sdg Interacting-Boson Model Applied to ^{168}Er

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The *sdg* interacting-boson model is applied to ^{168}Er. Energy levels and $E2$ transitions are calculated. This model is shown to solve the problem of anharmonicity regarding the excitation energy of the first $K^\pi = 4^+$ band relative to that of the first $K^\pi = 2^+$ one. The level scheme including the $K^\pi = 3^+$ band is well reproduced and the calculated $B(E2)$'s are consistent with the experimental data.

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Soon after almost all levels of ^{168}Er with $J < 6$ and $E_x < 2$ MeV were established,[1] Warner and Casten analyzed this nucleus in terms of the interacting-boson model 1 (IBM-1).[2] They used a Hamiltonian which has the pairing and quadrupole-quadrupole interactions and the usual L^2 term. Their calculation reproduced all positive-parity bands except the $K_i^\pi = 3_1^+$ band and the $K_i^\pi = 0_4^+$ band below the pairing gap and provided an excellent overall description of their decay properties.

On the other hand Bohr and Mottelson[3] pointed out that the *sd* IBM predicted the energy of the $K_i^\pi = 4_1^+$ bandhead at about 2 times as high as that of the $K_i^\pi = 2_1^+$ bandhead, while its experimental position is 2.5 times as high as that of the $K_i^\pi = 2_1^+$. Dumitrescu and Hamamoto[4] calculated the position of two-γ-phonon states, using the geometrical model and the microscopic random-phase approximation. Because the decay properties of any state below 2 MeV appeared to be inconsistent with identifying it as a two-γ-phonon state, they concluded that the bandhead energy of the two-γ-phonon state is more than 2.5 times larger than that of the γ band. This property is called anharmonicity in the γ motion. The only candidate for a two-γ-phonon state is the $K_i^\pi = 4_1^+$ state at 2050 keV. They suggested a large anharmonic γ potential to explain this experimental property.

Recently the hexadecapole degree of freedom has been found to be important both microscopically and phenomenologically in the IBM approach. Microscopically the nucleon pair coupled to $L = 4^+$ (G pair) is found to be important to reproduce $B(E2)$ and moment of inertia although 0^+ (S) and 2^+ (D) pairs are dominant over other nucleon pairs.[5] Phenomenologically many authors claimed the necessity of the hexadecapole degree of freedom in the deformed region and in the vibrational region.[6] Wu and Zhou[7] calculated the energy levels of ^{168}Er using perturbation theory in the *sdg* interacting-boson model (*sdg* IBM). This model incorporates g bosons on the same footing as s and d bosons. Although the overall agreement was good, they used the old experimental assignment. Later one of the 4^+ states at 2030 keV which had been assigned as the bandhead of the $K^\pi = 4^+$ band was shown to belong to the newly assigned $K_i^\pi = 0_4^+$ band.[8] They also failed in reproducing the anharmonic effect and the predicted $K_i^\pi = 1_1^+$ band was not observed by the (n,γ) experiment.

In the present paper we try to solve these problems phenomenologically in a consistent way. We take the *sdg* IBM because it is one of the natural extensions of the ordinary IBM and has some interesting features[9,10] which are not inherent in the *sd* model. In accordance with this extension the unitary group U(6) is replaced by the unitary group U(15). We diagonalize the *sdg* Hamiltonian and obtain $E2$ transitions using a one-body quadrupole operator. From now on, we use abbreviations g, β, and γ for the ground-state band, the $K_i^\pi = 0_2^+$ band, and the $K_i^\pi = 2_1^+$ band, respectively.

Applying the *sdg* IBM to ^{168}Er, where the number of active bosons is sixteen, we are forced to truncate the model space because the full space is too large for computer analysis. Since ^{168}Er is a well-deformed nucleus, we take the U(15)$\supset$SU(3) scheme and take as many as possible SU(3) basis states, which have large eigenvalues of the Casimir operator of SU(3). The basis states taken into account are those belonging to the SU(3) representations (64,0), (60,2), (58,3), $(56,4)^2$, (54,5), $(52,6)^3$, $(50,7)^2$, $(48,6)^4$, (55,3), (53,4), $(51,5)^2$, $(49,6)^2$, $(47,7)^3$, (58,0), (56,1), $(54,2)^3$, $(52,3)^3$, $(50,4)^6$, $(48,5)^6$, and $(46,6)^{10}$. Here the superscript indicates the multiplicity of the irredu-

cible representation. The SU(3) algebra used in this calculation is given by Draayer and Akiyama.[11]

We take two-body interactions with SU(3) tensor character (00), (22), and (06)+(60) only. Other interactions such as (44), (66), (82)+(28), (10,4)+(4,10), (12,0)+(0,12), and (88) tensors are not included in this analysis. This truncation of the interaction is due to the following reasons. First of all, for describing a strongly deformed nucleus, the main part of the Hamiltonian should consist of the Casimir operator of SU(3), the L^2 force, and possibly the SU(3) seniority interaction introduced in Ref. 9. Higher tensors admix more SU(3) representations simultaneously, destroying the rotational level structure. Thus such components, if they exist, should be small. Exclusion of higher tensors than (22) and (06)+(60) tensors should be a good first approximation. The model space introduced above is large enough to describe low-lying states such as the g, β, γ, and $K_i^\pi = 4_1^+$ states because all higher states which are directly connected by the interaction with those states are included.

We construct four two-body interactions V_1, V_2, V_3, and V_4 besides the Casimir operator of SU(3) and L^2 operators. In terms of them our Hamiltonian reads

$$H = a_1 V_1 + a_2 V_2 + a_3 V_3 + a_4 V_4 + a_5 C_{\mathrm{SU}(3)} + a_6 L^2,$$

where a's are parameters to be determined. The four interactions through V_1 to V_4 are the linear combinations of six (22) and four (06)+(60) tensors belonging to the symmetry $[42^{13}]$ of U(15) [see Refs. 9 and 10 for its classification in U(15)]. Among four interactions three interactions (V_1, V_2, V_3) are used for adjusting the position of the β, $K_i^\pi = 0_3^+$, and $K_i^\pi = 4_1^+$ bandheads. The V_1 produces the anharmonic effect. With the interaction V_1 only, the $K_i^\pi = 2_1^+$ and $K_i^\pi = 4_1^+$ bands are too low in energy compared with other bands. The interaction V_2 is used to lower the $K_i^\pi = 0_2^+$ band. The $K_i^\pi = 0_3^+$ band is adjusted by V_3. The interaction V_4 is used to make the eigenstates close to the SU(3) states (actually the strength of V_4 is related to the position of $K^\pi = 1^+$ bands). The detailed definition of V_1 through V_4 and the parameters will be given in a later publication.

Figure 1 shows the energy levels obtained by diagonalization of the Hamiltonian with an appropriate set of parameters. The solid lines show the theoretical energy levels and the dashed lines show the bandhead energies of experiment. We see that overall agreement with experiment is much improved compared to that of Ref. 7. The most remarkable feature is the reproduction of the anharmonicity, that is, $E(K_i^\pi = 4_1^+)/E(K_i^\pi = 2_1^+) = 2.5$, which is mainly due to the effect of configuration mixing. By first-order perturbation we cannot reproduce this strong anharmonicity.[7] This $K_i^\pi = 4_1^+$ band belongs mainly to $(56,4)^{W=0}$ representation where W is the SU(3) seniority quantum number.[9] Another $K^\pi = 4^+$ band arising from $(56,4)^{W=1}$ is pushed up at 3.8 MeV. This band would be degenerate in energy with the $K^\pi = 4^+$ band from $(56,4)^{W=0}$ in the case of a Hamiltonian consisting of only the Casimir operator of SU(3) and the L^2 operator. The predicted $K_i^\pi = 0_3^+$ band has the nature of a one-phonon state theoretically,[10] which is consistent with recent ^{167}Er(d,p) and ^{167}Er(t,d) reactions.[12] The $K_i^\pi = 3_1^+$ band is predicted at a reasonable position, which cannot be described in terms of the *sd* IBM-1. Below 4 MeV we have two $K^\pi = 1^+$ bands which are not shown in Fig. 1. One starts at 2.7 MeV and the other at 3.8 MeV.

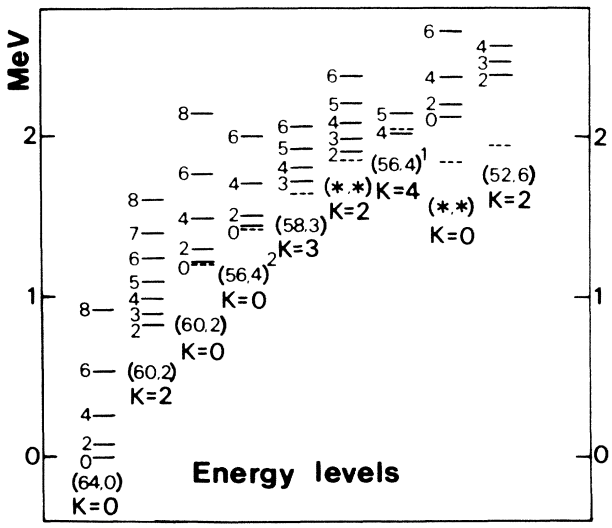

FIG. 1. Energy levels of all positive-parity bands below 2.4 MeV. Solid lines show theoretical ones and dashed lines show the experimental bandhead energies. The label below each bandhead represents an SU(3) representation (λ, μ) of its main component and K quantum number. States with asterisks cannot be assigned to definite SU(3) representations because the probability of their main components is less than 50%. The SU(3) labels $(56,4)^1$ and $(56,4)^2$ indicate $(56,4)^{W=0}$ and $(56,4)^{W=1}$, respectively.

The one-body $E2$ operator is given by four independent parameters in this model:

$$Q_\mu = e_1(s^\dagger \tilde{d}_\mu + \mathrm{H.c.}) + e_2(d^\dagger \tilde{d})^{(2)}_\mu + e_3[(d^\dagger \tilde{g})^{(2)}_\mu + \mathrm{H.c.}] + e_4(g^\dagger \tilde{g})^{(2)}_\mu.$$

This differs from the *sd* case where only two independent parameters exist. In terms of SU(3) tensors the *sdg* IBM has (11), (22), (33), and (44) tensors whereas the *sd* IBM has only (11) and (22) tensors. Bohr and Mottelson[3] analyzed interband $E2$ transitions among the lowest three bands, namely, g, γ, and β. Assuming that the intrinsic quadrupole moments of β and γ bands were the same as that of the ground band, they carried out Mikhailov plot analyses (MPA) for

the $\gamma \rightarrow g$, $\beta \rightarrow g$, and $\beta \rightarrow \gamma$ transitions.

With three M_0 values tabulated in Table III of Ref. 3 and the absolute $B(E2)$ of $L=2_1^+ \rightarrow L=0_1^+$, the determined four parameters are ($e_1=0.327$, $e_2=0.406$, $e_3=-0.420$, and $e_4=0.519$) whose ratios are close to $-2\sqrt{7}/\sqrt{5}:11/\sqrt{14}:-18/\sqrt{35}:3\sqrt{11}/\sqrt{7}$ given by a generator of SU(3). Figure 2 shows the MPA's for $\gamma \rightarrow g$, $\beta \rightarrow g$, and $\beta \rightarrow \gamma$. The general trend of the $E2$ transitions is well reproduced, that is, the theoretical slope of the $\gamma \rightarrow g$, $\beta \rightarrow g$, and $\beta \rightarrow \gamma$ MPA's is positive, positive, and negative, respectively. This is consistent with the experimental data. It should be noticed in Fig. 2 that the theoretical MPA's are not straight lines because of the effect of band mixing. The theoretical $B(E2)$ prediction gives

$$\frac{B(E2;K_i^\pi=4_1^+ \rightarrow K_i^\pi=2_1^+)}{B(E2;K_i^\pi=2_1^+ \rightarrow K_i^\pi=0_1^+)}=1.4.$$

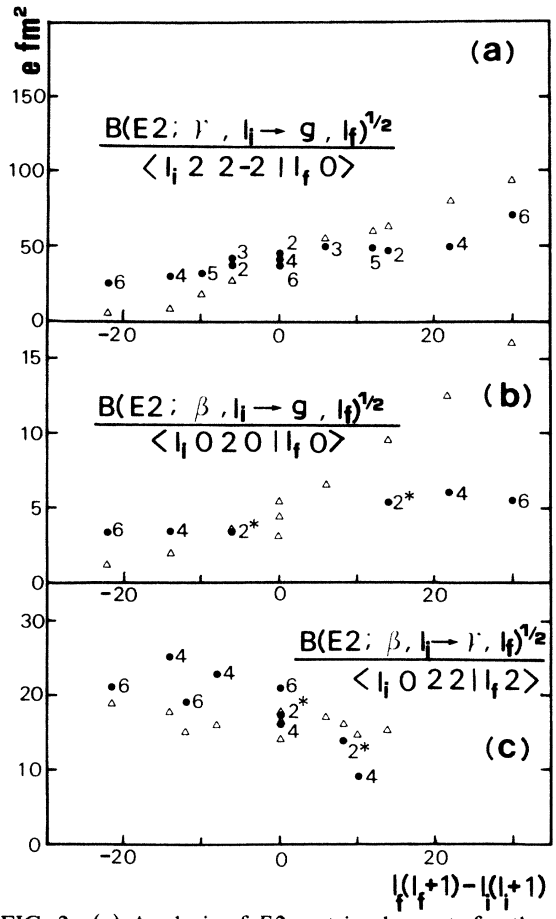

FIG. 2. (a) Analysis of $E2$ matrix elements for the γ to g transitions. Experimental values are shown by filled circles while theoretical ones are shown by open triangles. Points are labeled by I_f only for experiment. For more details see Ref. 3. (b) Analysis of $E2$ matrix elements for the β to g transitions. Asterisks indicate that for the $I=2_\beta$ states only relative intensities have been determined. (c) Analysis of $E2$ matrix elements for the β to γ transitions.

This indicates that the $K_i^\pi=4_1^+$ band has the nature of a 2γ band theoretically. The $E2$ transitions for $K_i^\pi=0_3^+ \rightarrow g$ are predicted to be as weak as for $\beta \rightarrow g$, which is also consistent with the experiment.

Concerning $M1$ transitions, there is a possibility to assign the theoretical 1_2^+ state to the 1^+ state at 3.39 MeV found by (e,e') experiment[13] although the theoretical state is higher in energy compared with experiment. The wave function of the 1^+ state at 3.8 MeV is mainly composed of the (58,3) representation whereas the one at 2.7 MeV is mainly a mixture of (55,3) and (56,1). The *sdg* IBM has (11) and (33) SU(3) tensors for a one-body operator of rank one, where (11) is just the angular momentum operator, being diagonal in O(3). The (33) tensor does induce interband $M1$ transitions. The calculated $B(M1;0_1^+ \rightarrow 1_2^+)$ is much larger than the $B(M1;0_1^+ \rightarrow 1_1^+)$ because (58,3) is connected with (64,0) by the (33) tensor while (55,3) and (56,1) are not. By adjusting the $M1$ one-body interaction, that is, the strength of the (33) tensor, we can reproduce the $B(M1;0_1^+ \rightarrow 1_2^+)_{\text{expt.}}$ which is 0.9(2) in μ_N^2. Using this $M1$ operator, one predicts that the typical value of the intraband $M1$ transitions from the gamma band in our present calculation is 10^{-4} single-particle units, which is consistent with the experiment.[2]

Our theoretical 1_2^+ state has a character different from the one possessed by the 1^+ state given[14] by the IBM-2; the former belongs to a neutron-proton-symmetric state, which can exist as a result of the hexadecapole degree of freedom, while the latter is a neutron-proton-asymmetric state. Currently we do not have any experimental information which distinguishes between these two interpretations of the experimental 1^+ state. Experiments on $M1$ decays from the 1^+ state to various 0^+ states are desirable, since the selection rules, or branching ratios, are different between these predictions.

In conclusion, we can say that many problems appearing in the low-lying positive-parity states of ^{168}Er are solved within the framework of the *sdg* IBM. It is significant to note that a study of the reaction ^{166}Er$(t,p)^{168}$Er carried out recently[15] seems to strongly support the *sdg* IBM. Details will be given in a subsequent note.

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