Flicker Noise in Frequency Fluctuations of Lasers

M. R. Sayeh and H. R. Bilger
School of Electrical and Computer Engineering, Oklahoma State University, Stillwater, Oklahoma 74078
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Measured frequency fluctuation spectra $S_{\delta f}$ in ring lasers show that $S_{\delta f}=h_{-1}/\nu+h_{-2}/\nu^2$ at low Fourier frequencies ν, while white noise prevails at higher frequencies. Experimentally, $S_{\delta f}=(\bar{A}f_0^2/Q^4)(1/\nu)$ has been found for the dependence of the $1/\nu$ noise (flicker noise) on the quality factor Q, with $\bar{A}\simeq 4$ ($f_0=$ laser frequency). The Q^{-4} dependence is readily explained by loss (or gain) fluctuations, through use of a Van der Pol oscillator.

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Noise in lasers has been a subject of research for the last two decades.[1-4] The simple equation is

$$S_{\delta f}(\nu)=hf_0^3/Q^2P \tag{1}$$

[h is Planck's constant, $f_0=\omega_0/2\pi$ is the (average) laser frequency, Q is the quality factor of the passive cavity, and P is the power loss per mode] for the one-sided power spectral density (PSD) $S_{\delta f}$ per mode versus the Fourier frequency ν. Here the frequency fluctuation $\delta f(t)$ due to quantum noise gives an approximate white-noise level which has been verified by experiment.[2,4] Equation (1) has been generalized.[3]

Let us consider a noise-driven Van der Pol oscillator as a model for a laser oscillator[4]:

$$\ddot{x}+[r-(g-\gamma x^2)]\dot{x}+\omega_0^2x=N(t), \tag{2}$$

where x is the mode amplitude, r is the energy decay rate, g is the unstaurated gain, γ is the saturation parameter, and $N(t)$ is the noise source due to the spontaneous-emission processes. From this model one can find the PSD of the frequency fluctuations as[5]

$$S_{\delta f}(\nu)=\frac{\gamma rhf_0}{8\pi^2(g-r)}\left[\frac{N_2}{N_2-N_1(g_2/g_1)}\frac{g(\nu+f_0)}{g(f_0)}+n_{\text{th}}\right], \tag{3}$$

where N_1 and N_2 are the populations of the lower and upper laser levels, respectively, g_1 and g_2 are the level degeneracies, $g(f)$ is the transition line shape, and n_{th} is the number of thermally emitted photons. Under limiting conditions ($N_2>>N_1$, $\nu<<f_0/Q$, $hf_0/kT>>1$), Eq. (3) converges to Eq. (1). The experimental work in Ref. 2 demonstrates that the Van der Pol model predicts the white noise correctly (within a factor 1.5) at the power levels involved in these experiments.

To realize that noises with frequency-dependent PSD, e.g., $S_{\delta f}(\nu)\propto 1/\nu$ (flicker noise), exist, we consider a noise mechanism from a different source. Let us assume that there exist fluctuations in the loss, r, independent of the existence of white noise, i.e., for the purpose of this derivation $N(t)$ in Eq. (2) is set equal to zero. Now we try to find the PSD due to loss fluctuations.

If $(g-r)$ is slowly varying, and furthermore, if $(g-r)/\omega_0<<1$, Poincaré's method[6] gives approximately

$$x(t)\simeq 2(g-r/\gamma)^{1/2}\cos(\xi\omega_0t), \tag{4}$$

where $\xi=1-[(g-r)/4\omega_0]^2$. The resonance frequency $\omega=\xi\omega_0$ therefore depends on the loss r, viz.,

$$\omega=\omega_0-(g-r)^2/16\omega_0. \tag{5}$$

By making use of $g\simeq r$ in a laser in steady-state operation, we find the fluctuation δf with respect to the fluctuation δr as

$$\delta f/f=(1/8Q^2)\delta r/r, \tag{6}$$

where $Q=\omega_0/r$.

The PSD of the fractional frequency fluctuation can be related to the PSD of the fractional loss fluctuation by using Eq. (6) as follows:

$$S_{\delta f/f}(\nu)=(1/64Q^4)S_{\delta r/r}(\nu), \tag{7}$$

i.e., the PSD is inversely proportional to Q^4. This has been observed experimentally in quartz oscillators over six decades of PSD.[7] The proportionality to Q^{-4} thus is independent of the specific assumptions on the type of loss fluctuations. As far as the $1/\nu$ spectrum is concerned, Handel has shown that the correct quantum-mechanic treatment of scattering cross sections will produce such a spectrum. A variety of papers in various disciplines of physics[8] provide quantitative support.

We consider it here as a plausible hypothesis. Our main objective in this Letter is, however, to link experimentally the $1/\nu$ spectrum of noise to a Q^{-4} dependence. We thus assume that

$$S_{\delta r/r}(\nu)=A/\nu, \tag{8}$$

where A is a constant depending on the nature of the interaction. Therefore, the PSD per mode can then be written as

$$S_{\delta f}(\nu)=(Af_0^2/64Q^4)(1/\nu). \tag{9}$$

A ring-laser gyro (RLG) can be used to study the frequency fluctuations in lasers. The technique[9] consists of the creation of two (or more) countercirculating nondegenerate modes which are superimposed at the ring output. The interfering beams create an intensity pattern which is time dependent, with a rate equal to the frequency difference between the modes. The frequency difference is relatively insensitive to changes of geometry which makes a ring laser a very useful instrument to measure frequency noise. Figure 1 shows a measured PSD of a four-mode RLG (the RLG was placed in a thermostat with 100-μK temperature deviation, precluding arguments based on noise induced by temperature fluctuation). The white noise dominates down to 6×10^{-4} Hz, flicker noise ($1/\nu$ noise) is from 6×10^{-4} to 4×10^{-5} Hz, and $1/\nu^2$ noise predominates at $\nu<4\times10^{-5}$ Hz. Generally it has been observed that the PSD of ring lasers obeys the following relation[4]:

$$S_{\delta f}(\nu)=h_{-2}\nu^{-2}+h_{-1}\nu^{-1}+h_0. \tag{10}$$

We have collected data from different types of RLG, including Fig. 1, and we verified that $1/\nu$ noise commonly occurs. The white-noise level h_0 in Eq. (10) was ascertanied.[4]

Figure 2 shows the measured values of h_{-1}, per mode, versus the quality factor Q, for the four-mode RLG's 1 and 2, and for the two-mode RLG 3. The least-squares fitted line in Fig. 2 gives a value of 256 for the constant A in Eq. (9). The experimental evidence therefore suggests for the $1/\nu$ noise

$$S_{\delta f}(\nu)\simeq 4(f_0^2/Q^4)(1/\nu). \tag{11}$$

The frequency ranges where $1/\nu$ noise dominates were vastly different from gyro to gyro. We are therefore led to believe that the existence of $1/\nu$ noise is not "accidental."[8]

The $1/f$ noise in laser systems is thus to be considered as a fundamental process which affects the output frequency of a laser. $1/f$ noise in the model is linked to laser loss mechanisms which are inflicted upon the photon field. In this sense, it is an even more basic noise process than white noise. Of course, if in the absence of loss the quality factor goes to infinity, there are no fluctuation effects and correspondingly there would not be any $1/f$ noise (there would also be no white noise but for different reasons). We note also that the quality factor enters the formula for white noise, Eq. (1), only indirectly because spontaneous

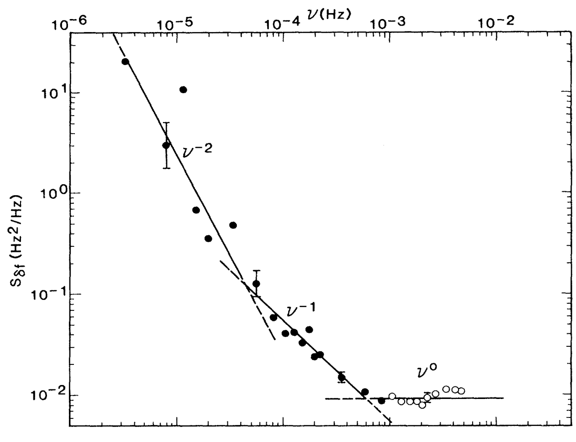

FIG. 1. Typical power spectral density of the frequency fluctuation $\delta f(t)$ vs Fourier frequency ν. There are three distinct regimes where $S\propto\nu^0$ (white noise), $S\propto\nu^{-1}$ (flicker noise), $S\propto\nu^{-2}$. The error bars indicate the statistical accuracy of the data. This noise spectrum was obtained on RLG 2 from a 5-day run with frequency measurements taken every 100 s.

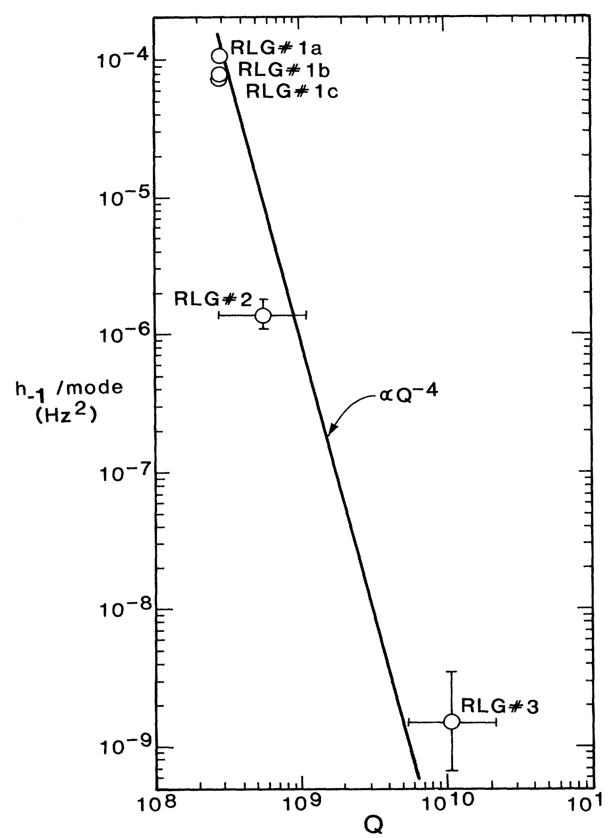

FIG. 2. Summary of $1/\nu$ amplitudes vs cavity quality factor Q obtained in three ring lasers, with the best-fitted line $\propto Q^{-4}$. All ring lasers use HeNe and operate at $f_0=474$ THz ($\lambda=633$ nm). The error bars of RLG 1 are of the same size as the circles.

emission is at the origin of white noise.

The $1/\nu$ noise is of great practical importance, since averaging of data containing $1/\nu$ noise does not significantly reduce the noise level, whereas averaging of data containing white noise over a time T reduces the noise level by a factor of $1/\sqrt{T}$. In the presence of $1/\nu$ and $1/\nu^2$ noise, the $1/\nu$ spectra density sets the lower limit of noise as the Allan variance of any oscillator[10] demonstrates.

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