Velocity of Longitudinal Sound and F_2^s in Liquid ^{3}He

B. N. Engel and G. G. Ihas
Department of Physics, University of Florida, Gainesville, Florida 32611
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Precise measurements of the velocity of 5-MHz longitudinal sound in liquid ^{3}He have been made over a broad range of temperatures (0.3 to 30 mK) and pressure (0.3 to 34 bars). The zero-temperature asymptotic velocity in the *B*-phase superfluid is measured and found to be the same as the hydrodynamic value in the normal Fermi liquid. The pressure dependence of F_2^s, the second symmetric Landau parameter for the Fermi liquid, is deduced.

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Liquid ^{3}He has become the archetypical "normal Fermi liquid" in which the physics of an interacting, degenerate system of fermions has been studied. The properties of such a system are dependent on the scattering amplitude, $F(\chi)$, between two quasiparticles (elementary excitations) near the Fermi surface, where χ is the angle between the momenta of the two particles. As was first described by Landau[1] and then generalized by Khalatnikov and Abrikosov,[2] it is convenient to expand $F(\chi)$ in spherical harmonics, the coefficients of the expansion (F_l^s and F_l^a) then appearing like molecular fields in calculations of the various quantities and response functions of the system.

Landau realized that, depending on the detailed nature of $F(\chi)$ and hence F_l^s and F_l^a (which cannot be known *a priori*), various new types of propagating collective modes might be observed in liquid ^{3}He and other similar systems. Originally he and others used measurements of the compressibility and heat capacity to determine F_0^s and F_1^s, which then led to the prediction of two kinds of "sound" in ^{3}He. One, ordinary or first sound, propagates when its period of oscillation $2\pi/\omega$ is long compared to the quasiparticle collision relaxation time τ ($\omega\tau << 1$). Another, zero sound, propagates in the opposite limit ($\omega\tau >> 1$). Zero sound exists in the long-mean-free-path limit, when collisions become unimportant (the collisionless limit), and can be generated by density fluctuations similar to ordinary sound. This mode can be thought of as an asymmetric distortion of the Fermi surface with a restoring force applied to each quasiparticle caused by the molecular fields (represented by the F_l^s) generated by all the other quasiparticles. It is therefore natural to use sound measurements to determine various of the F_l^s or Landau parameters.

We have measured the velocity of 5-MHz longitudinal sound in liquid ^{3}He over a wide range of temperature (0.3 to 30 mK) and pressure (0.3 to 34 bars). The velocity was measured relative to its temperature-independent first-sound value, c_1, in the normal Fermi liquid. At lower temperatures, but still in the normal Fermi liquid, τ increases such that $\omega\tau >> 1$ and zero sound propagates. The velocity of zero sound, c_0, is greater than that of first sound, c_1, by only a few percent and would be maintained to zero temperature if it were not for the onset of superfluidity. At the superfluid transition temperature, T_c, the zero-sound velocity begins to decrease back toward c_1 and approaches an asymptotic value for $T/T_c \lesssim 0.4$ at all pressures. We have measured the velocity well along this temperature-independent asymptote and found it to be that of first sound as theoretically expected for the fully developed condensate.[3] Using the difference between the zero- and first-sound velocities, we have deduced the $l=2$ symmetric Landau parameter, F_2^s, as a function of pressure. Our determinations are consistent with others[4,5] obtained from longitudinal-sound velocity measurements.

The sound measurements were made by propagating 5-MHz sound pulses between two *x*-cut quartz crystals. The helium sample was defined by an epoxy spacer with a low-temperature length of 6.25 mm and 5.0 mm inside diameter separating the quartz transducers. A 286-G magnetic field was applied to the helium parallel to the direction of sound propagation. The nuclear susceptibility at 250 kHz of ^{195}Pt powder immersed in the helium immediately next to the sound cell was used for thermometry with the superfluid transition, identified by the kink in the sound amplitude, serving as a fixed point.

The sound cell and thermometer were mounted on a heat exchanger[6] together with a strain gauge which was used in a feedback loop to determine and regulate the helium pressure. The pressure was regulated by use of the error signal from the strain gauge measurement to control the pressure of liquid ^{4}He contained in a bellows. This bellows was mechanically connected to a second bellows, which, being part of the ^{3}He container, could change the volume of the ^{3}He sample. Since the sound velocity is strongly pressure dependent, this pressure regulation of $\Delta P/P \sim 10^{-5}$ put an upper limit on the precision of the velocity measurements of $\Delta c/c \sim 10^{-5}$ in the worst case. Cooling was achieved by demagnetization of 60 moles of copper from 25 mK and 8.7 T. Many further experimental details are available in the work of Berg and Ihas.[7]

The received sound signal was split, with half detected and signal averaged for amplitude determination, and half sent to a phase-measurement circuit. The phase and hence sound velocity of the received signal were monitored by mixing the signal with the output of a 5-MHz synthesizer. The phase of the reference signal was modulated by the amplified output of the doubly balanced mixer. This was accomplished with use of a boxcar integrator to measure the filtered, mixed signal with a 1-μs-wide gate timed to coincide with the first received sound pulse. As the phase velocity changed, the output from the boxcar shifted the phase of the reference, tending to maintain the mixer output at a constant level. Hence, in this high-gain, phase-locked loop, the change in the boxcar output potential is directly proportional to the change in the phase of the sound signal over several factors of 2π. Then the relative change in phase velocity from c_1, used as a reference point, is

$$(c-c_1)/c_1 = (\Phi_1 - \Phi)c_1/\omega l.$$

$\Phi_1 - \Phi$ is the change in the phase relative to its value at high temperature, c_1 is the first-sound velocity, ω is the sound frequency, and l is the cell length.

The phase-measurement scheme used here is slightly dependent on the signal amplitude. Since this amplitude varied by more than 2 orders of magnitude over the temperature range studied, the precision of these measurements required that this effect be removed from the results. This was done by our calibrating the dependence of the phase on amplitude and then using the simultaneously measured amplitude to correct the phase measurement. This correction was always small (< 6%) for the measurements used here to determine F_2^s. However, since each calculation involves the difference of two nearly equal quantities, this correction is important.

Figure 1 shows the zero-temperature asymptotic velocity relative to c_1 plotted against pressure. The velocity was found to approach c_1 within the experimental precision. Note that the error in these points is greater than the 1 part in 10^5 capability of the measurement system. This is mainly due to the measured voltages proportional to these velocities being very near zero. At 0.32 bar we did not cool fully into the asymptotic region so that an extrapolation was necessary, resulting in increased error. These measurements thus verify the theoretical prediction that the velocity would return to the hydrodynamic value as the superfluid condensate fully developed. Also, because the velocity is identical to that in the normal Fermi liquid, the bulk compressibility and density thus remain constant to zero temperature, in spite of the intervention of the superfluid phase.

F_2^s can be deduced from the difference in first- and zero-sound velocities. If we ignore higher-order Landau parameters, we have

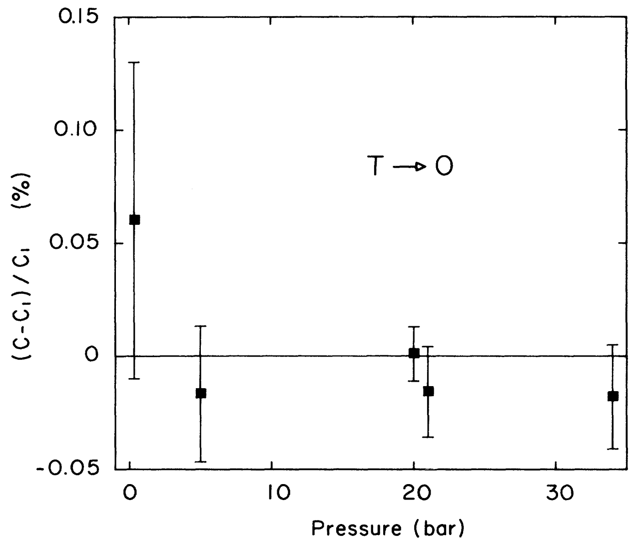

FIG. 1. Zero-temperature asymptotic velocity of sound measured relative to the first-sound velocity (as percentage) vs pressure.

$$\frac{c_0^2 - c_1^2}{c_1^2} = \frac{4}{5}\left[\frac{1+\frac{1}{5}F_2^s}{1+F_0^s}\right] + O\left(\left[\frac{v_F}{c_1}\right]^4\right), \qquad (1)$$

where F_0^s is the $l=0$ symmetric Landau parameter obtained from other experiments and v_F is the Fermi velocity. Zero sound is realized in the limit $\omega\tau >> 1$. Thus, for sufficiently high frequency the zero-sound velocity c_0 can be measured in the normal liquid just above T_c. However, for the 5-MHz sound used in this experiment, at T_c and high pressure this is not the case (e.g., at 34 bars $\omega\tau \approx 3$). Therefore, the velocity at T_c is somewhat less than c_0 and corrections for finite $\omega\tau$ must be invoked. For arbitrary frequencies the velocity of sound is given by[8]

$$c = c_1 + (c_0 - c_1)\mathrm{Re}\,\xi(\omega),$$

where

$$\xi(\omega) = (1 + i/\omega\tau_\alpha)^{-1},$$

and τ_α is the characteristic relaxation time for sound attenuation. The desired difference in the zero- and first-sound velocities may then be written

$$\frac{c_0 - c_1}{c_1} = \left(\frac{c-c_1}{c_1}\right)\left[1 + \frac{1}{(\omega\tau_\alpha)^2}\right],$$

where c is the actual measured velocity. This correction is the greatest at the melting pressure where it is ~ 12% and insignificant at low pressure where it is ~ 0.1%. The value of $\omega\tau$ at T_c as a function of pres-

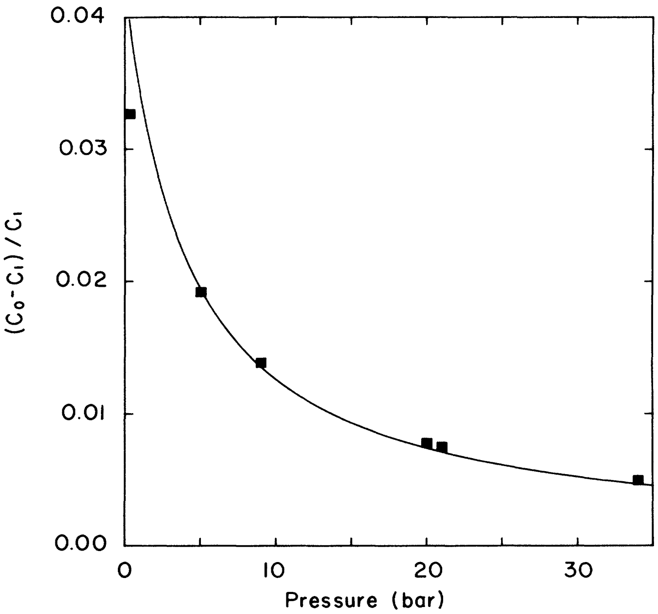

FIG. 2. Points are measured differences of zero- and first-sound velocities (c_1) normalized to c_1. Curve is the theoretical prediction [Eq. (1) in text] with $F_2^s \equiv 0$.

sure was obtained from $\tau_\eta T^2$ data tabulated by Wheatley[9] where τ_η is the characteristic relaxation time for viscosity. Comparison of τ_α measurements[4,5] with those of τ_η show them to be equal to within 5%, but because the τ_η data completely cover the full pressure range they were used in the correction in place of τ_α.

Because the velocity was found to return to c_1 at the lowest temperatures, we could increase precision in the determination of F_2^s by taking the phase difference from this asymptote and that at T_c to obtain $(c_0-c_1)/c_1$. Figure 2 is a plot of $(c_0-c_1)/c_1$ vs pressure. The points are our measurements corrected for finite $\omega\tau$ and the curve is the theory with $F_2^s=0$ and F_0^s as scaled by specific-heat measurements of Greywall and Busch.[10] Thus, the deviation of the points from the curve is a measure of F_2^s.

Figure 3 is a plot of F_2^s vs pressure. The points are our determinations as given by Eq. (1). The curve is a least-squares fit, described in the caption, and is presented as an aid in calculations involving F_2^s.

Figure 4 is a plot of F_2^s as calculated from our velocity measurements, but using the older values of F_0^s found in the Wheatley review article.[9] This was done to facilitate comparison with previous determinations of F_2^s, but it also shows the sensitivity of F_2^s to m^*/m determinations. As can be seen, good agreement is found with the earlier work in which F_2^s was obtained from longitudinal velocity measurements.[4,5] However, more recent values[11] obtained from absolute attenuation measurements are not consistent with our results.

For certain values[12] of Landau parameters, a transverse zero-sound (TZS) mode is predicted to exist: $F_1^s+3F_2^s/(1+\frac{1}{5}F_2^s)>6$. Various attempts[12,13] to

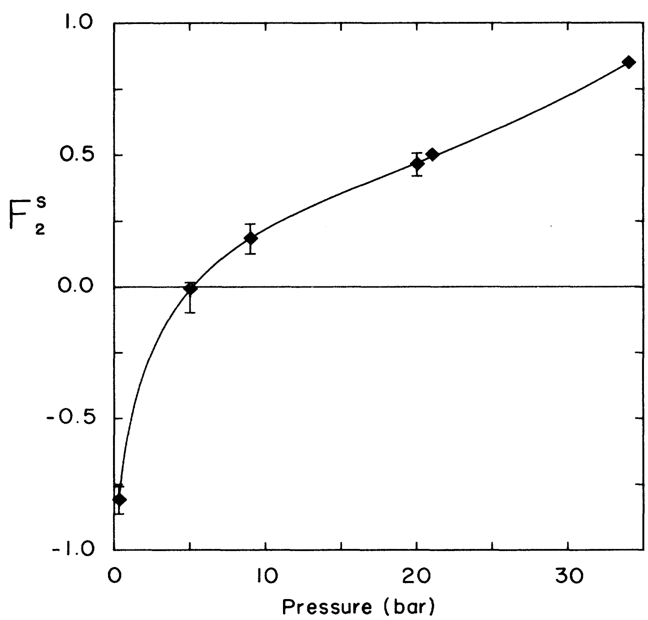

FIG. 3. Points are F_2^s values obtained from our velocity measurements and Eq. (1) with F_0^s from Ref. 10. The curve is a cubic least-squares fit in $P^{1/2}$:

$$F_2^s=-1.264+0.896P^{1/2}-0.187P+0.0163P^{3/2}.$$

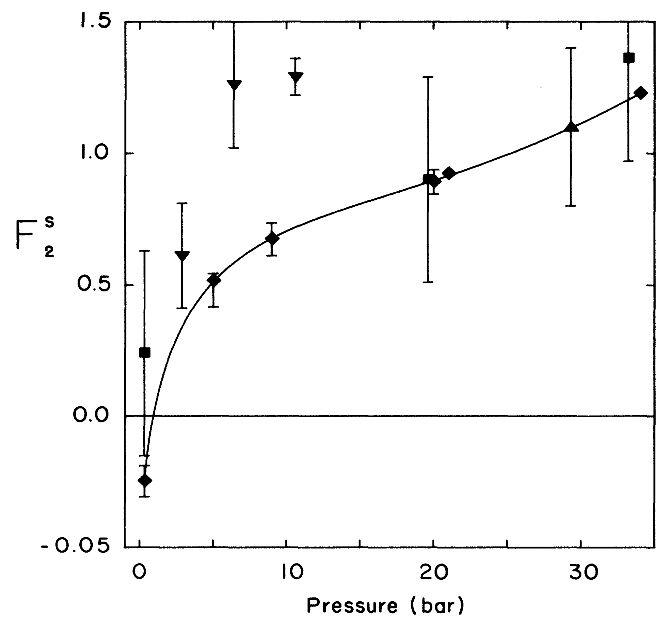

FIG. 4. F_2^s as determined by various workers, with F_0^s from Ref. 9. This work, diamonds; Ref. 4, squares; Ref. 5, triangles; Ref. 11, inverted triangles. The curve is a least-squares fit to our data.

detect this mode by acoustic impedance techniques have resulted in determinations of F_2^s which are mostly negative, except at very low pressure. Our values of F_2^s, together with the above expression, would preclude the existence of TZS at pressures below 3.4 bars (using scaled values[10] of F_1^s). Another feature of the TZS determinations of F_2^s is that they appear to be frequency dependent. The above circumstances lead to the conclusion[14] that these acoustic impedance measurements cannot be analyzed as pure TZS, casting doubt on their determination of F_2^s.

In conclusion, the sound measurements in liquid ^{3}He presented here show the following: (1) The velocity asymptotically approaches c_1, the hydrodynamic velocity, as the temperature approaches zero. Hence, the sound propagation is a collective mode of a fully developed BCS condensed state. Also, as expected, the superfluid transition has no effect on the density and compressibility of the liquid. (2) Because of (1), the difference between the zero- and first-sound velocities $(c_0 - c_1)$ can be measured quite precisely, yielding rather good determinations of the Landau parameter F_2^s. In turn, F_2^s can now be used to calculate better the properties and responses of liquid ^{3}He (e.g., sound velocity and attenuation, frequencies of the superfluid gap modes, and the pressure range of the existence of transverse zero sound). (3) The results agree with other similar, less precise measurements, but the new values of F_2^s disagree with those recently obtained from longitudinal-sound attenuation measurements and transverse zero-sound (TZS) acoustic-impedance experiments. In the longitudinal-sound case, this is probably due to the increased difficulty of making precise absolute attenuation measurements which depend on the temperature scale used. In the case of TZS, the disagreement may arise from the highly (over?)damped nature of the mode.

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