Semirelativistic Behavior of Electrons in InSb in Crossed Magnetic and Electric Fields

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Cyclotron-resonance experiments on conduction electrons in InSb are performed in crossed magnetic and electric fields with use of a metal-oxide–semiconductor structure. The data are successfully described on the basis of a three-level $\mathbf{k}\cdot\mathbf{p}$ band model. The electron behavior in crossed fields is interpreted as a spectacular example of an analogy between electrons in narrow-gap semiconductors and relativistic electrons in vacuum.

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It was shown some time ago that semiconductor electrons in the presence of crossed magnetic and electric fields offer interesting physical possibilities.[1] The crossed-field configuration is fundamental for classical and quantum transport phenomena in solids and it has attracted a renewed interest in connection with the discovery of the quantum Hall effect.[2] It has been predicted theoretically that in narrow-gap semiconductors the crossed-field case could serve as an example of a "semirelativistic" electron behavior, governed by the energy-momentum relation analogous to that for free electrons in vacuum.[3]

However, the experimental work with the use of crossed fields has been limited to germanium, which is not a narrow-gap semiconductor, and to interband magneto-optical experiments, whose interpretation has been obscured by the degenerate character of the valence band in this material.[4] Only recently has it become possible to apply high electric fields to narrow-gap semiconductors in crossed-field configuration, because of improved technology of metal-oxide semiconductor (MOS) structures.[5] An additional important feature, which has made possible a clear interpretation of intraband cyclotron resonance experiments in the Voigt configuration, is the absence of the plasma shift for the electron gas in this structure.[6]

Here we report cyclotron-resonance (CR) experiments performed on InSb in crossed magnetic and electric fields, their theoretical description based on the three-level $\mathbf{k}\cdot\mathbf{p}$ model, and finally their physical interpretation in terms of the semirelativistic analogy.

Our experiments have been carried out on Ge-doped ($N_A = 10^{14}$ cm^{-3}) InSb(111) platelets with SiO_2 gate insulators and semitransparent NiCr gates. The magnetic field was directed parallel to the interface. The change of transmission ΔT due to the inversion electrons has been measured at fixed laser energies $\hbar\omega$ and inversion electron densities n_s in a sweep of the magnetic field. The far-infrared light has been incident perpendicular to the interface and to the magnetic field (Voigt configuration) and it was polarized perpendicularly to the magnetic field (see inset in Fig. 1). Thus cyclotron-resonance transitions[5] could be induced.

Examples of transmission spectra, taken for one laser energy and different surface-electron densities n_s (or electric fields), are shown in Fig. 1. Clear observation of cyclotron resonance for the light polarization parallel to the interface indicates that we deal with

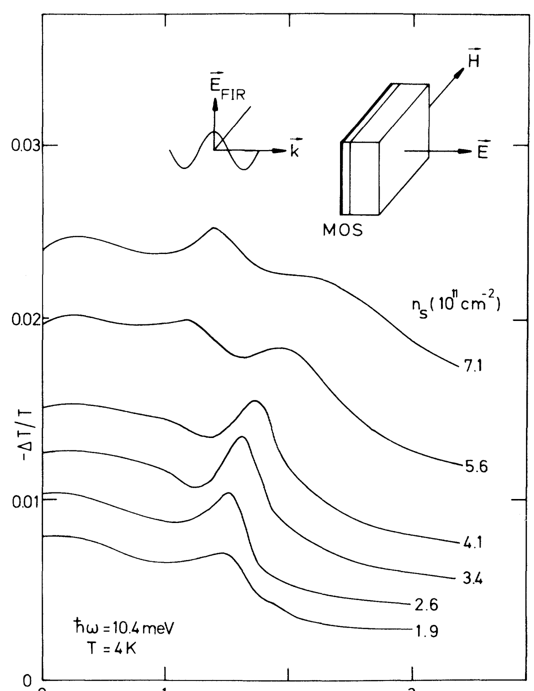

FIG. 1. Cyclotron-resonance spectra of inversion electrons vs magnetic field parallel to the inversion layer at a fixed laser energy $\hbar\omega$ (crossed-field configuration, see inset). Spectra are shown for various electron densities n_s.

electrons away from the barrier, i.e., with those possessing essentially three-dimensional character.[5,7] As the electric field increases the cyclotron-resonance position shifts to higher magnetic fields and finally the line disappears. This indicates that nonparabolic effects in the conduction band of InSb come strongly into play.

To treat the problem theoretically we consider the electron in a periodic potential $V_0(\mathbf{r})$ in the presence of an external magnetic field H and a constant electric field E,

$$[(1/2m_0)P^2+V_0+e\mathbf{E}\cdot\mathbf{r}+H_{\mathrm{s.o.}}]\Psi=\epsilon\Psi, \tag{1}$$

where $\mathbf{P}=\mathbf{p}+(e/c)\mathbf{A}$ in the standard notation and $H_{\mathrm{s.o.}}$ is the spin-orbit interaction. We look for solutions in the form $\Psi=\sum f_n(\mathbf{r})u_n(\mathbf{r})$ where f_n are envelope functions, u_n are the Luttinger-Kohn[8] periodic functions at $k=0$, and the index n runs over the bands.

Considering a narrow-gap semiconductor of the InSb type we take into account three levels at $k=0$: a Γ_6 conduction level separated from a Γ_8 valence level by the gap energy ϵ_g, this in turn separated from a Γ_7 valence level by the spin-orbit energy Δ. We neglect the free-electron term as it gives only a small contribution to the effective mass, and we assume $\Delta >> \epsilon_g$, which is approximately the case in InSb. The choice $\mathbf{E}=(0,E,0)$ and $\mathbf{A}=(-Hy,0,0)$ allows us to separate the variables: $f_n(\mathbf{r})=\exp(ik_xx+ik_zz)\phi_n(y)$. The final effective equation for the envelope functions $\phi_\pm(y)$ related to the S-like conduction level is

$$\left(-\frac{\hbar^2}{2m_0^*}\frac{\partial^2}{\partial y^2}-\alpha y+\frac{m_0^*}{2}\omega_{\mathrm{eff}}^2y^2\right)\phi_\pm=\lambda_\pm\phi_\pm, \tag{2}$$

where $\omega_{\mathrm{eff}}^2=\omega_c^2-2e^2E^2/m_0^*\epsilon_g$, $\alpha=\hbar\omega_ck_x-2eE\epsilon/\epsilon_g$, and

$$\lambda_\pm=\frac{\epsilon^2}{\epsilon_g}-\frac{1}{4}\epsilon_g\mp\frac{1}{2}g_0^*\mu_{\mathrm{B}}H-\frac{\hbar^2k_x^2}{2m_0^*}-\frac{\hbar^2k_z^2}{2m_0^*}.$$

Here $\omega_c=eH/m_0^*c$ is the cyclotron frequency, and m_0^* and g_0^* are the effective mass and the spin-splitting factor at the band edge, respectively. To arrive at Eq. (2) we have neglected a term resulting from the noncommutation of P_y and eEy operators. This term is responsible for the Zener tunneling between the bands[9] and it is of no importance for the electron energies as long as the energy gap is not very small.[10]

Equation (2) is similar to the eigenvalue problem for the harmonic oscillator with an effective frequency $\omega_{\mathrm{eff}}=\omega_c(1-\delta^2)^{1/2}$, where $\delta^2=(cE/H)^2(\epsilon_g/2m_0^*)^{-1}$. The term αy determines the position of the quadratic potential well, while $\hbar\omega_{\mathrm{eff}}$ determines the quantization energy and the character of motion. For $\omega_{\mathrm{eff}}^2>0$ one deals with quantized levels. As the electric term in ω_{eff} becomes larger than the magnetic one there is no potential well any more, i.e., no magnetic quantization. In other words, a sufficiently strong transverse electric field destroys the Landau levels.[11]

The quantized levels in crossed fields (for $\delta<1$) are derived from Eq. (2) by completing the square and carrying out the harmonic oscillator quantization. For $k_z=0$ we obtain

$$\epsilon_{l,k_x}^\pm=\frac{cE}{H}\hbar k_x+(1-\delta^2)^{1/2}\left[\left(\frac{\epsilon_g}{2}\right)^2+\epsilon_gD_l^\pm\right]^{1/2}, \tag{3}$$

where

$$D_l^\pm=\hbar\omega_c(1-\delta^2)^{1/2}(l+\tfrac{1}{2})\pm\tfrac{1}{2}g_0^*\mu_{\mathrm{B}}H. \tag{4}$$

For $E=0$ Eqs. (3) and (4) reduce to the well-known formulas for Landau levels in narrow-gap materials.[12]

Figure 2 shows experimental and theoretical results for the cyclotron mass, defined as $m^*=e\hbar H/(\epsilon_{l+1}^+-\epsilon_l^+)c$. Experimental errors for the mass m^* and the density n_s are $\Delta m\lesssim2\times10^{-4}m_0$ and $n_s\simeq0.1\times10^{11}$ cm^{-2}, respectively. Solid lines are calculated from Eq. (3) for $l=0^+$ to $l=1^+$ transitions. The momentum $\hbar k_x$ is conserved in a cyclotron-resonance transition.

The points corresponding to the lowest line ($E=0$) have been measured on a bulk InSb sample with $n=6\times10^{13}$ cm^{-3}, in which only $0^+\rightarrow1^+$ transitions are possible. For the $E=0$ case one can use a model

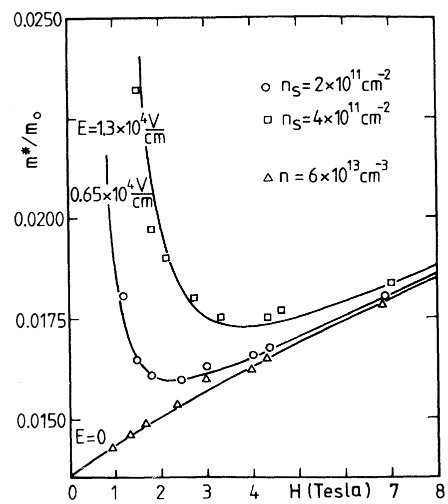

FIG. 2. Electron cyclotron masses in InSb in crossed electric and magnetic fields for two inversion-electron densities n_s, i.e., electric field strengths E. Volume masses of n-type InSb ($E=0$) are included for comparison. Solid lines are calculated. A steep increase of the observed and calculated values at low magnetic fields for $E\neq0$ is caused by the semirelativistic enhancement of the mass, as the drift velocity in crossed fields becomes comparable to the maximum velocity possible in the conduction band.

with final Δ value.[12] The data are then very well described with the realistic InSb parameters: $m_0^* = 0.0136 m_0$, $g_0^* = -51.3$, $\epsilon_g = 0.236$ eV, and $\Delta = 0.81$ eV. However, since for the crossed fields we use a somewhat simplified model (which assumes large Δ), we use the same scheme for $E=0$. This requires a slight modification of the gap value to $\epsilon_g = 0.250$ eV. The solid line for $E=0$ has been calculated under this procedure.

The $E \neq 0$ points have been measured on the MOS structure described above. In their description we use Eqs. (3) and (4) with the same band parameters (i.e., $\epsilon_g = 0.250$ eV). Since we assume the electric field to be constant, which is not the case in a real MOS structure, the effective field intensities indicated in Fig. 2 have been treated as an adjustable parameter. For the electron densities shown in Fig. 2 the fields are found to be a factor of about 3 less than the fields just inside the semiconductor: The three-dimensional electrons away from the barrier feel an electric field that is screened by the electrons closer to the interface. A similar screening factor has previously been reported for space-charge layers on HgCdTe.[13] It can be seen that the ratio of adjusted field intensities corresponds to the measured ratio of the surface electron densities for the two cases shown in Fig. 2. The two curves $E \neq 0$ can be interpreted in the following way. At high magnetic fields the electric field term in ω_{eff} is unimportant and the mass increase is similar to the $E=0$ case. As the magnetic field decreases, however, the electric term in ω_{eff} becomes important: $\omega_{\text{eff}} < \omega_c$, and the mass measured by the cyclotron resonance increases. This is in sharp contrast to the $E=0$ case.

The inhomogeneity of the electric field seems to be responsible for the appearance of the second peak at higher electric fields seen in Fig. 1, which we interpret as $l=1$ to $l=2$ CR transition. Real behavior of the electric potential in a MOS structure is sublinear, so that higher magnetic states are subjected to a weaker electric field, which qualitatively agrees with the observations. Disappearance of the CR peak in Fig. 1 corresponds to the condition $\omega_{\text{eff}} = 0$, if the scattering is neglected, or, more realistically, to the condition $\omega_{\text{eff}} \tau < 1$, where τ is the electron relaxation time.

The increase of the cyclotron mass with increasing electric field and the disappearance of cyclotron resonance are spectacular manifestations of the semirelativistic behavior of conduction electrons in InSb. In the absence of external fields the dispersion relation for two interacting bands is given by the simplified Kane[14] formula

$$\epsilon = [(\epsilon_g/2)^2 + \epsilon_g p^2/2m_0^*]^{1/2}. \tag{5}$$

It has the form of the relativistic relation for electrons in vacuum, with $2m_0c^2$ replaced by ϵ_g and m_0 by m_0^*. A maximum velocity u in the two-band model can easily be deduced by analogy: $c = (2m_0c^2/2m_0)^{1/2}$ is to be replaced by $u = (\epsilon_g/2m_0^*)^{1/2} = 1.3 \times 10^8$ cm s^{-1}.[15]

According to the special theory of relativity,[16] the drift velocity in crossed fields is $v_{\text{dr}} = cE/H$, as long as $cE/H < c$. (The same is true for the two-band system, as long as $cE/H < u$.) If a Lorentz transformation is made to a coordinate frame moving with v_{dr} with respect to the laboratory frame, the electric field in the moving frame disappears and the magnetic field is weakened: $H' = H(1 - E^2/H^2)^{1/2}$. This amounts to the proportional decrease of the cyclotron frequency. In a vacuum there is $E^2/H^2 = v_{\text{dr}}^2/c^2$, which in the two-band model is to be replaced by $v_{\text{dr}}^2/u^2 = (cE/H)^2(\epsilon_g/2m_0^*)^{-1}$. Thus the semirelativistic analogy gives $H' = H(1-\delta^2)^{1/2}$, which corresponds precisely to $\hbar\omega_c(1-\delta^2)^{1/2}$ in Eq. (4). (The spin splitting behaves somewhat differently since it is governed by the spin-orbit interaction of atomic origin, which does not have correspondence in the free-electron case.) Thus, in the moving system the electron energy is given by the square root of the square bracket in Eq. (3). However, the observation is made in the laboratory so that the energy has to be Lorentz transformed back to that system. This is done with the four-vector $(\mathbf{p}, \epsilon/c)$, involving the energy as the fourth component. This gives the complete expression (4), when the above mentioned replacements are made.

It is of interest that the difference of $\epsilon_{l+1,k_x}^+ - \epsilon_{1,k_x}^+ = \hbar\omega$ can be interpreted as the transverse Doppler shift (TDS) of the radiation frequency ω_0 emitted by a moving source.[16] This is related to the Voigt geometry of our experiment. In the special theory of relativity TDS is described by $\omega = \omega_0(1 - v^2/c^2)^{1/2}$, which corresponds to the $(1-\delta^2)^{1/2}$ term in front of the square root in Eq. (3). It should be emphasized that the increase of the mass shown in Fig. 2 is, in fact, approximately proportional to $(1-\delta^2)^{-1}$, as follows from Eq. (3) in the limit of $\hbar\omega_c \ll \epsilon_g$. The truly relativistic transverse Doppler shift is rather difficult to observe and it was first measured in 1938 (see Ives and Stilwell[17]). In the theory of relativity TDS is considered to be a direct manifestation of the time dilatation.

It should be borne in mind that we not deal in reality with truly relativistic drift velocities of conduction electrons: $v_{\text{dr}} = 0.85 \times 10^8$ cm s^{-1} for the highest point on the upper curve. The semirelativistic increase of the cyclotron mass is so well observable because u^2 in the semiconductor is almost 10^5 times smaller than c^2.

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