Growth and Quasistabilization of Large-Scale Spikes on Laser Beams in Self-Focusing Media

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We present numerical calculations concerning the growth of a large-scale Gaussian spike riding axially over a Gaussian-profile intense laser beam. Incorporation of both self-focusing and depletion of the laser beam leads to a criticality with regard to the spike's initial relative intensity due to the counteracting behavior of these two effects, thereby determining the net power transfer to the spike and its quasistabilization.

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In experimental situations where intense laser beams traveling through nonlinear self-focusing media ($\epsilon=\epsilon_0+\epsilon_2|E|^2$) result in multiple-filament formation, there is a one-to-one correspondence between filaments and intensity spikes riding with the incident laser beam.[1] Linearized instability theories[2,3] predict that the transverse sizes of the spikes determine their exponential growth coefficients and the optimal-size spikes grow fastest, leading to filament formation first. By incorporation of energy conservation in the theory,[4,5] the stabilization of fastest-growing spikes was adequately explained and correct estimates for filament-formation thresholds were obtained. The accurate theoretical estimate for filament diameters was made by considering the nonlinearized instability growth[6] and by phenomenologically including avalanche ionization[7] as a saturation mechanism in the theory.

In all of these theoretical treatments,[4–7] since the results had to be compared quantitatively with the experimentally observed distances for the first appearance of filaments and the observed filament diameters near filament-formation thresholds, the theories only considered optimal-sized spikes. In view of this, hardly any theoretical treatment has considered the development of spikes significantly larger than optimal size even though large-scale filaments were reported in some earlier experimental investigations.[8] In this paper we consider growth of large-scale spikes and find that the relative intensity in spikes becomes a critical parameter of their development due to the counteracting behavior of self-focusing and depletion effects of the main beam. Thus, the initial relative intensity in the spike determines, in a critical way, the final energy, size, and quasistabilization. Here, the word "quasistabilization" is used to mean the stabilization in an approximate way of spike size and/or its field.

In the two-Gaussian model[5] used here, we consider a complex perturbation (e_1+ie_2) riding on a Gaussian background field A_0. The total electric field amplitude of the laser beam is taken as

$$A=(A_0+e_1+ie_2)e^{-iks}, \tag{1}$$

where s is the eikonal of the wave and the fields in the main beam and perturbation are

$$A_0=(E_0/f)\exp(-r^2/2r_0^2f^2), \tag{2}$$

and

$$e_{1,2}=e_{10,20}e^{\alpha(z)}\exp[-r^2/2b^2(z)], \tag{3}$$

with

$$f^2(z)=1-z^2/z_{\mathrm{sf}}^2. \tag{4}$$

Here, z_{sf} is the self-focal distance for the smooth Gaussian main beam. The total field amplitude of Eq. (1) satisfies the quasioptic equation

$$2ik\,\partial A/\partial z=\nabla_\perp^2 A+(\epsilon_2/\epsilon_0)k^2|A|^2A. \tag{5}$$

For the two-Gaussian model,[5] Eqs. (1) to (5) lead to two coupled-differential equations for the rate of change of the growth parameter $\alpha(z)$ and of the size of the spike $b(z)$ within the framework of linearized instability theory. These equations are

$$\frac{d\alpha}{dz}=\frac{1}{k}\left[\left(\frac{1}{b^2}-\frac{1}{r_0^2f^2}\right)\left\{\frac{\epsilon_2}{\epsilon_0}k^2\frac{E_0^2}{f^2}-\left(\frac{1}{b^2}-\frac{1}{r_0^2f^2}\right)\right\}\right]^{1/2}-\beta(z), \tag{6}$$

$$\frac{db}{dz}=\frac{b^3}{2k}\left[\left\{\left(\frac{1}{b^4}-\frac{1}{r_0^4f^4}\right)\left[\frac{2\epsilon_2}{\epsilon_0}\frac{k^2}{r_0^2}\frac{E_0^2}{f^2}-\left(\frac{1}{b^4}-\frac{1}{r_0^2f^4}\right)\right]\right\}^{1/2}+\frac{2k\beta}{b^2}\right], \tag{7}$$

where $\beta=f^{-1}\,df/dz$ is the inverse of the radius of curvature of the laser wave front. As the spike grows in the nonlinear medium, it draws energy from the background laser field, thus depleting the energy of the background. Here, the depletion of the background energy will be introduced phenomenologically in the theory as an afterthought, as was done in the previous cases.[4–6] This amounts to the replacement of E_0^2 by $E_0^2(1-\delta^2e^{2\alpha}b^2/r_0^2)$ in Eqs. (6) and (7), as was shown in Ref. 5. Here, δ^2 is the relative initial intensity in the spike, and is given by

$$\delta^2=(e_{10}^2+e_{20}^2)/|E_0|^2. \tag{8}$$

The resulting equations are difficult to solve analytically. Only under specific conditions, viz. $f=1$, have analytical solutions been discussed earlier for optimal-sized spikes.[5] In this paper we present numerical solutions for $f\neq1$ (i.e., self-focusing effect of the laser beam included in the analysis) and for large-scale spikes.

For the computer solutions of Eqs. (6) and (7) with self-focusing and depletion of the main laser beam considered simultaneously, we have chosen the nonlinear medium with $\epsilon_0=2.418$ and $\epsilon_2=4.373\times10^{-11}$ esu. We take the main laser beam electric field to be $E_0=2000$ esu, its diameter 2 mm, and the diameter of the spike $2b_0=400$ μm initially at $z=0$. The numerical solutions are presented in Figs. 1 to 4. In Fig. 1(a) it is seen that initially $\alpha(z)$ varies linearly with z for all δ, which is typical of linearized instability theories. The initial exponential growth is then followed by a faster-than-exponential growth[5] (due to the effect of self-focusing of the main beam), and finally by a slowing down of growth due to the effective depletion of the main beam. The initial exponential growth suggests that the instability development is not significantly affected by self-focusing or depletion. The faster-than-exponential growth suggests that the instability growth is enhanced by self-focusing and depletion is not yet significant. The slowing down of instability growth at the final stages of the development of the spike suggests the reduction of the self-focusing effect of the main beam by the counteracting strong depletion of its energy.

The typical variation of the spike size $b(z)$ is shown in the inset of Fig. 1(b) for $\delta=10^{-3}$. The spike size increases in the initial region, symptomatic of diffraction for a weak intensity spike. As the spike grows in intensity, it experiences self-focusing of its own size due to the self-focusing of the main beam and subsequently shows quasistabilization due to the counteract-

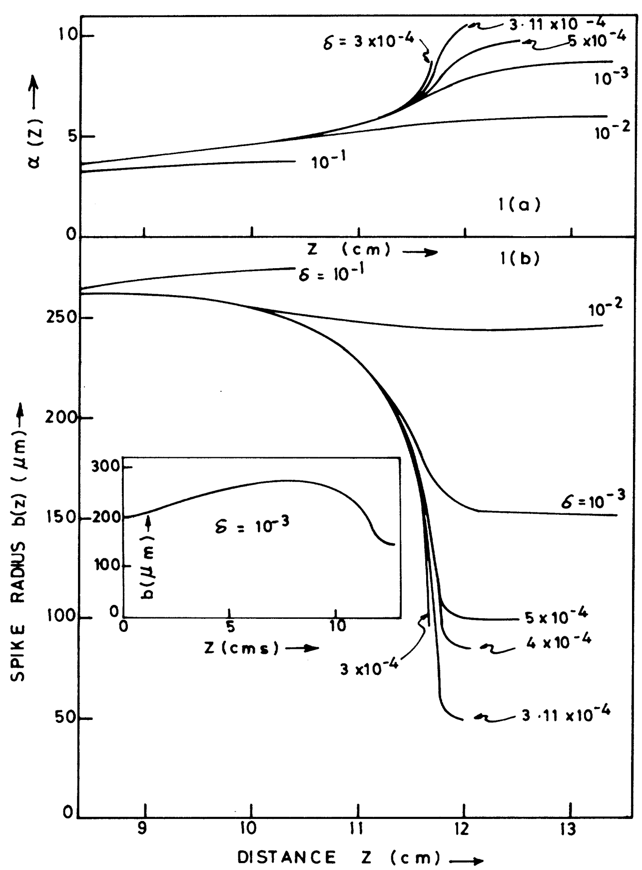

FIG. 1. Variation of (a) the growth parameter $\alpha(z)$, and (b) the transverse size of the spike $b(z)$, for various values of δ with distance on an expanded scale. The inset shows typical overall variation of $b(z)$ for $\delta=10^{-3}$. The main Gaussian beam radius is 1000 μm and the peak electric field is $E_p=2000$ esu with $\epsilon_0=2.418$, $\epsilon_2=4.373\times10^{-11}$ esu, and $k=1\times10^5$ cm^{-1}.

ing effect of main-beam depletion against its self-focusing. The variation of spike size $b(z)$ is shown in Fig. 1(b) for various values of δ on an expanded scale. For $\delta = 10^{-1}$, the depletion of the main beam is strong enough to prevent the self-focusing of the spike totally, although the main beam does experience a shrinkage of its size initially to some extent (not shown in the figure). For δ values in the range of $\delta \cong 10^{-2}$ to δ_c $(= 3.098 \times 10^{-4})$, the spike size experiences the shrinkage before showing the quasistabilization in all cases. For $\delta < \delta_c$, the depletion of the main beam is not strong enough to prevent the self-focusing of both the spike and the main beam, and the quasistabilization of the spike is never achieved.

Figure 2 shows the computer-generated profiles of the main beam and the spike for two values of δ, one greater than δ_c, viz. $\delta = 4 \times 10^{-4}$, and the other smaller than δ_c, viz. $\delta = 3 \times 10^{-4}$. For $\delta > \delta_c$, the depletion of the main beam by the growing spike is able to prevent the catastrophic self-focusing of the main beam, resulting in shrinkage and quasistabilization of the spike's size (except for the case $\delta = 10^{-1}$ where shrinkage is not obtained). For $\delta < \delta_c$, the initial spike intensity is too small to draw sufficient energy from the main beam and deplete it effectively. The catastrophic self-focusing of the main beam and the spike is not prevented. It is not possible, in these cases, to carry out computer calculations beyond the self-focal distance z_{sf} defined for the main beam in Eq. (4) where the two sizes become equal.

In Fig. 3, we show the variation of the main beam and the spike electric fields for two values of δ, both greater than δ_c. For the value of δ nearer to δ_c (i.e., for $\delta = 4 \times 10^{-4}$), the field in the main beam first increases as a result of self-focusing and subsequently decreases rapidly as a result of depletion, indicating the prevention of catastrophic self-focusing. The corresponding field in the spike shows a steady growth followed by a rapid rise to a large value due to transfer of energy from the background. Thus, the values of δ from δ_c to $\delta = 4 \times 10^{-4}$ are suitable for obtaining very large values of fields in the spikes (about 60% to 75% of the field values in small-scale filaments which are close to the breakdown fields for the materials[6,7]) accompanied by a quasistabilization of their sizes. However, values of δ about an order of magnitude larger than the above values (viz. $\delta = 5 \times 10^{-3}$) are suitable for obtaining moderate field values in the spikes with quasistabilization of both fields and sizes.

In Fig. 4, we show the variation of the power or energy in the quasistabilized region of the spike and the corresponding power or energy in the main beam for various values of δ. It can be clearly seen that for any significant transfer of power into the spike from the main beam δ should be larger than a critical value δ_c, already mentioned earlier in the text. For $\delta < \delta_c$, there is no significant transfer of power into the spike. This shows the criticality of the parameter δ. For δ sufficiently large, there is almost total transfer of power from the main beam to the spike, but the field in the spike is not very large since the spike does not experience significant shrinkage in size. For $\delta > \delta_c$ but close to δ_c, there is less transfer of power to the spikes but the field in the spike obtains a large value

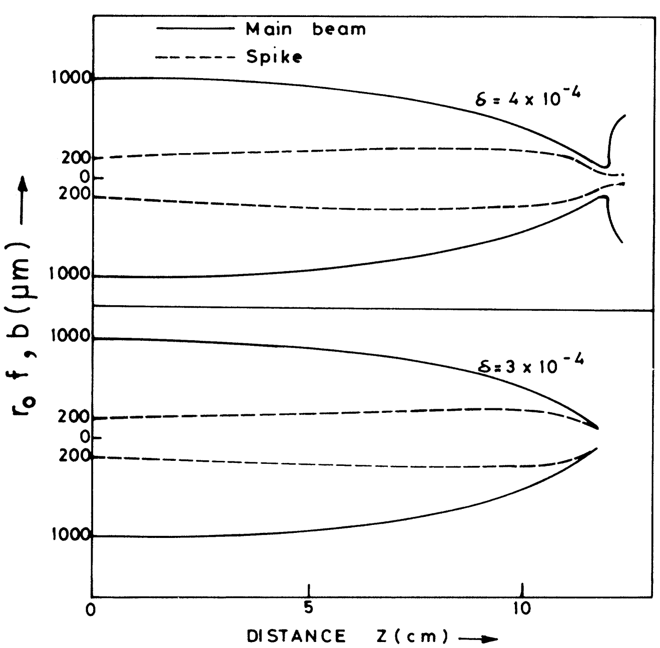

FIG. 2. Computer-generated profiles of the main beam and the spike for $\delta = 4 \times 10^{-4}$ and $\delta = 3 \times 10^{-4}$. Numerical values of the different parameters are the same as in Fig. 1.

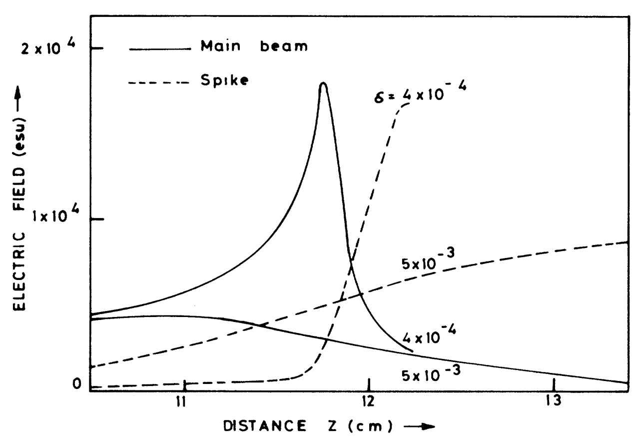

FIG. 3. Electric fields in the main beam and the spike vs distance for two values of δ both greater than δ_c: $\delta = 4 \times 10^{-4}$ and $\delta = 5 \times 10^{-3}$. Other parameters are the same as in Fig. 1.

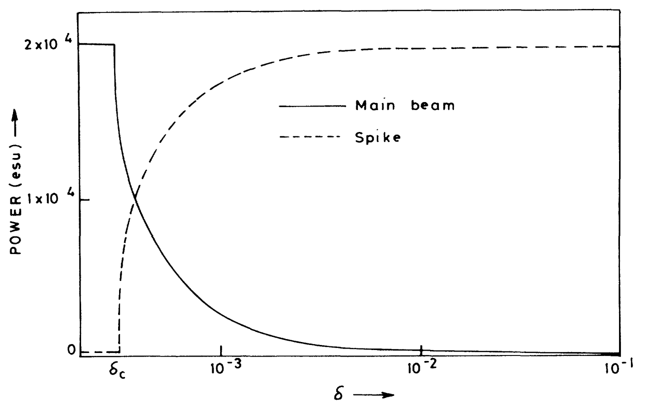

FIG. 4. Powers in the spike and the main beam when quasistabilization is reached as a function of δ. The critical value of δ is marked on the graph. Other parameters are the same as in Fig. 1.

since the spike experiences significant shrinkage in its size.

We have also done numerical calculations for other values of b_0 and E_0. Different values of E_0 simply scale the distances while giving a similar qualitative picture. Different values of b_0 also give a similar qualitative picture. However, Eq. (7) suggests that calculations for b_0 less than a certain $b_{\min}$ cannot be performed, where $b_{\min}$ is given by

$$b_{\min} = \left(\frac{2\epsilon_2}{\epsilon_0} k^2 \frac{E_0^2}{r_0^2} + \frac{1}{r_0^4} \right)^{-1/4}. \tag{9}$$

The optimal-sized spikes have a very fast growth and they stabilize to form small-scale filaments at distances much smaller than $z_{\rm sf}$ for the smooth Gaussian main beam. Hence, self-focusing does not play a significant role in their development. The growth of large-scale spikes is relatively slow and their development is not complete even at $z = z_{\rm sf}$. Thus, self-focusing and depletion of the main beam should be simultaneously taken into account while studying the growth of the large-scale spikes. This, in turn, leads to the criticality in δ due to the counteracting behavior of self-focusing and depletion effects of the main beam. Similar criticality in δ does not appear in the theoretical treatments for optimal-sized spikes since self-focusing of the main beam does not play a significant role. Also, the calculations for $b_0 = 150$ μm suggest that $\delta_c = 1.7954 \times 10^{-5}$. Thus, reduction of b_0 causes a reduction in δ_c which explains the decreasing importance of self-focusing for small-scale filaments.

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