Semiclassical Theory of Light Detection in the Presence of Feedback

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The usual open-loop semiclassical theory of light detection is extended to include closed-loop operation in which there is feedback from the detector to the source. The revised theory takes the form of a self-exciting point process. It admits photocount statistics that are associated with non-classical light in open-loop configurations, e.g., sub-shot-noise spectra and sub-Poisson photocounts. The extended theory is used to explain two recent closed-loop experiments in which sub-Poisson photocount statistics were produced.

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The usual formulations of the quantum[1–5] and semiclassical[5–8] theories of photodetection presume open-loop configurations, i.e., that there are no feedback paths leading from the output of the photodetector to the light beam impinging on that detector. In such configurations, the qualitative and quantitative distinctions between the quantum and semiclassical theories are well understood. In the quantum theory, photocurrent and photocount randomness arise from the quantum noise in the illumination beam, whereas in the semiclassical theory the fundamental source of randomness is associated with the excitations of the atoms forming the detector. Nevertheless, the quantum theory subsumes the semiclassical theory in a natural way.[3–5]

In this Letter, we examine the semiclassical theory of light detection for closed-loop configurations. We demonstrate that the unmistakable signatures of non-classical light associated with open-loop detection do not carry over to closed-loop systems. To illustrate this most clearly, we begin with a recounting of the open-loop results.

Consider an open-loop photodetection system in which quasimonochromatic light of nominal frequency ν_0 illuminates the active region of the detector. The quantities of interest at the output are the photocurrent i_t and the photocount record N_t. The former is a train of impulses (each of area q with q the electronic charge) located at the photodetection event times. The latter counts the number of such events that have occurred in the time interval $[0,t)$, and is given by

$$N_t = (1/q)\int_0^t i_\tau \, d\tau. \tag{1}$$

In the open-loop semiclassical theory, the event times underlying both i_t and N_t comprise a doubly stochastic Poisson point process (DSPP).[5,7–9] The conditional probability per unit time that there be a photodetection event at the time t, given the power illuminating the detector P_t, is known as the conditional rate function μ_t. For the process at hand,

$$\mu_t = \eta P_t / h\nu_0, \tag{2}$$

where η is the quantum efficiency of the detector, P_t is the (possibly random) power illuminating the detector, and $h\nu_0$ is the photon energy. If P_t is a stationary random process it follows that the mean value and the bilateral covariance (noise) spectrum of the photocurrent are $\langle i \rangle = q\eta\langle P\rangle / h\nu_0$ and

$$S_{ii}(f) = q\langle i\rangle + (q\eta/h\nu_0)^2 S_{PP}(f),$$

respectively, in terms of the like quantities for the illumination power P. Physically, the first term on the right-hand side of the expression for $S_{ii}(f)$ is the usual shot noise and the second term is the (nonnegative) power-randomness excess noise associated with the semiclassical theory. For arbitrary P_t, the semiclassical theory yields Mandel's rule[6] for the photocount distribution and always leads to super-Poisson behavior, viz., $\mathrm{Var}(N_t) \geqslant \langle N_t\rangle$.

In the open-loop quantum theory, the event times underlying i_t and N_t form a self-exciting point process (SEPP) rather than a DSPP.[3,9] The kth-order multicoincidence rate (MCR) then satisfies[1-4]

$$w_k(t_1,t_2,\ldots,t_k)=\eta^k\,\mathrm{Tr}\left\{\hat\rho\int_{A_d}d^3x_1\cdots\int_{A_d}d^3x_k\left[\prod_{i=1}^{k}\hat E^\dagger(\mathbf{x}_i,t_i)\right]\left[\prod_{i=1}^{k}\hat E(\mathbf{x}_i,t_i)\right]\right\},\quad k=1,2,\ldots,\tag{3}$$

where $\hat\rho$ is the density operator for the field, $\hat E(\mathbf{x},t)$ is the positive-frequency photon-units field operator, and A_d represents the active area of the photodetector. Multiplying the kth-order MCR by Δt^k and letting $\Delta t\to 0$ yields the joint probability that events are registered in time intervals Δt about the times $t_1, t_2,\ldots,t_k$. In the special case of a classical state, Eq. (3) yields the semiclassical results because[3]

$$w_k(t_1,t_2,\ldots,t_k)=\left\langle\prod_{i=1}^{k}(\eta P_{t_i}/h\nu_0)\right\rangle_{\text{P-rep}},\tag{4}$$

where

$$P_t=h\nu_0\left\langle\boldsymbol{\alpha}\left|\int_{A_d}d^3x\,\hat E^\dagger(\mathbf{x},t)\hat E(\mathbf{x},t)\right|\boldsymbol{\alpha}\right\rangle,\tag{5}$$

with $|\boldsymbol{\alpha}\rangle$ the multimode coherent state.

The class of SEPPs is broader than that of DSPPs,[9] so that when the density operator $\hat\rho$ does not represent a classical state, the quantum theory permits photodetection statistics that are impossible in the semiclassical construct. In particular, for stationary processes, the quantum theory can give rise to a sub-shot-noise spectrum

$$S_{ii}(f)<q\langle i\rangle\tag{6}$$

and a sub-Poisson photocount distribution

$$\mathrm{Var}(N_t)<\langle N_t\rangle,\tag{7}$$

the latter even if the stationarity assumption is relaxed. In open-loop photodetection, either of these conditions provides an unambiguous signature of quantum (i.e., nonclassical state) light. Such nonclassical light has recently been observed in two open-loop experiments (albeit with some difficulty).[10,11] In each case, the physical locus for the SEPP behavior can be readily identified. In the Short-Mandel experiment,[10] it is the dead time associated with successive atomic emissions. In the Teich-Saleh experiment,[11] it is the space charge associated with the electron beam. Indeed, a renewal point process (which is a special case of a SEPP) was used to model the statistics of this beam.[12]

The theoretical results for open-loop systems do not carry over to closed-loop photodetection. We proceed now to reexamine the semiclassical and quantum photodetection properties of i_t and N_t when the output of the photodetector is permitted to affect the light at its input through a causal, but possibly nonlinear, feedback loop. In *both* the semiclassical and quantum formulations it then turns out that the photodetection event times form a SEPP. The most convenient way to specify the SEPP statistics is via the conditional rate function[9] $\mu_t(\{i_\tau:\tau<t\})$ representing the conditional probability per unit time that there be a photodetection event at time t, given the past history of these events. In the semiclassical theory a simple relationship prevails,

$$\mu_t(\{i_\tau:\tau<t\})=\eta P_t(\{i_\tau:\tau<t\})/h\nu_0,\tag{8}$$

where the power P_t depends explicitly on the event history through the feedback law. It is this explicit dependence that renders the closed-loop semiclassical-theory SEPP more general than the DSPP. In the quantum theory the appropriate relation is

$$\mu_t(\{i_\tau:\tau<t\})=\eta\,\mathrm{Tr}\{\hat\rho_t(\{i_\tau:\tau<t\})\int_{A_d}d^3x\,\hat E^\dagger(\mathbf{x},t)\hat E(\mathbf{x},t)\},\tag{9}$$

where it is the density operator that carries the explicit feedback information. Although general expressions are available[9] for the photocurrent moments of stationary SEPPs and the photocount distributions of arbitrary SEPPs (i.e., the replacement for Mandel's rule), they are not sufficiently explicit to warrant inclusion here.

As a first example, we consider the nonparalyzable dead-time-modified Poisson process (DTMPP). Explicit counting statistics are available for the DTMPP, which is a renewal process and a SEPP.[9,12-15] For a detector with fixed dead time τ_d, illuminated by nonrandom light of constant power P, the photocount distribution $P_n(N_t=n)$ is[14,15]

$$P_n(N_t=n)=\begin{cases}\displaystyle\sum_{k=0}^{n}p_0\{k,\mu[t-n\tau_d]\}-\sum_{k=0}^{n-1}p_0\{k,\mu[t-(n-1)\tau_d]\}, & n<t/\tau_d,\\ \displaystyle 1-\sum_{k=0}^{n-1}p_0\{k,\mu[t-(n-1)\tau_d]\}, & t/\tau_d\le n<t/\tau_d+1,\\ 0, & n\ge t/\tau_d+1,\end{cases}\tag{10a}$$

where

$$p_0\{k,a\} = a^k e^{-a}/k! \text{ and } \mu = \eta P/h\nu_0. \tag{10b}$$

The results are valid for a detector that is unblocked at the beginning of each counting interval, although exact number distributions for counters that are blocked and for equilibrium counters are also available. In the usual situation, the mean count is much greater than unity in which case the differences arising from the three initial conditions are insubstantial and a simple approximation for the photocount distribution suffices.[15] The photocount mean and variance then take the asymptotic forms[12]

$$\langle N_t \rangle = \mu t/(1+\mu\tau_d) \tag{11}$$

and

$$\mathrm{Var}(N_t) = \langle N_t \rangle/(1+\mu\tau_d)^2, \tag{12}$$

representing sub-Poisson behavior for all values of $\mu\tau_d$.

The DTMPP results are relevant to the experiments recently carried out by Walker and Jakeman.[16] The simplest form of their experimental arrangement is illustrated in Fig. 1(a). The registration of a photoevent at the detector operates a trigger circuit that causes an optical gate to be closed for a fixed period of time τ_d following the time of registration. During this period, the power P_t of the (He-Ne) laser illuminating the detector is set precisely equal to zero so that no photodetections are registered. The arrangement is therefore equivalent to that illustrated in Fig. 1(b) in which the gating is electronic rather than optical, at least as far as the photocount statistics are concerned. This latter arrangement was used by Teich and Vannucci.[15] Sub-Poisson photocounts were observed in both cases. This is because the point process seen by the counter in these experiments is the DTMPP considered above.

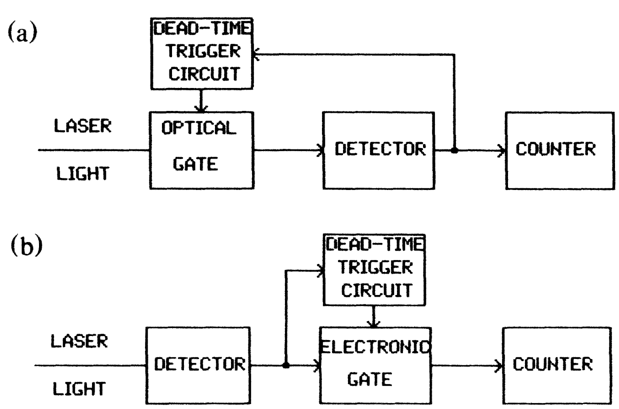

FIG. 1. (a) Closed-loop photocounting experiment carried out by Walker and Jakeman (Ref. 16). (b) Photocounting experiment carried out by Teich and Vannucci (Ref. 15).

As Walker and Jakeman appreciated, an explanation for their observations does not require the quantum theory of photodetection; under closed-loop conditions, sub-Poisson photocounts are possible within the semiclassical framework.

The second example of closed-loop photodetection is the semiclassical analysis of a system incorporating negative linear feedback, as illustrated explicitly in Fig. 2. The optical power emerging from the intensity modulator, given the event history at the in-loop detector, obeys the negative-feedback law

$$P_t(\{i_\tau : \tau < t\}) = P_0 - (P_1/q)\int_{-\infty}^{t} i_\tau \exp[-(t-\tau)/\tau_f]\,d\tau, \tag{13}$$

where P_0 is the nonrandom optical power entering the modulator, τ_f is the time constant of the assumed single-pole feedback filter, and P_1 is the feedback power constant. Because $P_t(\{i_\tau : \tau < t\}) \geqslant 0$ must prevail under all circumstances, Eq. (13) can only be employed as an approximation whose validity is guaranteed by ensuring that[17] $\eta P_0\tau_f/h\nu_0 >> 1 + \eta P_1\tau_f/h\nu_0$.

By use of Eq. (13), a set of integral equations can be developed for the mean, autocovariance, and cross-covariance functions of the in-loop and out-of-loop photocurrents i_t and i_t', respectively. For an ideal beam splitter transmitting 50% of the light and reflecting the other 50%, and with the assumption that $\eta P_1\tau_f/h\nu_0 >> 1$, these equations yield the following mean currents and noise spectra[17]:

$$\langle i \rangle = \langle i' \rangle = qP_0/P_1\tau_f, \tag{14}$$

$$S_{ii}(f) = q\langle i \rangle\{1 - [1+(4\pi f/\mu_1)^2]^{-1} + (2/\mu_1\tau_f)^2\}, \tag{15}$$

and

$$S_{i'i'}(f) = q\langle i' \rangle\{1 + [1+(4\pi f/\mu_1)^2]^{-1}\}, \tag{16}$$

where $\mu_1 = \eta P_1/h\nu_0$. Equations (15) and (16) show

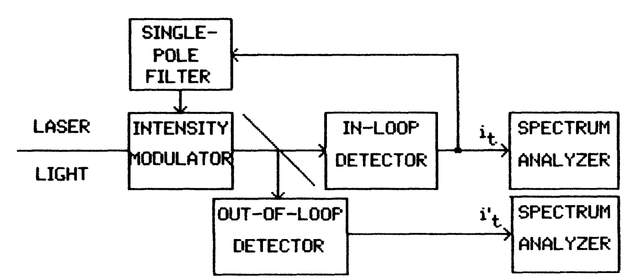

FIG. 2. Closed-loop photodetection experiment with negative linear feedback. This arrangement is a good representation for the beam-splitter experiments carried out by Yamamoto, Imoto, and Machida (Refs. 18 and 19), and by Walker and Jakeman (Ref. 16).

that the in-loop detector produces a sub-shot-noise spectrum within the closed-loop bandwidth $\mu_1/4\pi$ Hz, while the out-of-loop detector produces a super-shot-noise spectrum at all frequencies.

These results bear on the recent experiments of Yamamoto, Imoto, and Machida.[18,19] These authors used a GaAs/AlGaAs injection laser diode to generate light and a Si *p-i-n* photodiode to detect it. Negative electrical feedback from the detector was provided to the current driving the laser diode. As in the Walker-Jakeman experiments, two experimental arrangements were used. Operation in a configuration analogous to that of Fig. 1(a) led to a sub-shot-noise spectrum and sub-Poisson photocount statistics. The second experimental arrangement is identical to that shown in Fig. 2 if it is assumed that intensity modulation of the laser light properly represents the negative electrical feedback from the detector to the laser diode. The results of their experiments were such that the in-loop detector showed a sub-shot-noise spectrum (and sub-Poisson photocounts) while the out-of-loop detector showed a super-shot-noise spectrum (and super-Poisson photocounts).

It follows that the observations of Yamamoto, Imoto, and Machida can be understood within the context of semiclassical closed-loop photodetection theory. This is because a sub-shot-noise spectrum emerges from the SEPP at the in-loop detector, whereas a super-shot-noise spectrum is required for the DSPP at the out-of-loop detector. Furthermore, the similarity in the experimental results reported by Walker and Jakeman and by Yamamoto, Imoto, and Machida can be understood from a physical point of view. In the configuration used by the latter authors, the injection-laser current (and therefore the injection-laser light output) is reduced in response to peaks of the in-loop photodetector current i_t. This is essentially the same effect produced in the Walker-Jakeman experiment where the He-Ne–laser light output is reduced (in their case to zero) in response to photoevent registrations at the in-loop photodetector. The feedback acts like a dead time, suppressing the emission of light in a manner that is correlated with photoevent occurrences at the in-loop detector.

It is, by now, apparent that the signature of nonclassical light in closed-loop photodetection is far less obvious than it is in open-loop photodetection. It turns out, however, that open-loop nonclassical light can be generated with closed-loop photodetection if photon pairs are available or, alternatively, if the in-loop photons are not destroyed. Saleh and Teich[20] recently proposed the use of selective deletion using pairs of photons from cascaded atomic emissions. Jakeman and Walker[21,22] suggested a similar scheme, using the photon pairs generated in parametric down conversion. Yamamoto, Imoto, and Machida[18] proposed the use of a quantum nondemolition measurement[23] to avoid the destruction of the in-loop photons in their experiment.

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