Precise Determination of $\sin^2\theta_W$ from Measurements of the Differential Cross Sections for $\nu_\mu p \to \nu_\mu p$ and $\bar{\nu}_\mu p \to \bar{\nu}_\mu p$

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This paper describes measurements of the semileptonic weak-neutral-current reactions $\nu_\mu p \to \nu_\mu p$ and $\bar{\nu}_\mu p \to \bar{\nu}_\mu p$ which yield the absolute differential cross sections $d\sigma(\nu_\mu p)/dQ^2$ and $d\sigma(\bar{\nu}_\mu p)/dQ^2$. The weak-neutral-current parameter, $\sin^2\theta_W$, is determined to be $\sin^2\theta_W = 0.220 \pm 0.016(\text{stat.})^{+0.023}_{-0.031}(\text{syst.})$.

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With the discovery of the intermediate vector bosons,[1] the W and Z, the basic validity of the standard $SU(2) \otimes U(1)$ electroweak model[2] is now firmly established. It remains of interest, however, to seek possible extensions of that model both by searching for new phenomena and by making more precise measurements of known phenomena. In the latter category are measurements of the fundamental weak-neutral-current parameter, $\sin^2\theta_W$, in all processes in which that parameter can be determined accurately. In this paper we describe measurements of the semileptonic elastic reactions $\nu_\mu p \to \nu_\mu p$ and $\bar{\nu}_\mu p \to \bar{\nu}_\mu p$ from which the differential cross sections $d\sigma(\nu_\mu p)/dQ^2$ and $d\sigma(\bar{\nu}_\mu p)/dQ^2$ are obtained.[3] These in turn provide precise determinations of the axial-vector form factor $G_A(Q^2)$ and of $\sin^2\theta_W$ which are based on empirically specified quantities, and are largely independent of assumptions concerning the quark constituents of the proton.[4]

Data were obtained on the interactions $\nu_\mu p \to \nu_\mu p$ and $\bar{\nu}_\mu p \to \bar{\nu}_\mu p$ as part of a systematic study of neutrino interactions[5,6] using a 170-metric-ton, high-resolution target-detector[7] in horn-focused wide-band neutrino (antineutrino) beams of mean energy 1.3 (1.2) GeV at the Brookhaven Alternating-Gradient Synchrotron (AGS). Separate exposures were made of 0.55×10^{19} and 2.5×10^{19} protons of 28 GeV on the AGS production target, with positive (ν) and negative ($\bar{\nu}$) horn focusing, respectively.

Neutral-current elastic proton-scattering candidates were single tracks fully contained within the detector and not accompanied by any additional track. The minimum length of an accepted track yielded up to nine measurements of dE/dx [three in liquid scintillator and six in proportional drift tubes (PDT)], and corresponded to $Q^2 = 2MT \geqslant 0.35$ $(\text{GeV}/c)^2$ for elastic proton scatters. The vertex of all event candidates was restricted to a fiducial volume equal to 0.2 of the total detector volume. The measured energy and angular resolutions for protons at a typical kinetic energy $T = 250$ MeV were $\sigma_T = 15$ MeV and $\sigma_\theta = 0.030$ rad.

To separate protons from pions (muons), separate confidence levels (C.L.) for each hypothesis were constructed for each candidate event from the observed energy depositions in the scintillators and PDT. The result is shown in Fig. 1. Protons are defined to be tracks with confidence levels in the region above the contour in Fig. 1(c) and for which scintillator and PDT pion confidence levels are less than 0.2. This particle-identification procedure (PID), which results in negligible pion contamination, has been verified quantitatively by use of tagged test-beam protons in a similar detector.[8]

After proton candidates were selected, further criteria were applied to suppress background events exhibiting relatively large in-time extra energy depositions while maintaining high detection efficiency for the signal. Three limitations were placed on the observed energy depositions in the scintillator cells: (i) The visible energy in the scintillator cell at the event vertex had to be less than 50 MeV, (ii) the sum of the energy in a volume 5 cells by 5 cells centered on the vertex cell (excluding cells on tracks) had to be less than 30 MeV, and (iii) the total energy within 5 m of the vertex cell (again excluding cells on the tracks) had to be less than 60 MeV. Variation of these limits did not significantly affect the results given below.

The resulting samples contain backgrounds from

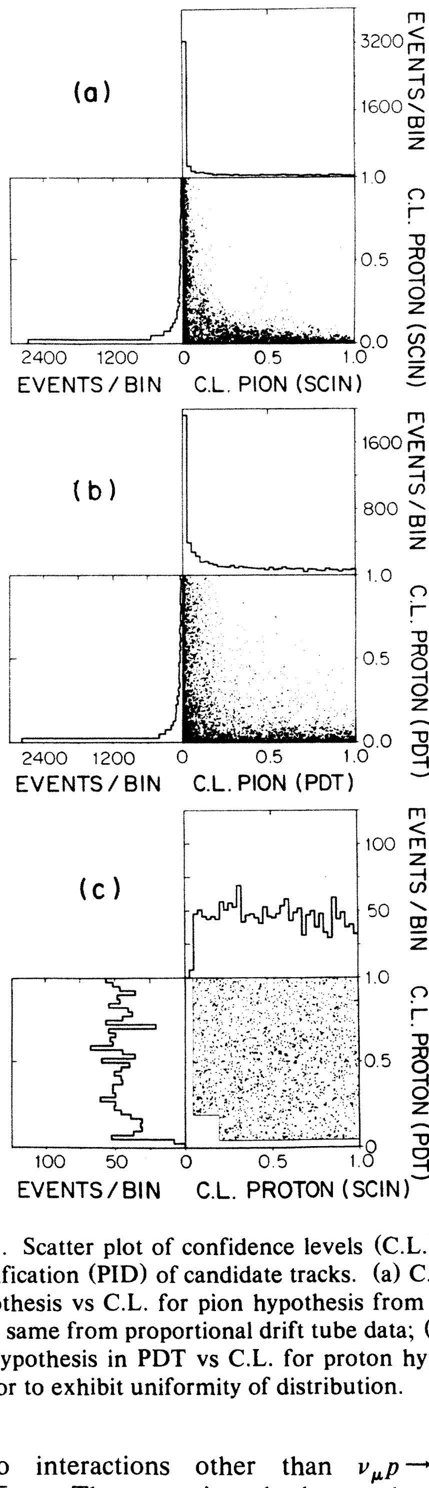

FIG. 1. Scatter plot of confidence levels (C.L.) for particle identification (PID) of candidate tracks. (a) C.L. for proton hypothesis vs C.L. for pion hypothesis from scintillator data; (b) same from proportional drift tube data; (c) C.L. for proton hypothesis in PDT vs C.L. for proton hypothesis in scintillator to exhibit uniformity of distribution.

neutrino interactions other than $\nu_\mu p \to \nu_\mu p$ and $\bar{\nu}_\mu p \to \bar{\nu}_\mu p$. The neutrino backgrounds were (a) $\nu_\mu n \to \nu_\mu n$, where the final-state neutron produced a single detected proton, (b) neutral-current single-pion production, where the charged pion or the two photons from $\pi^0 \to \gamma\gamma$ were undetected, and (c) a small contribution from the quasielastic reaction $\nu_\mu n \to \mu^- p$ in which the muon was undetected. Also, in each sample corrections were necessary for elastic proton scatters produced by the measured contamination of opposite-helicity neutrinos in the incident beam. The event-vertex and event-timing distributions showed that the background in the fiducial volume from interactions of neutrons which entered the detector was negligible.

The backgrounds in which a μ or π^+ was observed to decay near the event vertex were directly subtracted. A Monte Carlo calculation[9] was used to estimate the remaining backgrounds. After all subtractions, which are listed in Table I, final samples of 951 $\nu_\mu p \to \nu_\mu p$ and 776 $\bar{\nu}_\mu p \to \bar{\nu}_\mu p$ events remain.

The experimental acceptance and efficiency factors for the $\nu_\mu p$ and $\bar{\nu}_\mu p$ samples are shown in Table II. The magnitudes of these factors were determined from Monte Carlo calculations and have been confirmed by measurements using test-beam data or visual scans of the neutrino-induced data. The Monte Carlo calculation was also tested against a large sample of $\nu_\mu n \to \mu^- p$ events in which the protons exhibited kinematics similar to the protons in the elastic data.

To determine the differential cross sections $d\sigma(\nu_\mu p)/dQ^2$ and $d\sigma(\bar{\nu}_\mu p)/dQ^2$, we have indirectly measured the integrated incident neutrino fluxes using quasielastic events, $\nu_\mu n \to \mu^- p$ and $\bar{\nu}_\mu p \to \mu^+ n$, with small momentum transfers, $\langle Q^2 \rangle \simeq 0.2$ (GeV/c)^2. The quasielastic events were extracted from the same data samples that yielded the $\nu_\mu p$ and $\bar{\nu}_\mu p$ events, and were selected by the requirements that the outgoing muon angle with respect to the incident neutrino beam be less than 350 mrad, and that the muon kinetic energy be greater than 0.3 GeV. Backgrounds in the normalization samples, which arise from the presence of charged-current, soft, single-pion and multipion events,[10] were determined by Monte Carlo calculation. A summary of the corrections necessary to determine the absolute numbers of $\nu_\mu n \to \mu^- p$ and $\bar{\nu}_\mu p \to \mu^+ n$ events is presented in Tables I and II.

The data in Tables I and II yield the absolute differential cross sections $d\sigma(\nu_\mu p)/dQ^2$ and $d\sigma(\bar{\nu}_\mu p)/dQ^2$ shown in Fig. 2, and the total quasielastic rates. From these quantities and the known quasielastic differential cross sections, one finds the ratios for $0.5 \leq Q^2 \leq 1.0$ (GeV/c)^2:

$$R_\nu = \frac{\sigma(\nu_\mu p \to \nu_\mu p)}{\sigma(\nu_\mu n \to \mu^- p)} = 0.153 \pm 0.007(\text{stat.}) \pm 0.017(\text{syst.}),$$

$$R_{\bar{\nu}} = \frac{\sigma(\bar{\nu}_\mu p \to \bar{\nu}_\mu p)}{\sigma(\bar{\nu}_\mu n \to \mu^+ n)} = 0.218 \pm 0.008(\text{stat.}) \pm 0.023(\text{syst.}), \quad (1)$$

The 11% systematic uncertainty[11] in R_ν and $R_{\bar{\nu}}$ in Eq. (1) is the absolute scale uncertainty of each of the individual differential cross sections in Fig. 2.

To reduce the overall systematic errors in R_ν and $R_{\bar{\nu}}$

TABLE I. Evolution of the final data samples. Numerical entries are numbers of events observed or calculated.

		Neutrino	Antineutrino
Elastic sample after PID and energy cuts		1686	1821
Decay subtraction:	μN $\nu N\pi^+$	146	160
Monte Carlo subtraction:	$\nu N\pi$	283	297
	$\nu N\pi\pi$	38	41
	νn	268	196
Wrong-helicity beam contamination[a]		Negligible	351
Final elastic sample		951	776
Quasielastic sample after angle and energy cuts		20 102	32 936
Monte Carlo subtraction:	$\mu N\pi$	6945	7888
	$\mu N\pi\pi$	776	879
Wrong-helicity beam contamination[a]		704	3623
Final quasielastic sample		11 677	20 546

[a]There is a measured $(8.7 \pm 1.3)\%$ contamination of ν_μ in the $\bar{\nu}_\mu$ beam, and a $(2.4 \pm 0.5)\%$ contamination of $\bar{\nu}_\mu$ in the ν_μ beam. These lead, for example, to ν_μ-induced events from $\nu p \to \nu p$, $\nu n \to \nu n$, and $\nu N \to \nu N\pi$ during data taking in the dominantly $\bar{\nu}_\mu$ beam. Corresponding contamination occurs in the quasielastic channels.

and the quantities derived from them, R_ν and $R_{\bar{\nu}}$ have been calculated over the limited interval $0.5 < Q^2 < 1.0$ $(\text{GeV}/c)^2$ as given in Eq. (1), while $d\sigma(\nu_\mu p)/dQ^2$ and $d\sigma(\bar{\nu}_\mu p)/dQ^2$ have been fitted simultaneously over the interval $0.4 < Q^2 < 1.1$ $(\text{GeV}/c)^2$. Furthermore, some of the uncertainties in the experimental and theoretical quantities in Tables I and II correlate when we treat ν_μ and $\bar{\nu}_\mu$ data together, which results in a smaller systematic uncertainty in quantities extracted from the combined ν_μ and $\bar{\nu}_\mu$ data.[11]

The differential cross section for $\nu_\mu p$ depends primarily on $\sin^2\theta_W$, while the differential cross section for $\bar{\nu}_\mu p$ is particularly sensitive to the axial-vector form factor, $G_A(Q^2)$. If it is assumed[4] that $G_A(Q)^2$ is given by the axial-vector isovector dipole form factor, $G_A^3(Q^2) = \frac{1}{2} \times 1.26/(1+Q^2/M_A^2)^2$, with no corrections for heavy quark currents and no axial-vector isoscalar term, then extraction of $\sin^2\theta_W$ and M_A from $d\sigma(\nu_\mu p)/dQ^2$ and $d\sigma(\bar{\nu}_\mu p)/dQ^2$ yields[12] $\sin^2\theta_W = 0.218^{+0.039}_{-0.047}$ and $M_A = 1.06 \pm 0.05$ GeV, where the errors represent a 67% rectangular confidence area. This value of M_A is in good agreement with the present world-average value,[13] $M_A = 1.032 \pm 0.036$

TABLE II. Acceptances and efficiencies applied to the final data samples of Table I. The errors shown are systematic.

	Neutrino	Antineutrino
	Elastic Analysis	
Q^2 acceptance[a]	0.207	0.115
Extra-track cut	0.81 ± 0.02	0.90 ± 0.01
Track-finding efficiency	0.86 ± 0.03	0.91 ± 0.03
Insufficient information	0.95 ± 0.02	0.95 ± 0.02
Confidence-level requirement on PID	0.75 ± 0.03	0.77 ± 0.03
Combined extra-energy cuts	0.73 ± 0.01	0.78 ± 0.01
	Quasielastic analysis	
Q^2 acceptance[a]	0.263	0.455
Track-finding efficiency	0.83 ± 0.02	0.85 ± 0.02

[a]Fractional acceptance of a single track of length ≥ 3 modules corresponding to $Q^2 \geq 0.35$ $(\text{GeV}/c)^2$.

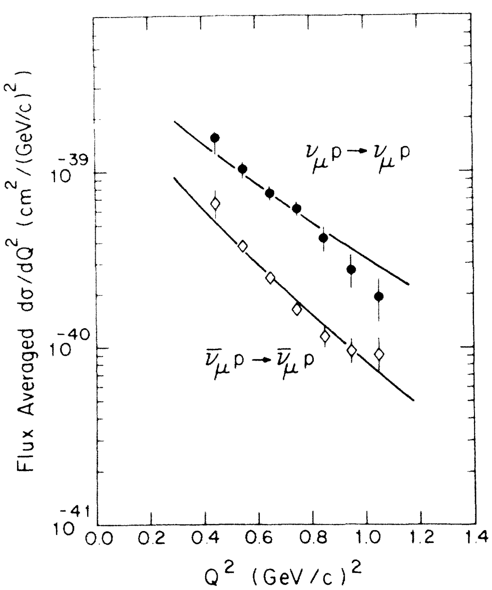

FIG. 2. The data points are the measured flux-averaged differential cross sections for $\nu_\mu p \to \nu_\mu p$ and $\bar{\nu}_\mu p \to \bar{\nu}_\mu p$ from this experiment. The solid curves are best fits to the combined data with the values $M_A = 1.06$ GeV and $\sin^2\theta_W = 0.220$. This fitting procedure imposes adjustment of the solid curves by scale factors of 1.05 for $\nu_\mu p$ and 1.09 for $\bar{\nu}_\mu p$, consistent with the absolute scale uncertainty of approximately 11% in each of the individual cross sections which was included in the fitting procedure (see Ref. 11). The error bars represent statistical errors and also include systematic errors of 18% and 8% on the points at $Q^2 = 0.45$ and 0.55 $(\mathrm{GeV}/c)^2$ to reflect the effect on those points of uncertainties in tracking efficiency and in the low-energy region of the incident neutrino spectra.

GeV.

Alternatively, if $\sin^2\theta_W$ is fixed at the value 0.22 and M_A constrained to the world-average value, a search may be made for additional terms in $G_A(Q^2)$ due to neutral-current contributions from heavy quark currents or a "nonstandard" isoscalar axial-vector current.[4] Writing $G_A(Q^2) = G_A^3(Q^2)(1+\eta)$, one finds $\eta = 0.12 \pm 0.07$, or equivalently, $0.00 < \eta < 0.25$ at 90% C.L. This result for η is independent of the numerical value assumed for $\sin^2\theta_W$.

To exhibit the internal consistency of the data, and to extract the most precise value of $\sin^2\theta_W$, we constrain M_A at the world-average value and calculate $\sin^2\theta_W$ in all ways that combined the ν_μ and $\bar{\nu}_\mu$ data. The resulting three values are shown in Table III. These values are insensitive to the choice of any value of η in the region of $0.00 < \eta < 0.25$. If we take into account the increased information in the differential cross sections, the most precise value of $\sin^2\theta_W$ from this experiment is

$$\sin^2\theta_W = 0.220 \pm 0.016(\mathrm{stat.})^{+0.023}_{-0.031}(\mathrm{syst.}). \quad (2)$$

TABLE III. Values of $\sin^2\theta_W$ obtained from the combined ν_μ and $\bar{\nu}_\mu$ data with the axial-vector isovector form-factor mass constrained to be $M_A = 1.032 \pm 0.036$ GeV.

Measured quantities	$\sin^2\theta_W$	χ^2/d.o.f.
R_ν and $R_{\bar{\nu}}$	$0.207 \pm 0.016^{+0.032}_{-0.048}$	1.1/1
$\dfrac{\sigma(\bar{\nu}_\mu p \to \bar{\nu}_\mu p)}{\sigma(\nu_\mu p \to \nu_\mu p)}$ [a]	$0.228 \pm 0.016^{+0.028}_{-0.037}$	0.0/0
$d\sigma(\nu_\mu p)/dQ^2$ and $d\sigma(\bar{\nu}_\mu p)/dQ^2$	$0.220 \pm 0.016^{+0.023}_{-0.031}$	16/15

[a] $\sigma(\bar{\nu}_\mu p \to \bar{\nu}_\mu p)/\sigma(\nu_\mu p \to \nu_\mu p)$ is given by

$$\frac{R_{\bar{\nu}} \int_{0.5}^{1.0} [d\sigma(\bar{\nu}_\mu p \to \mu^+ n)/dQ^2] dQ^2}{R_\nu \int_{0.5}^{1.0} [d\sigma(\nu_\mu n \to \mu^- p)/dQ^2] dQ^2} = 0.302 \pm 0.019 \pm 0.037.$$

In an earlier paper[5] we reported a value of

$$\sin^2\theta_W = 0.209 \pm 0.029(\mathrm{stat.}) \pm 0.013(\mathrm{syst.}),$$

obtained from measurement of the ratio of the cross sections for the purely leptonic reactions $\nu_\mu e \to \nu_\mu e$ and $\bar{\nu}_\mu e \to \bar{\nu}_\mu e$ with the same apparatus and using the identical normalization procedure as in the present experiment. There is good agreement between that value[14] and the value in Eq. (2) obtained from the semileptonic elastic reactions $\nu_\mu p \to \nu_\mu p$ and $\bar{\nu}_\mu p \to \bar{\nu}_\mu p$. Furthermore, these values are in good agreement with the values of $\sin^2\theta_W$ determined from the masses of the W and Z bosons,[15] from inelastic electron-deuteron scattering,[16] and from deep-inelastic neutral-current neutrino reactions.[17] Hence, over a wide range of Q^2 [10^{-2} to 10^4 $(\mathrm{GeV}/c)^2$], and with significantly different assumptions and corrections in the various experiments, the weak-neutral-current parameter $\sin^2\theta_W$ is, within present experimental errors of about 10%, a universal constant.

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[11]The systematic uncertainty of 11.2% (ν) or 10.4% ($\bar{\nu}$) consists of a contribution of 7.0% (ν) or 6.5% ($\bar{\nu}$) from the errors on the acceptance and inefficiency factors in Table II, and a contribution of 8.7% (ν) or 8.1% ($\bar{\nu}$) (in quadrature) from the following factors: energy scale; pion (muon) decay detection effeciency; detector resolution; Fermi momentum; Pauli exclusion principle; scattering, charge exchange, and absorption of pions and nucleons internal and external to the target nucleus; neutrino flux uncertainties; and uncertainties in the cross section for single-pion and multipion production. The positive correlation coefficient of the systematic errors above is

$$\rho = \sigma^2_{\nu\bar{\nu}}/\sigma_\nu\sigma_{\bar{\nu}} = (0.076)^2/(0.112)(0.104) = 0.50.$$

[12]A radiative correction amounting to a 2% reduction is included in the values of R_ν and of $R_{\bar{\nu}}$ in Eq. (1). The radiative correction to values of $\sin^2\theta_{\rm W}$ obtained from the combining of ν_μ and $\bar{\nu}_\mu$ data is negligible.

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