Comment on "New Ground State for the Splay-Fréedericksz Transition in a Polymer Nematic Liquid Crystal"

Lonberg and Meyer recently[1] discovered the existence of a critical value $r_c = 0.303$ for the ratio $r = K_2/K_1$ between the twist and splay Frank elastic constants of nematic liquid crystals. For $r < r_c$ a periodic splay-twist distortion has lower free energy than the well-known aperiodic distortion appearing in a splay-Fréedericksz transition. A few comments are given here, in order to show that the discovery could be of greater practical interest than originally supposed.

As a preliminary comment, I note that the critical value obtained by the authors from a numerical computation of the distortion is very close to the exact one, which is

$$r_c = [(\tfrac{1}{8}\pi^2 - 1)^2 + \tfrac{1}{8}\pi^2 - 1]^{1/2} - \tfrac{1}{8}\pi^2 + 1$$
$$= 0.303\,25\ldots.$$

This value is the positive root of the equation

$$r_c^2 + 2ar_c - a = 0, \tag{1}$$

where $a = \frac{1}{8}\pi^2 - 1$. Equation (1) is valid under the same hypotheses as given in the Letter. The simplest way to deduce this equation is to consider the limit $r \to r_c$ in the general solution for f and g given in the Letter. This corresponds to taking $q \to 0$, where q is the wave vector associated to the periodicity. The other parameters are expanded in a power series of q up to terms of the order of q^2. From the bulk equations one immediately obtains

$$q_1 = qr^{-1/2};\quad q_2 = \frac{\pi}{d} + \frac{1-2r}{2r}\frac{d}{\pi}q^2;$$
$$\frac{A_1}{B_1} = (1-r)r^{-1/2}\left(\frac{d}{\pi}q\right)^2;\quad \frac{B_2}{A_2} = \frac{1-r}{r}\frac{d}{\pi}q, \tag{2}$$

where the symbols are the same as used in the Letter.

In the above approximation the critical magnetic field H_c must be considered as a constant, and equal to $H_{0c} = (\pi/d)(K_1/\Delta\chi)^{1/2}$. In fact in the Letter it is shown that $H_{0c} - H_c$ is of the order of $(r_c - r)^2$, and that $r_c - r$ is of the order of q^2.

By inserting Eq. (2) into the boundary conditions one obtains a relation between r and q, which in the limit $q \to 0$ reduces to Eq. (1).

Now I show that the limitations inherent to the low value of r_c may be partially removed when one or both of the following conditions are met: (a) the cell has a suitable weak anchoring; (b) the direction of the magnetic field is changed.

(a) A weak anchoring for the twist deformation is assumed. It may be accounted for by adding to the free energy the surface term $w_2 g^2/2$. It gives the boundary conditions $f(\pm d/2) = 0$ and

$$\mp K_2(g_z - f_y) + w_2 g = 0 \tag{3}$$

for $z = -d/2$ and $+d/2$, respectively. From Eqs. (2) and (3) one obtains, with the procedure used above, the same equation as (1), with

$$a = \frac{\pi^2}{8}\left(\frac{2}{d}\frac{K_2}{w_2} + 1\right).$$

This equation gives an r_c value which monotonically increases with decreasing w_2, up to 0.5.

(b) In a magnetic field parallel to the y axis the free energy is given by an expression which becomes identical to the previous one by interchange of f and g and of K_1 and K_2, and by consideration of weak anchoring for the splay distortion. This means that if a periodic solution for a given value of r exists, another solution exists for the inverse value of r. Periodic distortions may then be obtained for $r > 2$. Such r values are found near a nematic-smectic transition.

From the above considerations it appears that only the range $0.5 < r < 2$ seems forbidden for the periodic distortion.

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[1] F. Lonberg and R. B. Meyer, Phys. Rev. Lett. 55, 718 (1985).

