# Skyrmions in the Presence of Vector Mesons

U.-G. Meissner
*Institute of Theoretical Physics, University of Regensburg, D-8400 Regensburg, West Germany*

and

I. Zahed
*Department of Physics, State University of New York at Stony Brook, Stony Brook, New York 11794*
(Received 25 November 1985; revised manuscript received 28 January 1986)

The nonlinear $\sigma$ model with the Wess-Zumino term is extended to the low-lying vector meson resonances $\omega$, $\rho$, and $A_1$. These resonances are treated as chiral gauge multiplets in a minimally broken $SU(2)_L \otimes SU(2)_R \otimes U(1)_V$ gauge model. The bulk properties of hedgehog skyrmions are investigated. The results indicate a clear improvement over the conventional Skyrme model.

PACS numbers: 11.40.Fy, 11.10.Lm, 14.40.Cs

It is believed that a dynamical description of the light pseudoscalar mesons in a Skyrme-type scenario[1] including the proper chiral anomalies is equivalent to QCD at low energy. While the trivial chiral configurations should saturate low-energy theorems, their nontrivial counterparts should provide, via the soliton mechanism, the appropriate playground for meson-baryon dynamics.

The purpose of this Letter is to extend the nonlinear $\sigma$ model to the low-lying vector meson resonances such as the $\omega$, $\rho$, and $A_1$, in the presence of the non-Abelian anomlies.[2,3] We will omit the *ad hoc* Skyrme term from our discussion, since the topological configurations of this model are stabilized by vector-meson interactions at short distances, a well-known fact in nuclear physics. The reasons for this preliminary investigation are manifold. First, we know from the large-$N_c$ limit that QCD reduces to a weakly interacting theory of mesons (and glueballs), not only scalar but vector as well. Second, it is important to investigate the role of vector mesons on the structure and stability of baryons and their low-lying resonances. Their omission at the hadronic scale can hardly be justified. Third, they should be present in any serious attempt towards an understanding of the $NN$ interaction, given the phenomenological success of the boson-exchange models.[4] Finally, it is rather relevant to see how vector-meson dominance along Sakurai's precepts[5] fits into a soliton description of baryons.

While the nonlinear $\sigma$ model with the Wess-Zumino term provides an unambiguous realization of QCD at low energy based solely on chiral symmetry, the introduction of vector mesons is less straightforward. There is, however, a good reason to identify these mesons with gauge multiplets of a minimally broken $SU(2)_L \otimes SU(2)_R \otimes U(1)_V$ gauge model. Indeed, some years ago, Weinberg[6] and Callan *et al.*[7] have argued that an effective chiral description with external spin-1 gauge sources saturate to leading order the anomalous chiral Ward identities. In the heavy-mass limit, this argument gives a sizable credit to the above-mentioned procedure. Recently, there have been some attempts to incorporate vector mesons into an effective chiral description based on induced representations in the coset space of the pion. For further details, see Igarashi *et al.*[8] and Bando *et al.*[8]

Having said this, we consider $\omega$, $\rho$, and $A_1$ as gauge particles associated to $SU(2)_L \otimes SU(2)_R \otimes U(1)_V$ and define

$$A^\mu_{L,R} = \tfrac{1}{2}(\omega^\mu + \rho^\mu \pm a^\mu), \tag{1}$$

where $\omega_\mu = ig\omega_\mu$, $\rho_\mu = ig\rho^a_\mu \tau^a$, and $a_\mu = iga^a_\mu \tau^a$ are the gauge $\omega$, $\rho$, and $A_1$ fields, respectively. For convenience we have chosen the U(1) and SU(2) couplings to be identical. A minimally broken $SU(2)_L \otimes SU(2)_R \otimes U(1)_V$ Lagrangean for spin-1 mesons is given by

$$\mathscr{L}_1 = (1/8g^2)\mathrm{Tr}[a_{\mu\nu}a^{\mu\nu} + \rho_{\mu\nu}\rho^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu}] + (m^2/4g^2)\mathrm{Tr}[a_\mu^2 + \rho_\mu^2 + \omega_\mu^2], \tag{2}$$

where $m$ is the bare $\rho$-meson mass, and the vector and axial-vector field strengths are

$$\omega^{\mu\nu} = \partial^\mu\omega^\nu - \partial^\nu\omega^\mu, \tag{3a}$$

$$\rho^{\mu\nu} = \partial^\mu\rho^\nu - \partial^\nu\rho^\mu + \tfrac{1}{2}[\rho^\mu,\rho^\nu] + \tfrac{1}{2}[a^\mu,a^\nu], \tag{3b}$$

$$a^{\mu\nu} = \partial^\mu a^\nu - \partial^\nu a^\mu + \tfrac{1}{2}[\rho^\mu,a^\nu] + \tfrac{1}{2}[a^\mu,\rho^\nu]. \tag{3c}$$

The gauge-invariant nonlinear $\sigma$ model for spin-0 mesons is

$$\mathscr{L}_0 = \tfrac{1}{4}f_\pi^2\,\mathrm{Tr}[D^\mu U^\dagger D_\mu U], \tag{4}$$

where $U(x)$ is the usual SU(2) field, and $D^\mu$ the covariant derivative in the adjoint representation,

$$D^\mu U = \partial^\mu U + A_L^\mu U - U A_R^\mu . \tag{5}$$

The anomalous contribution due to the Wess-Zumino term has been discussed by Witten[2] and Kaymakcalan, Rajeev, and Schechter.[3] Since $SU(2)_L \otimes SU(2)_R$ is anomaly free, the gauged Wess-Zumino term in Bardeen's form[9] reduces to

$$\mathscr{L}_{WZ} = \frac{iN_c}{48\pi^2}\epsilon^{\mu\nu\alpha\beta}\omega_\mu \mathrm{Tr}[L_\nu L_\alpha L_\beta] + \frac{iN_c}{64\pi^2}\epsilon^{\mu\nu\alpha\beta}\omega_{\mu\nu}\mathrm{Tr}[\rho_\alpha(R_\beta - L_\beta) + a_\alpha(R_\beta + L_\beta) + \rho_\alpha a_\beta - \tfrac{1}{2}(\rho_\alpha - a_\alpha)U^\dagger(\rho_\beta + a_\beta)U] \tag{6}$$

where $N_c$ is the number of colors, and $L_\mu$ and $R_\mu$ are the left and right currents on $S^3$ in the Sugawara form,

$$L_\mu = U^\dagger \partial_\mu U = -U^\dagger R_\mu U. \tag{7}$$

It is clear from the above definitions that (6) vanishes for $U=1$. There is no topological contribution to (6), since $\pi_5(SU(2))=0$. The pertinent $\pi$-$\omega$-$\rho$-$A_1$ dynamics is described by Eqs. (2), (4), and (6). The baryons correspond to the nontrivial configurations of spin-0 mesons. Their stability is naturally insured by the repulsive character of the heavy spin-1 mesons at short distances. Thus, we no longer need a Skyrme fourth-order term.

Using the explicit form of the covariant derivative in (4) gives

$$\mathscr{L}_0 = -\tfrac{1}{4}f_\pi^2\,\mathrm{Tr}[R_\mu^2] + \tfrac{1}{4}f_\pi^2\,\mathrm{Tr}[\rho_\mu(R_\mu + L_\mu)] + \tfrac{1}{4}f_\pi^2\,\mathrm{Tr}[a_\mu(R_\mu - L_\mu)] - \tfrac{1}{4}f_\pi^2\,\mathrm{Tr}[a_\mu^2] + \tfrac{1}{8}f_\pi^2\,\mathrm{Tr}[a_\mu U\rho_\mu U^\dagger] - \tfrac{1}{8}f_\pi^2\,\mathrm{Tr}[\rho_\mu U a_\mu U^\dagger] + \tfrac{1}{8}f_\pi^2\,\mathrm{Tr}\{\rho_\mu[U,\rho_\mu]U^\dagger\} - \tfrac{1}{8}f_\pi^2\,\mathrm{Tr}\{a_\mu[U,a_\mu]U^\dagger\} \tag{8}$$

which shows that both the pion and $A_1$ fields mix up in the vacuum through

$$-\tfrac{1}{2}if_\pi\,\mathrm{Tr}[a_\mu\partial^\mu\pi], \tag{9}$$

requiring proper diagonalization. By suitably redefining the pion and $A_1$ fields such that

$$\tilde{a}_\mu = a_\mu - i(g^2\tilde{f}_\pi/m^2)\partial_\mu\tilde{\pi},\quad \tilde{\pi} = Z\pi, \tag{10}$$

where $\tilde{f}_\pi$ is the reduced pion decay constant and

$$Z = \tilde{f}_\pi/f_\pi = (1 + f_\pi^2 g^2/m^2)^{-1/2}, \tag{11}$$

we can eliminate the cross term (10). It follows that the $A_1$ field acquires a "renormalized" mass term of the form[3]

$$m_A = m/Z. \tag{12}$$

Although understood, the tilde will be omitted from now on for convenience. From the renormalized Lagrangean we have

$$\mathscr{L}_{\rho^0\to\pi^+\pi^-} = ig\rho_\mu^0(\pi^+\partial_\mu\pi^- - \pi^-\partial_\mu\pi^+). \tag{13}$$

Simple kinematics show[3] that $g^2/4\pi = f_{\rho\pi\pi}^2/4\pi \sim 3.0$. Since the $\omega$ field in (6) couples directly to the topological current, we conclude that $g_\omega^2/4\pi = N_c^2 g^2/16\pi \sim 6.5$, a bit too low compared with the experimental value of 10–12. Alternatively, one can use a different U(1) coupling relative to the SU(2) one, and choose to determine $g_\omega$ empirically by looking to the interaction between two nucleons. Adkins and Nappi[10] have recently investigated the $B=1$ hedgehog configuration when the $\rho$ and $A_1$ fields are switched off. They choose to fix $g_\omega$ by looking at $\omega\to 3\pi$, and obtain $g_\omega^2/4\pi \sim 19$. Their result constitutes an upper bound on $g_\omega$ since the decay rate of the $\omega$ is $\rho$ dominated.

To investigate the nontrivial sector (2)–(6), we will specialize to hedgehog skyrmions. As a result of the hedgehog symmetry and the intrinsic parity of the vector mesons, the most general form for the spin-1 fields is

$$\omega^\mu = ig\omega(r)\delta^{\mu 0}, \tag{14a}$$

$$\rho^\mu = ig\tau^a R(r)\epsilon^{aik}\delta^{\mu i}\hat{r}^k, \tag{14b}$$

$$a^\mu = ig\tau^a[a_1(r)\delta^{ai} + a_2(r)\hat{r}^a\hat{r}^i]\delta^{\mu i}. \tag{14c}$$

$\omega(r)$, $R(r)$, and $a_{1,2}(r)$ are continuous functions of $r$. In terms of (14), and the standard hedgehog configuration for the pion field,

$$U = \exp[i\tau\cdot\hat{r}f(r)], \tag{15}$$

the energy of a hedgehog skyrmion reads

$$E = E_\pi + E_{\pi\rho A} + E_{\pi A} + E_{\omega\rho A}^{(\mathrm{kin})} + E_{\omega\rho A}^{(\mathrm{mass})} + E_{WZ}. \tag{16}$$

The $E$'s are functionals of $F, R, \omega$, and $a_{1,2}$. They correspond to the various mesonic contributions discussed above. Their explicit form will be given elsewhere.[11] The functions $F, R, \omega$, and $a_{1,2}$ can be determined by functional minimization of (16). The resulting equations are precisely the Euler-Lagrange equa-

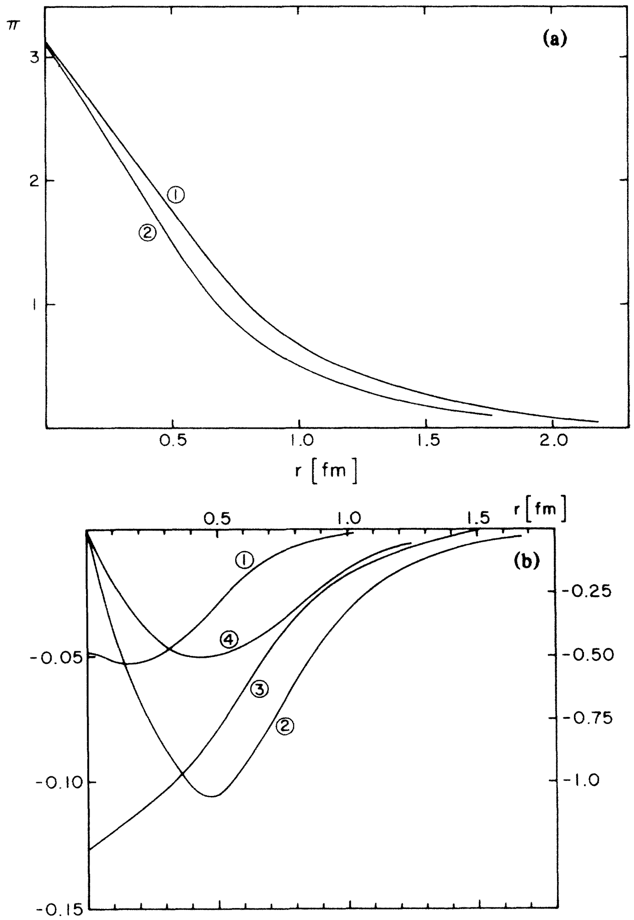


FIG. 1. (a) Chiral angle $F(r)$ for $g=5$ (curve 1) and $g=6$ (curve 2). (b) The vector-meson fields for $g=6$. Curve 1 presents the $\omega$ meson $\omega(r)$, curve 2 the $\rho$ meson $R(r)$, curve 3 the scalar part of the physical $A_1$ meson $A_1^s(r)$, and curve 4 the tensor part of the physical $A_1$ meson $A_1^t(r)$. The scale on the left-hand side stands for the $\rho$ meson, whereas the scale on the right-hand side corresponds to $\omega(r)$, $A_1^s(r)$, and $A_1^t(r)$.

tions. They can be solved by use of the appropriate boundary conditions on the meson fields which follow from their character and their behavior under symmetry transformations (parity, etc.). The only nontrivial boundary conditions are $F(0)=\pi$ and $F(\infty)=0$. They correspond to a skyrmion with unit baryon number. For $f_\pi=93$ MeV, $g=6$, $m_\pi=139$ MeV,

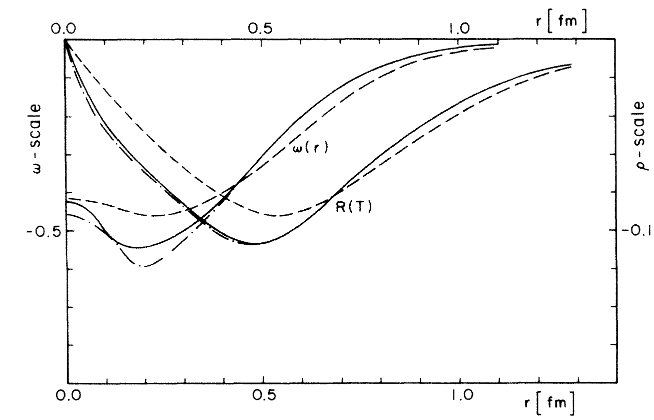


FIG. 2. The vector-meson fields $\omega(r)$ and $R(r)$ for different parameter sets. The solid lines represent $g=\tilde{g}=6$, $m_A=1115$ MeV; the broken lines $g=6$, $\tilde{g}=7.4$, and $m_A=960$ MeV; and the dashed-dotted lines $g=\tilde{g}=6$, $m_A=1275$ MeV. $g$ is the SU(2) coupling constant and $\tilde{g}$ is the U(1) coupling constant; $\tilde{g}=7.4$ corresponds to $g_{\omega NN}^2/4\pi=10$.

and $m=780$ MeV, the numerical solutions to the variational problem are shown in Fig. 1. In this case $g_\omega=9$, $m_A=960$ MeV, and the hedgehog mass $M_H=847$ MeV is considerably smaller than $M_H=1425$ MeV as originally discussed by Jackson and Rho.[12] Notice that one has to multiply the hedgehog mass of 865 MeV of Adkins, Nappi, and Witten[13] by 1.46 for a consistent comparison. Remember that they fit the nucleon and $\Delta$-isobar masses, while we are using $f_\pi$ and $g_A$ as input parameters. The mean square radius is $r_H=0.64$ fm. The hedgehog parameters are found to be very stable in the range $g\sim 4$–6 as shown in Table I. Finally, notice that the pion-nucleon $\Sigma$ term,

$$\Sigma_{\pi N}=4\pi m_\pi^2 f_\pi^2 \int_0^\infty dr\, r^2(1-\cos F), \tag{17}$$

is of the order of 44 MeV for $g=6$, in good agreement with the mean experimental value $\Sigma_{\rm expt}=50\pm 20$ MeV. We have also checked the stability of our results against variations of the $\mathrm{U}(1)_V$ coupling constant and the mass of the $A_1$. The hedgehog mass turns out to be in the range of 850–900 MeV, and the isoscalar radius in the order of 0.60–0.65 fm. The only quantity which is a bit sensitive to $g$ is the pion-nucleon $\Sigma$ term. For $g_{\mathrm{U}(1)}=7.47$ (i.e., $g_\omega^2/4\pi=10$)

TABLE I. Various meson contributions to the hedgehog mass ($M_H$) and size ($r_H$), as defined in (16), for different values of the $\rho\pi\pi$ coupling $g$. All energies are given in megaelectronvolts.

| $g$ | $E_\pi$ | $E_{\pi\rho A}$ | $E_{\pi A}$ | $E_{\omega\rho A}^{(\rm kin)}$ | $E_{\omega\rho A}^{(\rm mass)}$ | $E_{WZ}$ | $M_h$ (MeV) | $r_H$ (fm) |
|---|---|---|---|---|---|---|---|---|
| 4 | 591 | −29 | −92 | 93 | −86 | 370 | 845 | 0.56 |
| 5 | 671 | −66 | −156 | 101 | −66 | 364 | 848 | 0.60 |
| 6 | 748 | −155 | −228 | 132 | −10 | 360 | 847 | 0.64 |

and $m_A = 1275$ MeV we find $\Sigma_{N\pi} = 59$ MeV.[13] Figure 2 shows the parameter dependence of the $\rho$ and $\omega$ meson fields $R(r)$ and $\omega(r)$, respectively.

The present analysis shows that the spin-1 mesons yield a considerable decrease in the hedgehog mass, an effect that will definitely improve the conventional results of the Skyrme model. A more thorough analysis of the present work, and further details on the static properties of baryons in this model will be given elsewhere.[11]

We would like to thank G. E. Brown and M. Rho for their interest in this work. After the completion of this Letter we were informed of a similar ongoing work by R. Vin Mau and collaborators at Orsay. This work was supported in part by Deutsche Forschungsgemeinschaft under Contract No. We 655/9-1, and in part by the U. S. Department of Energy under Contract No. DE-ACO2-76ER13001.

[1]For a recent review see I. Zahed and G. E. Brown, to be published, and references therein.

[2]E. Witten, in *Solitons and Elementary Particle Physics,* edited by A. Chodos, E. Hadjimichael, and C. Tze (World Scientific, Singapore, 1984), pp. 306–312.

[3]O. Kaymakcalan, S. Rajeev, and J. Schechter, Phys. Rev. D **30**, 594 (1984).

[4]R. Machleidt, in Proceedings of the LAMPF Workshop on Relativistic Dynamics and Quark Nuclear Physics, 2–14 June 1986 (to be published).

[5]J. J. Sakurai, *Currents and Mesons* (Univ. of Chicago Press, Chicago, 1969).

[6]S. Weinberg, Phys. Rev. **166**, 1568 (1968).

[7]C. G. Callan, S. Coleman, J. Wess, and B. Zumino, Phys. Rev. **177**, 2247 (1969).

[8]Y. Igarashi *et al.,* Nucl. Phys. **B259**, 721 (1985); M. Bando *et al.*, Phys. Rev. Lett. **54**, 1215 (1985); M. Bando *et al.,* Nucl. Phys. **B259**, 493 (1985).

[9]W. A. Bardeen, Phys. Rev. **184**, 1848 (1969).

[10]G. S. Adkins and C. R. Nappi, Phys. Lett. **137B**, 251 (1984).

[11]U.-G. Meissner and I. Zahed, to be published.

[12]A. D. Jackson and M. Rho, Phys. Rev. Lett. **51**, 751 (1983).

[13]G. S. Adkins, C. R. Nappi, and E. Witten, Nucl. Phys. **B228**, 552 (1983).