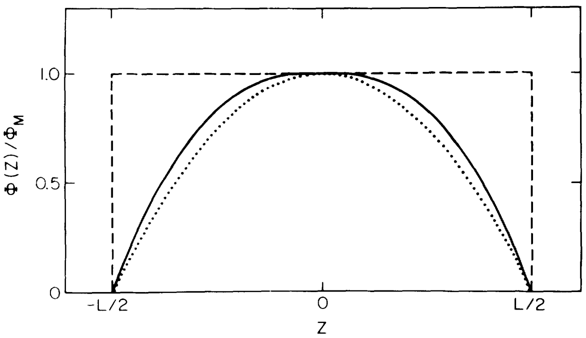

FIG. 1. Theoretical models of the electron-confining potential-well profile $\phi(z)$; ϕ_M is simply the maximum value. Dashed line is the square well, dotted line is the parabolic well, and the solid line is an intermediate well with $a=0.15$ and $b=0.12$.

potential by the ion energy analyzers and an emissive probe. For this value of ϕ/k_BT_e, electrostatic trapping is the primary confining mechanism.

The unstable DCLC waves had narrow spectral features at $f=1.1lf_{ci}$, $l=1,2,3,\ldots$, and they propagated in the ion diamagnetic direction with $\lambda_\perp \simeq 2.5$ cm and a radial structure peaked at a radius of 3–4 cm. The axial structure was carefully measured[15] to be an axial standing wave with $\lambda_\parallel \simeq 2L$; this deviation from flutelike mode structure is necessary for BRLD, since it indicates a nonzero $E_\parallel(t)$. It also justifies the use of $p=q=1$ in the theoretical model.

The evidence for the existence of BRLD was obtained by measurement of the mode power spectrum as the frequency of the mode was varied by varying the magnetic field. The amplitude of the relative rms density fluctuation $(\tilde{n}/n)$ in a small spectral bandwidth at f_{DCLC} was measured for a range of magnetic fields, for the two mirror cell lengths. These data were recorded at a radius of 3 cm (near maximum) and for each shot the amplitude was averaged over 10 μsec.

In Fig. 3(a) we show mode amplitude versus ω/ω_{BT} for a deuterium plasma with a mirror length of $L=92.5$ cm, where $\omega_{BT}=\pi(2k_BT_e/m_e)^{1/2}/L$; ω_{BT} is the bounce frequency of a thermal electron in a square well of length L and is a convenient normalization. Each datum point is the average of at least five shots and we see quite clearly a strong suppression of the mode centered near where $\omega \simeq \omega_{BT}$. In Fig. 3(b) we show data for deuterium and helium for the mirror length $L=60$ cm and we see again a very clear suppression region in fequency. In Fig. 3(c) we show the behavior of the electron bounce-resonance contribution to the damping rate as determined by Eqs. (3)–(6) and the experimental parameters, where a and b have been varied to obtain a best fit. The shape of the corresponding potential profile is shown in Fig. 1. Figure 3 shows very good correlation between the calculated damping rate and the mode-suppression windows with respect to both the location and width of the windows. The fact that good agreement was obtained for two different mirror lengths, but the same confining potential shape, gives us great confidence in the results.

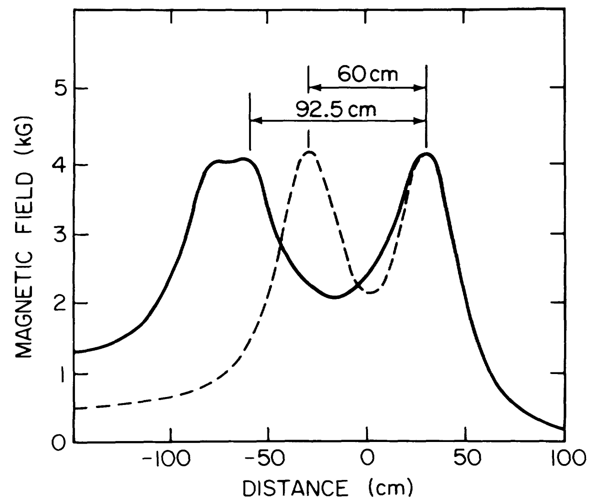

FIG. 2. Axial profile of magnetic field on axis in MIX 1, showing the two different length mirror cells.

As an additional point we note that the data shown

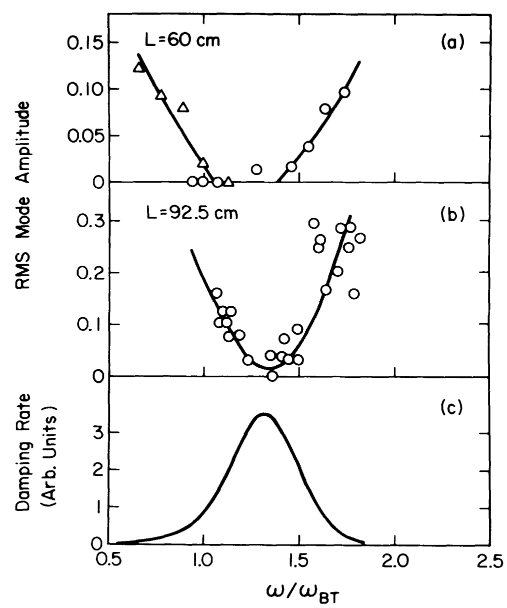

FIG. 3. (a) Variation of measured DCLC mode amplitude with normalized mode frequency, ω/ω_{BT}, for deuterium in the 92.5-cm-long mirror cell, with $T_e=8.6$ eV. Solid line is to guide the eye. (b) Same as (a) for deuterium (circles) and helium (triangles) in the 60-cm-long mirror cell, with $T_e=5.5$ eV for deuterium and 3.3 eV for helium. (c) Relative linear damping rate from Eq. (3) for the experimental parameters and $a=0.15$ and $b=0.12$.

here pertain to instability at the fundamental ion cyclotron frequency. In some cases DCLC in MIX 1 is unstable at many harmonics of f_{ci} and for these cases any harmonic with $\omega \simeq \omega_{BT}$ is strongly suppressed.

Turning to alternative explanations, we note that measurements of the plasma properties, in particular ion energies and ambipolar potential, do not show any change as B is varied which could account for the observed suppression. In addition, we have calculated the DCLC growth rate versus B from a slab-model code excluding BRLD, and no damping window appears in this theory. Moreover, data taken with hydrogen for the short mirror show no dependence of mode amplitude on magnetic field for $2.5 < (\omega/\omega_{BT}) < 3.8$. Finally, we have calculated the dominant nonlinear saturation mechanism, which is wave-induced quasilinear diffusion, and have verified experimentally that this is the most important nonlinear process; these results do not indicate any suppression window as described here.

In conclusion, for the first time, the phenomenon known as bounce-resonance damping has been experimentally verified. This damping severely reduces the amplitude of the DCLC mode for frequencies comparable to the axial bounce frequency of thermal electrons. Calculated damping rates accurately predict the location and width of the so-called suppression window in frequency for two mirror lengths and two ion species.

It is a pleasure to acknowledge useful discussions with Dr. Michael J. Gerver concerning the phenomenon of bounce-resonance damping. This work was supported by the National Science Foundation.

[a]Present address: Department of Nuclear Engineering, University of Washington, Seattle, Wash. 98195.

[b]Present address: Department of Nuclear Engineering, University of Wisconsin, Madison, Wis. 53706.

[1]J. E. Drummond, R. A. Gerwin, and B. G. Springer, J. Nucl. Energy, Part C **2**, 98 (1961).

[2]H. L. Berk and L. D. Pearlstein, Phys. Fluids **14**, 1810 (1971).

[3]A. N. Kaufman and T. Nakayama, Phys. Fluids **13**, 956 (1970).

[4]J. M. Berger, W. A. Newcomb, J. M. Dawson, E. A. Frieman, R. M. Kulsrud, and A. Lenard, Phys. Fluids **1**, 301 (1958).

[5]B. I. Meerson, P. V. Sasorov, and A. V. Stepanov, Sol. Phys. **58**, 165 (1978).

[6]V. I. Karpman, B. I. Meerson, A. B. Mikhailovsky, and O. A. Pokhotelov, Planet. Space Sci. **25**, 573 (1977).

[7]K. Nishihara, A. Hasegawa, C. G. Maclennan, and L. J. Lanzerotti, Planet. Space Sci. **20**, 747 (1972).

[8]P. Rutherford, M. Rosenbluth, W. Horton, E. Frieman, and B. Coppi, in *Proceedings of the Third International Conference on Plasma Physics and Controlled Nuclear Fusion Research, Novosibirsk, U.S.S.R., 1968* (International Atomic Energy Agency, Vienna, Austria, 1969), Vol. 1, p. 367.

[9]J. G. Cordey, Phys. Fluids **12**, 1506 (1969).

[10]W. M. Sharp, H. L. Berk, and C. E. Nielsen, Phys. Fluids **22**, 1975 (1979).

[11]L. D. Landau, J. Phys. (Moscow) **10**, 25 (1946).

[12]A. B. Mikhailovskii, Dokl. Akad. Nauk. SSSR **169**, 554 (1966) [Sov. Phys. Dokl. **11**, 603 (1967)].

[13]R. F. Post and M. N. Rosenbluth, Phys. Fluids **9**, 730 (1966).

[14]J. E. McCune, Oak Ridge National Laboratory Report No. TM-3970, 1972 (unpublished).

[15]M. E. Koepke, Ph.D. dissertation, Department of Physics, University of Maryland, 1984 (unpublished).