Atomic Effects and Heavy-Neutrino Emission in Beta Decay

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Conventional treatments of atomic effects in beta decay may fail for electron momenta comparable in magnitude to the inverse of the Bohr radius. Exchange corrections are shown to produce a distortion of the tritium beta spectrum similar in shape to that for heavy-neutrino emission, though significantly smaller. The limitations of standard screening corrections are discussed.

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Recently Simpson[1] observed a distortion in the β-decay spectrum of tritium for electron kinetic energies $\epsilon < 1.5$ keV. An analysis indicated that the distortion is consistent with the emission of a heavy neutrino with a mass of 17.1 keV and a mixing probability of approximately 3%.[1,2]

A number of conventional approximations in treating final-state Coulomb effects should fail for small β energies, where the Compton wavelength of the electron is comparable to the atomic size. The relevant scale governing atomic excitations by the β ray is $\eta = -2/pa_0$, where p is the electron momentum and $a_0/2$ is the Bohr radius of the daughter He ion. Thus $\eta(1\ \text{keV}) \sim -0.24$. A particular class of the neglected atomic effects, those corresponding to exchange terms in the sudden approximation,[3] are shown to generate corrections of order η^4 to the standard Coulomb function, producing a distortion of the β spectrum qualitatively similar to that observed by Simpson. Similarly, the standard treatment of screening corrections becomes unreliable whenever η is not small. Thus it is possible that a complete treatment of atomic effects will provide a conventional explanation of the observed distortion.

Consider the allowed decay of atomic tritium in its ground ($1s$) state, producing a β ray of momentum p and kinetic energy ϵ. In the present treatment the nuclear finite size, relativistic corrections to the electron wave functions, and nuclear recoil will be neglected. The β-decay amplitude depends on the matrix element of the weak leptonic current evaluated at the nucleus,

$$\langle f | j_\mu^{\text{lep}}(0) | i \rangle, \tag{1}$$

where $|i\rangle$ is the initial tritium $1s$ wave function and $|f\rangle$ is the final wave function for the two electrons and the noninteracting neutrino.

The Hamiltonian for the final two-electron state is $H = H_0 + H_1$ where

$$H_0 = \sum_{i=1}^{2} \left[\frac{p_i^2}{2m} - \frac{2e^2}{r_i} \right], \qquad H_1 = \frac{e^2}{|\mathbf{r}_1 - \mathbf{r}_2|}. \tag{2}$$

If H_1 is neglected, the two-body wave function can be written as a product of the single-particle wave functions of H_0. The leptonic matrix element becomes

$$l_\mu(0,s_e;\mathbf{p}_\nu,s_\nu)\phi_e^*(0)\delta_{s_n,s_i}\int d^3r\ \phi_n^*(\mathbf{r})\chi_{1s}(\mathbf{r}) - l_\mu(0,s_n;\mathbf{p}_\nu,s_\nu)\phi_n^*(0)\delta_{s_e,s_i}\int d^3r\ \phi_e^*(\mathbf{r})\chi_{1s}(\mathbf{r}), \tag{3}$$

where l_μ is $\gamma_\mu(1-\gamma_5)$ evaluated between electron and neutrino spinors and ϕ_e, ϕ_n, and χ_{1s} are the wave functions for the more energetic final electron, the second (bound or continuum) electron, and the initial tritium $1s$ electron, respectively. The corresponding spins are denoted by s_e, s_n, and s_i. The electron has been treated nonrelativistically. The principal contribution to the second (exchange) term in Eq. (3) comes from β-ray emission into bound levels in He^+ while the spectator $1s$ electron is knocked into the continuum by the sudden change in the nuclear charge.

Using Eq. (3) in a standard calculation of the differential β-decay rate yields[4]

$$\frac{d\omega}{dp} = \frac{G^2}{2\pi^3}\cos^2\theta_c p^2 \frac{2\pi\eta}{e^{2\pi\eta}-1}|M|^2 \times \left\{ \sum_{n=1}^{\infty} 2\left[W_0 - \epsilon_b\left(1-\frac{1}{n^2}\right) - \epsilon\right]^2 \left[256 n^5 \frac{(n-2)^{2n-4}}{(n+2)^{2n+4}} + \frac{\alpha^2(n)}{n^3} - 16n\frac{(n-2)^{n-2}}{(n+2)^{n+2}}\alpha(\eta)\right] + \frac{1}{\pi}\int_{-\infty}^{\eta}\frac{d\eta'}{\eta'^4}\left[W_0 - \epsilon_b\left(1+\frac{1}{\eta'^2}\right) - \epsilon\right]^2 \frac{2\pi\eta'}{e^{2\pi\eta'}-1}\left[\alpha^2(\eta') + \alpha^2(\eta) - \alpha(\eta')\alpha(\eta)\right] \right\}, \tag{4}$$

where

$$\alpha(\eta)=\eta^4\exp[2\eta\arctan(-2/\eta)]/(1+\eta^2/4)^2,$$

$G\cos\theta_c$ is the weak coupling constant, M is the usual nuclear matrix element in the allowed approximation, $\epsilon_b=54$ eV is the $1s$ binding energy in $^3He^+$, and $W_0\approx 18.6$ keV is the maximum electron kinetic energy. A massless neutrino has been assumed. The first and second terms are the contributions from bound and continuum states, respectively. For small η the exchange terms generate order-η^4 corrections to the direct terms. The dominant contribution to Eq. (4) produces $^3He^+$ in its ground state ($n=1$); for this state the direct and exchange terms interfere constructively. The relative contributions of bound and continuum states for $\epsilon=1$ keV are given in Table I.

In the standard treatment of β decay the exchange terms are neglected and the sum over bound and continuum atomic states is completed by closure. (Thus W_0 becomes an effective maximum energy representing an average over atomic states.) This yields

$$\frac{d\omega}{dp}=\frac{G^2}{2\pi^3}\cos^2\theta_c p^2\frac{2\pi\eta}{e^{2\pi\eta}-1}|M|^2(W_0^{\text{eff}}-\epsilon)^2. \tag{5}$$

Apart from a screening correction discussed below, this is the formula used in Ref. 1. From Eq. (4) one finds $W_0^{\text{eff}}=18.588$ keV. However, the values deduced by Simpson from fits to the β spectrum range from 18.7 to 19.3 keV. As he notes, one possible source of this discrepancy could be an inadequate screening function. This point is discussed more fully below.

The Kurie function is defined as

$$K(\epsilon)=\left[\frac{d\omega}{dp}\left(\frac{G^2}{2\pi^3}\cos^2\theta_c p^2\frac{2\pi\eta}{e^{2\pi\eta}-1}|M|^2\right)^{-1}\right]^{1/2}. \tag{6}$$

The quantity of interest is

$$[K^{\text{d+e}}(\epsilon)-K^{\text{d}}(\epsilon)]/K^{\text{d}}(\epsilon)=\Delta K/K,$$

where $K^{\text{d+e}}(\epsilon)$ and $K^{\text{d}}(\epsilon)$ are the Kurie functions for Eq. (4) with and without exchange terms, respectively. The results are compared to Simpson's data in Fig. 1. The theoretical spectrum was convoluted with a Gaussian resolution function whose width is 220 eV,[5] the value given in Ref. 1, though this proved to have a negligible effect on $\Delta K/K$.

The energy dependence of the calculated and measured values of $\Delta K/K$ are similar; the theoretical curves fall about a factor of 6 below experiment. Although these results do not rule out a massive neutrino whose parameters are similar to those discussed by Simpson, it does suggest that the observed distortion could have a more conventional origin. A strong conclusion requires a quantitative investigation of several additional approximations. The nuclear finite size, nuclear recoil, relativistic corrections to the electron wave functions, and forbidden contributions to the β-decay amplitude have been ignored.[6] However, none of these is associated with any scale that can generate the abrupt end-point behavior noted by Simpson.

TABLE I. Contributions of the direct and direct plus exchange terms to $d\omega/dp$ as percentages of the closure result for $\epsilon=1$ keV.

State	Direct	Direct + exchange
$1s$	70.23	70.42
$2s$	25.00	24.96
$3s$	1.27	1.27
$4s\to\infty$	0.86	0.86
Continuum	2.62	2.58
Total	100.00	100.09

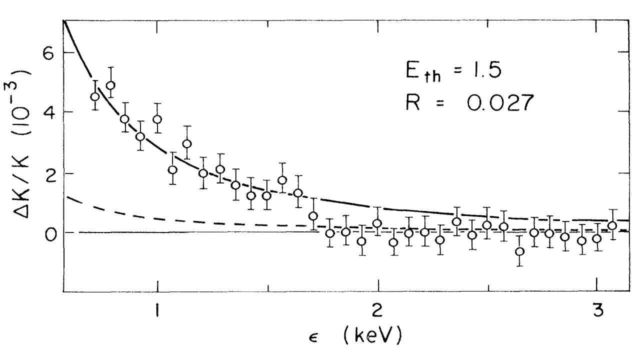

FIG. 1. The data of Ref. 1 (for the run that included active pileup rejection) are compared to the theoretical deviation $\Delta K/K$ (dashed line) attributable to the neglect of exchange corrections in the standard treatment of Coulomb distortions. The solid line is the theoretical result multiplied by six.

In contrast, atomic excitations do play an important role in the final-state electron-electron interaction.[7,8]

In Ref. 1 the electron-electron interaction was taken into account by use of Rose's estimate of the screening correction.[9] Because WKB wave functions are inaccurate near the origin, Rose developed a procedure for matching the asymptotic WKB wave function to the interior Coulomb solution appropriate for a constant interaction potential. This determines an energy shift for the interior solution. The procedure, which accounts only for mean-field effects, is valid provided that the matching radius $r_0 \approx p^{-1} << a_0$. That is, one must require $\eta << 1$. (Durand demonstrated this explicitly for s-wave states of a Hulthén screened Coulomb field.)[10] Thus the screening correction used in Ref. 1 is probably not valid near the low-energy end point of the β spectrum.

A more reliable approach is that of Williams and Koonin,[8] who iterated the Lippmann-Schwinger equation to first order (in H_1) to obtain the rescattering corrections for the high-energy tail of the tritium β spectrum. This procedure also takes into account inelastic final-state interactions. An extension of the work of Ref. 8 to order η^4, when compared to the Rose prescription, would determine any corrections that will compete with the exchange terms evaluated here.

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[1]J. J. Simpson, Phys. Rev. Lett. **54**, 1891 (1985).

[2]A factor of $\frac{1}{2}$ is missing from the formula in Ref. 1 that relates the deviation of the Kurie plot ($\Delta K/K$) to the mixing probability R. This is a misprint and does not affect the numerical results of Ref. 1 (J. Simpson, private communication).

[3]For earlier discussions of exchange or overlap corrections, see J. N. Bahcall, Phys. Rev. **129**, 2683 (1963); J. Law, Phys. Rev. D **7**, 3314 (1973); K. E. Bergkvist, Phys. Scr. **4**, 23 (1971); M. Fukugita and K. Kubodera, Z. Phys. C **9**, 365 (1981).

[4]The factor $(n-2)^{n-2}$ is defined as unity for $n=2$.

[5]This is the full width at half maximum. The Gaussian is convoluted with $d\omega/d\epsilon$ to determine an effective differential rate.

[6]Other corrections can depend on the nature of the detector. Simpson's detector is essentially a calorimeter, so that the decay energy of excited $^3He^+$ states or of secondary ejected electrons will be recorded. The simple relationship between the decay rate and Kurie function is then lost. Instead, the natural experimental quantity is $d\omega/d\epsilon_\nu$, where ϵ_ν is the neutrino energy. In the present work and in Ref. 1 the associated corrections to the standard Kurie function treatment are not discussed.

[7]E. L. Feinberg, J. Phys. (Moscow) **4**, 423 (1941); I. S. Batkin, I. V. Kopytin, Yu. G. Smirnov, and T. A. Churakova, Yad. Fiz. **33**, 48 (1981) [Sov. J. Nucl. Phys. **33**, 25 (1981)]; R. L. Intemann, Phys. Rev. A **26**, 3012 (1982).

[8]R. D. Williams and S. E. Koonin, Phys. Rev. C **27**, 1815 (1983).

[9]M. E. Rose, Phys. Rev. **49**, 727 (1936).

[10]L. Durand, Phys. Rev. **135**, B310 (1964); L. S. Brown, Phys. Rev. **135**, B314 (1964).