Spin-Dependent Forces in Heavy-Quark Systems

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The spin-dependent potentials in the formalism of Eichten and Feinberg and of Gromes are generalized. Consistency of the observed spin splittings in the J/ψ and Y systems with QCD imposes stringent constraints on the type of nonperturbative spin-dependent forces allowed.

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In spite of the existence of a fairly extensive literature on the spin-dependent forces in heavy-quark systems[1-5] our understanding of these forces is less than satisfactory. The usual approach is to combine the Breit-Fermi interaction with an *ad hoc* long-range interaction to obtain the spin-dependent potential V_{SD}. The value of the running coupling constant α_S used is (unjustifiably) taken from the spin-independent potential or elsewhere, although different physical processes in general have different values for α_S (the common parameter is $\Lambda_{\overline{MS}}$, where $\overline{MS}$ denotes the modified minimal-subtraction scheme, not α_S). Moreover, most of the analyses of V_{SD} do not incorporate the QCD radiative corrections. Such corrections are rather significant since they contribute to the spin-dependent forces in a form different from the tree-level terms; also, the inclusion of the one-loop effects is necessary for relating V_{SD} to $\Lambda_{\overline{MS}}$.

In this Letter we will use the one-loop QCD corrections to show explicitly that the $1/m$ expansion for V_{SD} in the formalism of Eichten and Feinberg[2] and Gromes[3] (EF&G) has to be appropriately generalized to include the quark-mass dependence. To confront the experimental data, we shall take the following approach. Since we know that QCD radiative corrections must be included while the nonperturbative long-range spin forces are not known, we use the perturbative expression (tree level plus one loop) for V_{SD} together with various forms of long-range spin-orbit interactions. This allows us to make eight independent determinations of the parameter $\Lambda_{\overline{MS}}$ from the current data on the spin spin (V_S), the spin orbit (V_L), and the tensor (V_T) terms from the J/ψ and Y systems. Consistency of the observed spin splittings will impose stringent constraints on the type of long-range spin-dependent forces allowed. Our results are summarized in Figs. 1 and 2. We observe that for the ψ system some additional effects, such as relativistic corrections,

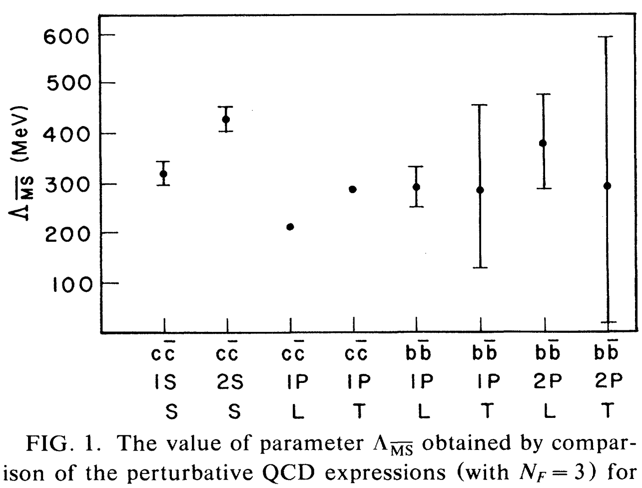

FIG. 1. The value of parameter $\Lambda_{\overline{MS}}$ obtained by comparison of the perturbative QCD expressions (with $N_F = 3$) for the hyperfine (S) term, the spin-orbit (L) term, and the tensor (T) term with the experimental values for the $\psi - \eta_c(1S)$, $\psi' - \eta_c(2S)$, the χ_c, the χ_b, and the χ_b' splittings.

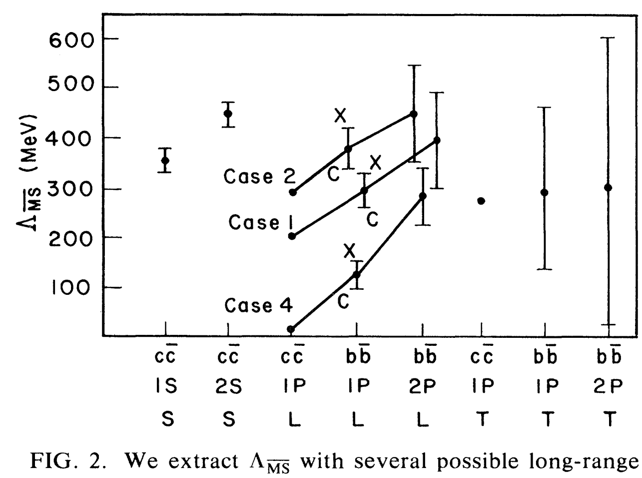

FIG. 2. We extract $\Lambda_{\overline{MS}}$ with several possible long-range forces added to the spin-orbit potential, as explained in the text. The × stands for the data from Crystal Ball Collaboration (see Ref. 9), and the c stands for the data from Ref. 7. The spin-spin, spin-orbit, and tensor terms are denoted by S, L, and T, respectively. The spin-independent potential $[E(r)]$ used is consistent with $\Lambda_{\overline{MS}} = 0.30$ GeV at short distances.

are required. On the other hand, one can account for the fine structures for the Υ system with the inclusion of the one-loop QCD corrections and a choice of $\Lambda_{\overline{\mathrm{MS}}} = 0.30 \pm 0.06$ GeV but *no* long-range (nonperturbative) spin-dependent force. The existence of some (probably small) long-range spin interactions is not ruled out; should nonperturbative spin effects be necessary for the Υ system, better data will certainly provide valuable hints on the form of such forces.

Irrespective of any other corrections, the spin-dependent potential V_{SD} definitely receives contributions from QCD radiative corrections. To include such contributions the EF&G classification of the spin-dependent forces must be generalized. For unequal quark and antiquark masses ($m_1 \neq m_2$), with incorporation of one-loop QCD effects[4,6] the effective potential is

$$V(r) = E(r) + \left(\frac{\mathbf{S}_1}{m_1^2} + \frac{\mathbf{S}_2}{m_2^2}\right)\cdot\mathbf{L}\frac{1}{r}\left[\frac{1}{2}\frac{dE}{dr} + \frac{dV_1}{dr}\right] + \left(\frac{\mathbf{S}_1+\mathbf{S}_2}{m_1 m_2}\right)\cdot\mathbf{L}\frac{1}{r}\frac{dV_2}{dr} + \frac{1}{m_1 m_2}\left[\hat{\mathbf{r}}\cdot\mathbf{S}_1\hat{\mathbf{r}}\cdot\mathbf{S}_2 - \frac{1}{3}\mathbf{S}_1\cdot\mathbf{S}_2\right]V_3$$
$$+\frac{1}{3}\frac{\mathbf{S}_1\cdot\mathbf{S}_2}{m_1 m_2}V_4 + \left[\left(\frac{\mathbf{S}_1}{m_1^2} - \frac{\mathbf{S}_2}{m_2^2}\right)\cdot L + \left(\frac{\mathbf{S}_1-\mathbf{S}_2}{m_1 m_2}\right)\cdot L\right]V_5, \qquad (1)$$

$$\bar{E}(r) = -\frac{1}{r}C_F\alpha_{\overline{\mathrm{MS}}}(\mu)\left\{1 + \frac{\alpha_S}{\pi}\left[\frac{b_0}{2}(\ln\mu r + \gamma_E) + \frac{5}{12}b_0 - \frac{2}{3}C_A\right]\right\},$$

$$V_1 = -\frac{1}{r}\frac{1}{2}C_F\frac{\alpha_S^2}{\pi}\{C_F - C_A[\ln(m_1 m_2 r)^{1/2} + \gamma_E]\},$$

$$V_2 = -\frac{1}{r}C_F\alpha_{\overline{\mathrm{MS}}}(\mu)\left[1 + \frac{\alpha_S}{\pi}\left\{\frac{b_0}{2}(\ln\mu r + \gamma_E) + \frac{5}{12}b_0 - \frac{2}{3}C_A + \frac{1}{2}\{C_F - C_A[\ln(m_1 m_2 r)^{1/2} + \gamma_E]\}\right\}\right],$$

$$V_3 = \frac{1}{r^3}3C_F\alpha_{\overline{\mathrm{MS}}}(\mu)\left[1 + \frac{\alpha_S}{\pi}\left(\frac{b_0}{2}\left(\ln\mu r + \gamma_E - \frac{4}{3}\right) + \frac{5}{12}b_0 - \frac{2}{3}C_A + \frac{1}{2}\left\{C_A + 2C_F - 2C_A\left[\ln(m_1 m_2 r)^{1/2} + \gamma_E - \frac{4}{3}\right]\right\}\right)\right],$$

$$V_4 = 8\pi C_F\alpha_{\overline{\mathrm{MS}}}(\mu)\left\{\left\{1 + \frac{\alpha_s}{\pi}\left[\frac{5}{12}b_0 - \frac{2}{3}C_A - C_A + C_F - \frac{3}{4}\left(C_F\frac{m_1-m_2}{m_1+m_2} + \frac{1}{2}(C_A - 2C_F)\frac{m_1+m_2}{m_1-m_2}\right)\ln\frac{m_2}{m_1}\right]\right\}\delta^3(r)\right.$$
$$\left.+\frac{\alpha_S}{\pi}\left[-\frac{b_0}{4}\frac{1}{2\pi}\nabla^2\left(\frac{\ln\mu r + \gamma_E}{r}\right) + \frac{7}{8}C_A\frac{1}{2\pi}\nabla^2\left(\frac{\ln(m_1 m_2 r)^{1/2} + \gamma_E}{r}\right)\right]\right\},$$

$$V_5 = \frac{1}{r^3}\frac{1}{4}C_F C_A\frac{\alpha_S}{\pi}\ln\frac{m_2}{m_1}, \qquad (2)$$

where $E(r)$ is the phenomenologically determined quarkonium potential while $\bar{E}(r)$ is the perturbative QCD potential; $b_0 = \frac{11}{3}C_A - \frac{4}{3}T_F N_f$ with N_f being the number of light quarks, μ is the renormalization scale, and $\gamma_E = 0.5772\ldots$ is the Euler constant. T_F, C_F, and C_A are gauge-group factors. In QCD they read $T_F = \frac{1}{2}$, $C_F = \frac{4}{3}$, and $C_A = 3$. Each $V_i(m_1,m_2,r)$ is a guage-invariant quantity. The potential $V_5(m_1,m_2,r)$ is the new one that we have added to the EF&G formalism. For the equal-mass case ($m_1 = m_2 = m$) the $V_5(m,r)$ term vanishes and $V_4(m_1,m_2,r)$ receives additional contributions from the annihilation diagrams,

$$\delta V_4(m,r) = 12(1-\ln 2)C_F T_F\alpha_S^2\delta^3(r). \qquad (3)$$

The EF&G formalism assumes that the V_i are flavor independent. Since V_i are proportional to $\ln m$, which is not analytic as $m \to \infty$, it is not surprising that the EF&G method does not include this term. However, Eq. (3) shows that, even in the equal-mass case where $V_5(m,r)$ disappears, the other $V_i(m,r)$'s still contain logarithms of $(q/m)^2$. These logarithms are not present in QED for which C_A vanishes. The presence of these $\ln(q/m)$ terms show that these forces are actually flavor dependent.

Applying Lorentz invariance Gromes has derived the relation[3]

$$(d/dr)[E(r) + V_1(m_1,m_2,r) - V_2(m_1,m_2,r)] = 0. \qquad (4)$$

The perturbative expressions for $\bar{E}(r)$, $V_1(m_1,m_2,r)$, and $V_2(m_1,m_2,r)$ satisfy this identity at the tree level and at the one-loop level for both the unequal- and the equal-mass cases. The Gromes relation implies that V_1 and/or V_2 must contain some long-range contributions similar to $E(r)$, but the relation gives no information about the net long-range spin-orbit interaction. We also note that invariance under infinitesimal boosts dictates that $(m_1^{-2}\mathbf{S}_1 - m_2^{-2}\mathbf{S}_2)\cdot\mathbf{L}$ and $(m_1m_2)^{-1}\times(\mathbf{S}_1-\mathbf{S}_2)\cdot\mathbf{L}$ must be described by the same potential factor. This also is satisfied at the one-loop level and we have incorporated it into our formalism by defining only one new potential in Eq. (1), namely, V_5.

Recently many new experimental results on the fine and hyperfine structure in the ψ and Υ systems have become available.[7–12] For the $c\bar{c}$ 1^3P_J states we use $M_J = 3555.8\pm0.6$, 3510.0 ± 0.6, 34315.0 ± 1.0 MeV, respectively, for $J=2,1,0$. The $c\bar{c}$ hyperfine splittings are $\Delta E = 113\pm5$ and 92 ± 5 MeV, respectively, for $1S$ and $2S$ states. For the photon energies (E_γ) in the $b\bar{b}$ transition $2^3S_1 \to 1^3P_J+\gamma$, we use[7–11] $E_\gamma = 109\pm1$, 129 ± 1, 155 ± 4 MeV, and in $3^3S_1\to 2^3P_J$, $E_\gamma = 84.2\pm2.2$, 101.4 ± 3.2, 122.1 ± 5.5 MeV for $J=2,1,0$, respectively.

Models of heavy quarkonium start with a nonrelativistic, spin-averaged Hamiltonian that is solved exactly for wave functions and energy levels.[13–15] The spin splittings are treated as small perturbations. To leading order

$$\begin{aligned} M(n\,^3S_1) - M(n\,^1S_0) &= \langle V_S\rangle, \\ M(n^1P_1) &= M_0 - \tfrac{3}{4}\langle V_S\rangle, \\ M(n^3P_J) &= M_0 + \tfrac{1}{4}\langle V_S\rangle + l_J\langle V_L\rangle + t_J\langle V_T\rangle, \end{aligned} \tag{5}$$

where $l_J = 1, -1, -2$ and $t_J = -\frac{1}{10}, \frac{1}{2}, -1$, respectively, for $J=2,1,0$; M_0 is the spin-averaged mass, and expectation values are denoted by angular brackets. The potentials are obtained from Eq. (1):

$$\begin{aligned} V_S &= \frac{1}{3}\frac{1}{m^2}V_4(m,r), \quad V_T = \frac{1}{3}\frac{1}{m^2}V_3(m,r), \\ V_L &= \frac{1}{m^2}\frac{1}{r}\frac{d}{dr}\left[\frac{1}{2}E(r) + V_1(m,r) + V_2(m,r)\right]. \end{aligned} \tag{6}$$

To extract the QCD scale parameter $\Lambda_{\overline{\rm MS}}$ from the energy-level splittings we first invert Eq. (5) to get the experimental values for $\langle V_S\rangle$, $\langle V_L\rangle$, and $\langle V_T\rangle$. (The dependence on M_0 cancels out in mass differences.)

For the spin-dependent partial $V_{\rm SD}$ we use the QCD perturbation (tree level plus one loop) expression plus some long-range spin-dependent forces. The Gromes relation provides one constraint on the three potentials $E(r)$, $V_1(r)$, and $V_2(r)$ that appear in the spin-orbit interactions. With $E^N(r)$, the nonperturbative part of $E(r)$, approximately known, we need one assumption in order to determine fully the long-range part of the spin-orbit interaction. We will discuss the following four representative cases (other cases can be considered in a similar manner): (1) $V_1^N = -3V_2^N = -\frac{3}{4}E^N$, such that there is no nonperturbative spin interactions (the superscript N denotes nonperturbative, $E^N = E-\bar{E}$); (2) $V_2^N = 0$; (3) $V_1^N=0$; (4) $V_1^N = -V_2^N = -\frac{1}{2}E^N$. For each case, three independent determinations of the parameter $\Lambda_{\overline{\rm MS}}$ can be made from the current data on the fine structures from the J/Ψ and Υ systems. Irrespective of which type of long-range spin-orbit interaction one considers, the current data on V_S and V_T terms provide five more determinations of the parameter $\Lambda_{\overline{\rm MS}}$. Consistency demands that for one particular type of long-range spin-dependent forces we obtain a common value of $\Lambda_{\overline{\rm MS}}$ (within errors). We emphasize that, in contrast to the usual approach which assumes a certain form of the long-range spin forces and then confronts experimental data, we use the data to extract the parameter $\Lambda_{\overline{\rm MS}}$ and deduce the long-range spin-dependent potentials.

Now we consider the four types of long-range spin interactions separately.

Case 1: $V_1^N = -3V_2^N = -\frac{3}{4}E^N$, such that $V_{\rm SD}$ is taken entirely to be the expression given by the perturbative QCD calculations. For this case the spin-orbit potential is given by

$$\begin{aligned} V_L &= [2/m^2r^3]\alpha_{\overline{\rm MS}}(\mu)[1+\alpha_S/\pi A], \\ A &= \tfrac{1}{2}b_0(\ln\mu r + \gamma_E - 1) + \tfrac{5}{12}b_0 + \tfrac{8}{9} - 2(\ln mr + \gamma_E). \end{aligned} \tag{7}$$

By use of Eq. (7) for V_L and the one-loop expressions [Eq. (2)] to describe V_S and V_T we are left with $\alpha_{\overline{\rm MS}}(\mu)$ as the only unknown parameter. All other quantities that the spin splittings depend on, such as the quark mass and the wave function, can be determined from other properties of the quarkonium spectrum.[13–15]

We choose the prescription advocated by Grunberg[16] to specify the mass scale μ. To calculate the wave-function factors we use the potential given in Ref. 15. After solving for $\alpha_{\overline{\rm MS}}(\mu)$ we extract $\Lambda_{\overline{\rm MS}}$ via the two-loop β function.[15] The results[17,18] for $\Lambda_{\overline{\rm MS}}$ shown in Fig. 1 are consistent with one another within the experimental and theoretical uncertainties.[6] We estimate $\Lambda_{\overline{\rm MS}} = 0.30\pm0.06$ GeV, a value consistent with other determinations of $\Lambda_{\overline{\rm MS}}$. We conclude that current data indicate no appreciable long-range (nonperturbative) spin-dependent force or any other correction for the Υ system. For the charmonium system, some additional effects, such as relativistic corrections, are required. Future (better) data can certainly check if this approach for the Υ system is correct.

Case 2: $V_2^N=0$ so that $V_1^N=-E^N$. This case is equivalent to the assumption that the spin-averaged confining potential comes from an attractive scalar exchange.[5,3] A phenomenological expression for $E(r)$ (which is consistent with $\Lambda_{\overline{\rm MS}}=0.30$ GeV at short distances) is used to add a long-range part to the interaction. We extract $\Lambda_{\overline{\rm MS}}$ from the remaining perturbative, nonphenomenological part of the expression. The results are shown in Fig. 2.

Case 3: $V_1^N=0$ so that $V_2^N=E^N$. For this case

$$V_L(m,r)=\frac{1}{m^2r}\frac{d}{dr}\left[\frac{3}{2}E+(-\bar{E}-V_1+V_2)\right]. \quad (8)$$

But the combination $(-\bar{E}+V_1+V_2)$ is already of order α_S^2. To extract $\Lambda_{\overline{\rm MS}}$ we need to calculate E, V_1, and V_2 to order α_S^3.

Case 4: $V_1^L=-V_2^L$ so that $V_1^N=-\frac{1}{2}E^N$, $V_2^N=\frac{1}{2}E^N$. The assumption, made in Ref. 2, that only the color electric field contributes to the long-range part of the static energy is phenomenologically equivalent to this case. The results for $\Lambda_{\overline{\rm MS}}$ are shown in Fig. 2 with the other cases. Since there is a significant difference between the Klopfenstein *et al.* (CUSB Collaboration)[7] data and the Irion *et al.* (Crystal Ball Collaboration)[9] data for the $1P$ splitting, Fig. 2 also presents separately the $\Lambda_{\overline{\rm MS}}$ extracted from each data set.

Figure 2 shows that the electric-confinement model is disfavored. Although we have not extracted $\Lambda_{\overline{\rm MS}}$ for the $V_1^N=0$ case, the trend indicates that this model gives values of $\Lambda_{\overline{\rm MS}}$ below those of the electric-confinement model, and is probably ruled out. More details as well as predictions can be found in Ref. 6. Better data will be able to distinguish the model of short-range spin forces from the model of scalar long-range force and other models.

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