Dilaton and Chiral-Symmetry Breaking

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The spontaneous breaking of chiral symmetry in certain gauge models may also imply the spontaneous breaking of an approximate scale symmetry. This breaking will produce the dilaton as a pseudo-Goldstone boson of spontaneously broken scale invariance.

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In four dimensions, Yang-Mills gauge theories possess a classical scale symmetry as there are no dimensional parameters associated with the classical formulation of the theory. In perturbation theory, the quantum fluctuations of the gauge fields produce an explicit breaking of the scale symmetry and cause the running of the gauge coupling constant through the effects of renormalization. In quantum chromodynamics, this running is responsible for the asymptotic freedom and the observed logarithmic scaling violations of deep-inelastic processes. These scaling violations also lead to the nonperturbative aspects of confinement which produce the hadron mass scale through the mechanism of dimensional transmutation. In the confinement process, the effects of the explicit breaking of scale symmetry are dominant, and there is no remaining consequence of the original classical scale symmetry.

The addition of fermions in low representations of the gauge group does not seem to alter this situation. The confinement process in QCD seems to trigger the spontaneous breaking of chiral symmetry for the light quarks, which are in the fundamental triplet representation of the color gauge group. This spontaneous breaking produces the observed Goldstone bosons of chiral symmetry, the pions.

However, there may be other situations where the spontaneous breaking of chiral symmetry occurs when the explicit breaking of scale symmetry is not a dominant effect. In this situation the spontaneous breaking of chiral symmetry may imply the spontaneous breaking of an approximate scale symmetry. There is an indication from numerical studies of lattice gauge theory that the scale of chiral-symmetry breaking for fermions in higher representations of the gauge group can be relatively short compared to the confinement scale.[1] If the chiral condensation occurs at a sufficiently short distance scale, then the slow logarithmic running of the coupling constant at that scale may reflect only a weak breaking of the scale symmetry of the gauge interactions compared to the large spontaneous breaking of scale symmetry associated with the chiral condensation.

In terms of the one-gluon-exchange approximation, the attractive force causing the fermion condensation has a strength characterized by the product $c_2(f) \times g^2(\mu)$, where $c_2(f)$ is the quadratic Casimir operator of the fermion representation and $g(\mu)$ is the running coupling constant of the underlying gauge theory. The rough estimate of the chiral symmetry breaking scale is provided by the criterion that $c_2(f)g^2(\mu)$ must reach a certain critical value for the condensation to begin. If the gauge coupling constant is in a domain where $g^2(\mu)$ varies only logarithmically with energy, it follows that small changes in the value of the Casimir, $c_2(f)$, can lead to very different scales of chiral-symmetry breaking.

Marciano has made the interesting suggestion[2] that exotic quarks belonging to higher-dimensional representations (e.g., sextets) of the $SU(3)_c$ color gauge group might form chiral-symmetry–breaking condensates at a scale of order 100 GeV. If these exotic quarks were also to carry the appropriate weak charges of the standard model, then the condensates would dynamically break the electroweak $SU(2) \otimes U(1)$ gauge symmetries and generate masses for the W and Z bosons with the usual relation, $M_W = M_Z \cos\theta_W$. This dynamical Higgs mechanism can thus serve as an alternative to the hypercolor scenario[3] in which entirely new gauge interactions are introduced to provide the dynamical symmetry breaking required to give the W and Z bosons mass. Of course, there may be other situations where fermions in higher representations of the gauge group produce a hierarchy of condensation scales at distances short compared to the confinement scales and where the explicit breaking of scale symmetry is suppressed.

We wish to study the consequences of the approximate scale symmetry associated with the chiral condensates which occur in the region where the gauge coupling constants are slowly running but where the effective fermion coupling constants, $c_2(f)g^2(\mu)$, have reached the critical value. In such cases, it should be possible to approximate the gauge theory by an effective theory with a fixed but critical coupling constant; the running coupling is only required to assure that the effective coupling reaches the critical value. The theory with the fixed coupling constant now possesses an exact scale invariance in the chiral-symmetry limit. When the chiral symmetry is spontaneously broken, the scale invariance is also broken spontaneously, resulting in the appearance of a massless, fermion-antifermion scalar bound state, the dilaton, which is the Goldstone boson of scale symmetry. Of course, the actual coupling constant is not fixed, and the scale symmetry is explicitly broken by the effects of renormalization. Consequently, the dilaton associated with these condensates should appear as a pseudo-Goldstone boson with a mass of order of the scale of the running coupling constant, which is roughly the confinement scale in QCD, $\Lambda_{\rm QCD}$.

We wish to examine the fixed–coupling-constant theory for evidence of spontaneously broken scale symmetry. Since the gauge coupling constant is small compared to the effective fermion coupling constant in the theories described above, it is probably sufficient to consider a further truncation of the non-Abelian theory to an effective Abelian theory with fixed coupling constant which we will analyze in the one-gluon-exchange, planar or "ladder," approximation. This planar approximation may actually correspond to a large-N limit of the non-Abelian theory where the Casimir operator for the fermions, $c_2(f)$, is also taken to infinity.

Chiral-symmetry breaking has been extensively studied in the one-gluon-exchange approximation. Johnson, Baker, and Willey[4] first obtained the chiral-symmetry–breaking fermion self-energy function as a solution of the homogeneous, massless Schwinger-Dyson equation. However, these solutions do not correspond to spontaneous breaking of chiral or scale symmetry even though no explicit mass term was apparently included. The scaling behavior of these solutions reflects the fact that the fermion mass operator, $\bar{\psi}\psi$, has a finite anomalous dimension for fixed coupling constant.[5] The situation concerning these solutions was clarified by Maskawa and Nakajima[6] who carefully studied the theory using cutoffs to control the short-distance divergences. They found that there was no spontaneous breaking of chiral symmetry for weak coupling as all nontrivial solutions of the Schwinger-Dyson equation required an explicit bare-mass term with a cutoff dependence which reflected the anomalous dimension of the mass operator. They also found evidence for a spontaneously broken phase at strong coupling.

Fukuda and Kugo[7] have computed a complete set of numerical solutions to the Schwinger-Dyson equation for both weak and strong coupling. The strong-coupling solution for the self-energy function in the massless case requires the solution of the gap equation,[8]

$$0=\sin\{(\alpha/\alpha_c-1)^{1/2}\ln[e^{\delta}\Lambda/\Sigma(0)]\}+(\alpha/\alpha_c-1)^{1/2}\cos\{(\alpha/\alpha_c-1)^{1/2}\ln[e^{\delta}\Lambda/\Sigma(0)]\}, \quad (1)$$

where $\alpha=g^2/4\pi$ is the effective gauge coupling constant, $\alpha_c=\pi/3$ is the critical coupling for the strong-coupling phase ($\alpha>\alpha_c$), and $\delta\approx 0.55$ is a parameter in the asymptotic expansion of the fermion self-energy function $\Sigma(p)$. This equation must be solved for the fermion mass scale, $\Sigma(0)$. Although Eq. (1) has an infinite number of solutions for $\Sigma(0)$, only the largest value of $\Sigma(0)$ corresponds to the ground state as all other solutions have higher vacuum energy. The ground-state solution for $\Sigma(p)$ is also the only solution without nodes. For $\alpha/\alpha_c\approx 1$, the relevant solution is given by

$$0=\pi-(\alpha/\alpha_c-1)^{1/2}\ln[e^{\delta}\Lambda/\Sigma(0)]-(\alpha/\alpha_c-1)^{1/2} \quad (2a)$$

or

$$\Sigma(0)=e^{\delta+1}\Lambda\exp[-\pi/(\alpha/\alpha_c-1)^{1/2}]. \quad (2b)$$

This result appears to be a disaster as the fermion mass scale diverges with the cutoff, and all the dynamics associated with the spontaneous chiral-symmetry breaking occurs at the cutoff scale, Λ. This conclusion was also reached in the numerical studies of Bartholomew *et al.*[1]

However, this conclusion may not be the only interpretation of these solutions of the gap equation. Miransky[9] has argued that the critical coupling α_c should be viewed as an ultraviolet fixed point of the strong-coupling phase. We must then require that the gauge coupling constant α approaches the critical value as the cutoff tends to infinity. His limiting behavior is given by

$$\alpha/\alpha_c=1+\pi^2/\ln^2(\Lambda/\kappa), \quad \Lambda\to\infty. \quad (3)$$

This behavior for the gauge coupling constant generates an infrared scale proportional to κ with the fermion mass scale given as

$$\Sigma(0)=e^{\delta+1}\Lambda\exp\left[\frac{-\pi}{(\alpha/\alpha_c-1)^{1/2}}\right]=e^{\delta+1}\kappa. \quad (4)$$

Miransky's fixed-point interpretation appears to give a complete treatment of the strong-coupling phase. Chiral symmetry is spontaneously broken in this phase generating a finite fermion mass scale. The Bethe-Salpeter equations also have a massless solution for the pseudoscalar bound state, the Goldstone boson of spontaneous chiral-symmetry breaking.

Unfortunately, this picture appears to be incomplete as there is no associated scalar bound-state solution to the Bethe-Salpeter equations for the dilaton. Although the original theory was completely scale invariant, Miransky's fixed-point solution appears to break the scale symmetry explicitly. The possible resolution to this paradox comes from the existence of a different scale-invariant fixed-point solution for the theory.[8] This solution results from the observation that there are additional relevant operators in the strong-coupling phase which necessarily mix with the pure electromagnetic interactions. This mixing generates the new interactions and the scale-invariant fixed point occurs for nontrivial values of the new coupling constants. The dynamical generation of new interactions at the scale-invariant fixed point also occurred for the exact solution of the large-N limit of $\eta\phi^6$ theory[10] which exhibits spontaneous breaking of scale symmetry in a similar strong-coupling phase.

In our case the new interactions are just the four-fermion operators involving the scalar and pseudoscalar densities. We have previously observed that the fermion mass operator $\bar{\psi}\psi$ has an anomalous dimension at finite gauge coupling, being of dimension three at zero coupling and approaching dimension two at the critical point. In the planar limit we are using, the dimension of the four-fermion operator $(\bar{\psi}\psi)^2$ is just twice the dimension of the mass operator and approaches dimension four at the critical coupling for the gauge interactions, $\alpha=\alpha_c$. To study the fixed points of the vector-gluon theory we must include the relevant four-fermion interactions. These induced interactions must preserve the chiral symmetry of the vector-gluon interactions.

We may study the scale-invariant fixed point using the fermion Lagrangean,

$$L_f=\bar{\psi}\{i\gamma\cdot\partial-g\gamma\cdot A-\mu_0\}\psi+\tfrac{1}{2}G_0[(\bar{\psi}\psi)^2+(\bar{\psi}i\gamma_5\psi)^2], \tag{5}$$

where we have included a fermion mass term to provide explicit breaking in addition to the induced four-fermion interactions. The consistent treatment of the induced terms requires that we keep only the planar diagrams with the same structure as the ladder diagrams of the vector-gluon interactions. The new diagrams are just those of the large-N, chirally invariant Gross-Neveu model[11] except that we must now include the radiative corrections of the vector-gluon theory as shown in Fig. 1. These radiative corrections effectively make the Gross-Neveu model renormalizable in four dimensions.

The solutions for the vacuum structure of the modified theory can be obtained by use of exactly the same methods as used for the pure vector-gluon theory. The only modification of the Schwinger-Dyson equation depicted diagrammatically in Fig. 2 involves the replacement of the bare mass parameter m_0 by an effective bare mass which includes a term generated by the induced interactions:

$$m_0=\mu_0-G_0\langle\bar{\psi}\psi\rangle_0. \tag{6}$$

The vacuum expection value of the fermion bilinear must be computed self-consistently. Even in the chiral limit, $\mu_0=0$, we can expect that the effective bare mass will not vanish, $m_0\neq0$, due to the induced terms. This modification leads to a new gap equation,

$$\mu_0\Lambda=\tfrac{1}{2}\tilde{A}\Sigma^2(0)(\{[1-G_0(\alpha_c/\alpha)\Lambda^2/\pi^2]/(\alpha/\alpha_c-1)^{1/2}\}\sin\{(\alpha/\alpha_c-1)^{1/2}\ln[e^\delta\Lambda/\Sigma(0)]\}$$
$$+[1+G_0(\alpha_c/\alpha)\Lambda^2/\pi^2]\cos\{(\alpha/\alpha_c-1)^{1/2}\ln[e^\delta\Lambda/\Sigma(0)]\}), \tag{7}$$

where $\tilde{A}\approx1.2$ is another parameter of the asymptotic expansion of the self-energy function. The power dependence on the cutoff, Λ, implied for μ_0 and G_0 is precisely that expected from the anomalous dimensions of the mass and four-fermion operators. We must solve Eq. (7) to obtain the fermion mass scale, $\Sigma(0)$. As before only the largest-mass solution corresponds to the ground state, and we obtain the result

$$\Sigma(0)=e^\delta\Lambda\exp[-\theta/(\alpha/\alpha_c-1)^{1/2}], \tag{8}$$

where $0<\theta\leq\pi$. This result is similar to Miransky's except that the angle θ will not necessarily be near π and the approach to the critical point for the gauge coupling constant, i.e., the beta functions, will differ. Of course, it is clear that $\alpha=\alpha_c$ remains the ultraviolet critical point for the gauge coupling constant.

The gap equation, Eq. (7), has solutions for any value of the four-fermion coupling constant G_0. However, these solutions will generally be expected to break the scale symmetry unless G_0 approaches a fixed-point value. We can search for the scale-invariant fixed point by examining the fermion-antifermion scattering amplitudes for the poles associated with the Goldstone bosons of chiral and scale symmetry. The pure ladder diagrams will not generate even the pseudoscalar bound state as the effective bare mass m_0 will not vanish in the chiral limit except when

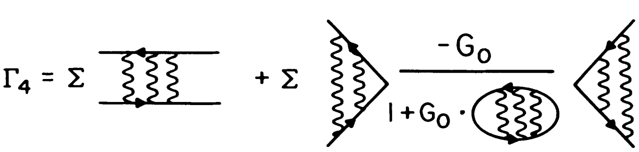

FIG. 1. The fermion-antifermion scattering amplitude.

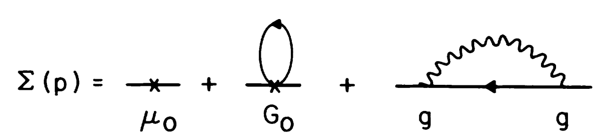

FIG. 2. The Schwinger-Dyson equation.

$G_0=0$. The poles will come instead from the bubble denominators of the induced diagrams as shown in Fig. 1. We calculate the bubble functions for both scalar and pseudoscalar channels in Ref. 8. As expected, the pseudoscalar denominator always vanishes in the chiral limit ($\mu_0=0$). The scalar denominator should also vanish, but only for a particular value of the four-fermion coupling constant $G_0\Lambda^2=\pi^2(\alpha/\alpha_c)$. This value determines the ultraviolet fixed point of the vector-gluon theory in the planar approximation. Hence, the vector-gluon theory would preserve the scale symmetry in the strong-coupling phase if the operator mixing is properly taken into account. The anticipated pole in the scalar amplitude is precisely the dilaton pole expected on the basis of our original speculations.

Actually, in our explicit calculations of Ref. 8, the scalar denominator does not completely vanish even at the fixed point. Instead, there remains a residual contribution which we believe is a reflection of the failure of the planar approximation to preserve the scale symmetry. We expect the scalar denominator to vanish at the fixed point of a consistent, scale-invariant approximation to QED as may be the case for the full, quenched approximation. Unfortunately, we have been able to perform an analytic analysis only for the planar approximation. We thus would encourage other approaches, such as lattice methods, to search for signals of the dilaton.

Explicit breaking of both the scale and chiral symmetries can be introduced through the fermion mass terms, $\mu_0\neq 0$. Using our methods, we can compute the effects of this symmetry breaking on the masses of the Goldstone bosons. As expected the masses are linear in μ_0 for small symmetry breaking.[8] In the actual application of our ideas to non-Abelian gauge theories, there will also be explicit breaking of scale symmetry coming from the slow running of the gauge coupling constant. The dilaton will get mass from these effects as well; unfortunately, we are not yet able to compute the impact of these corrections.[12]

We have speculated that spontaneous breaking of chiral symmetry in gauge theories can be accompanied by the spontaneous breaking of an approximate scale symmetry. In these situations, the dilaton will exist as a light scalar particle, the Goldstone boson of the scale symmetry. The coupling of the dilaton to the particles which become heavy at the scale of the chiral breaking will be dictated by the appropriate current-algebra relations. In the Marciano scheme, the dilaton interacts with particles of the 100-GeV scale, such as the W and Z bosons. This could have interesting consequences although the dilaton, like the physical Higgs particle, may be difficult to detect as it has vacuum quantum numbers. It may also be interesting to investigate the dilaton scenario in supersymmetric gauge theories where fermions transforming as higher-dimensional representations of $SU(3)_c$ occur naturally, namely, the gluinos. These aspects are currently under investigation.

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