# Relativistic Impulse Approximation for Meson-Nucleus Scattering in the Kemmer-Duffin-Petiau Formalism

B. C. Clark, S. Hama,[a] and G. R. Kälbermann
*Department of Physics, The Ohio State University, Columbus, Ohio 43210*
and
R. L. Mercer
*IBM Watson Research Laboratories, Yorktown Heights, New York 10598*
and
L. Ray
*Department of Physics, University of Texas, Austin, Texas 78712*
(Received 25 March 1985)

The first application of the Kemmer-Duffin-Petiau wave equation for spin-0 particles to meson-nucleus scattering is given. A relativistic impulse approximation using the Kemmer-Duffin-Petiau formalism is used to calculate meson-nucleus optical potentials. Comparison with the nonrelativistic impulse approximation is given.

PACS numbers: 24.10.Ht, 25.80.−e

The success of the relativistic impulse approximation (RIA) in describing proton-nucleus $(pA)$[1–4] scattering at intermediate energies suggests its use in treating nuclear reactions employing other probes. In this Letter we discuss a treatment of meson-nucleus elastic scattering[5] using the usual RIA as a guide in developing the parameter-free optical potentials. Standard optical-model treatments of meson-nucleus scattering have generally used the Klein-Gordon (KG) or Schrödinger equations as the relevant one-body wave equation. In this Letter we investigate an alternative approach using the first-order Kemmer-Duffin-Petiau (KDP)[6–8] wave equation which is similar in form to the Dirac equation. This approach is motivated by three general considerations. First, the equation is linear in energy which facilitates the development of impulse-approximation optical potentials in a manner analogous to the nucleon-nucleus RIA. Second, the richness of the KDP formalism regarding the introduction of interactions is intriguing.[9] For example, if the interaction has a conserved vector current then the KDP formalism gives identical results to the KG equation for spin-0 projectiles. If the interactions do not have a conserved current or if scalar interactions are considered, this is not necessarily the case.[9] Third, the KDP equation is appropriate for both spin-0 and spin-1 projectiles.

The free-particle KDP equation[6] is $(\hbar = c = 1)$

$$(i\beta^\mu \partial_\mu - m)\phi = 0, \tag{1}$$

where $\mu = 0,1,2,3$; $m$ is the mass parameter and the $\beta^\mu$ obey[10,11]

$$\beta^\mu\beta^\nu\beta^\lambda + \beta^\lambda\beta^\nu\beta^\mu = g^{\mu\nu}\beta^\lambda + g^{\lambda\nu}\beta^\mu. \tag{2}$$

The algebra generated by the four $\beta^\mu$'s has three irreducible representations of dimension one, five, and ten. The five-dimensional representation yields a spin operator whose eigenvalues are zero, the ten-dimensional case corresponds to spin one, and the one-dimensional case is trivial. The first component of the five-dimensional Kemmer wave function for the spin-0 case satisfies the Klein-Gordon equation for massive particles.

In order to apply the KDP formalism to meson-nucleus scattering we must introduce interactions in Eq. (1). If one writes

$$(i\beta^\mu \partial_\mu - m - U)\phi = 0, \tag{3}$$

the most general form for $U$ contains two scalar, two vector, and two tensor terms.[10] We omit the tensors to avoid noncausal effects.[10] For the spin-0 case the scalar operators are the unit operator $I$ and the $5\times 5$ operator $P$ whose elements are all zeros except the (1,1) element; thus $P$ acts as a projection operator onto the first component of $\phi$. The vector operators are $\beta^\mu$ and $\tilde{\beta}^\mu = P\beta^\mu - \beta^\mu P$. The form for $U$ is

$$U = U_s I + U_s^1 P + \beta^\mu U_v + \beta^\mu P U_v^1. \tag{4}$$

The last two terms may also be written as $\beta^\mu U_v + P\beta^\mu U_v^1$.

In order to construct impulse-approximation optical potentials consistent with Eq. (4) we need an invariant form for the meson-nucleon $t$ matrix. The choice for the invariant form used here is

$$t = I_N I t_s + I_N P t_s^1 + \gamma_\mu \beta^\mu t_v + \gamma_\mu \beta^\mu P t_v^1, \tag{5}$$

where $I_N$ and $\gamma_\mu$ are the unit and Dirac $\gamma$ matrices for the nucleon. As in the nucleon-nucleus RIA we equate the matrix elements of the empirical c.m. two-body scattering amplitude,

$$F(q) = f(q) + \boldsymbol{\sigma}\cdot\mathbf{n}\,g(q), \tag{6}$$

taken between Pauli spinors for the nucleon with the matrix elements of the invariant $t$ matrix between Dirac and Kemmer free-particle spinors. The scattering amplitude and the invariant $t$ matrix are related by a $2\times 4$ matrix, and thus certain choices need to be made in order to proceed. In fact, the choices could form the basis of a phenomenology using the KDP equation. In this work we limit the $t$ matrix to only two of the four possible terms in Eq. (5). Thus, we consider models with two scalars, two vectors, or a vector-scalar mixture. For each choice a transformation matrix $K$ relates $t$ to $F$. For example, for a scalar-vector mixture

$$\begin{pmatrix} t_s \\ t_v \end{pmatrix} = -\frac{2\pi s^{1/2} K^{-1}}{Mm}\begin{pmatrix} f \\ g \end{pmatrix}, \tag{7}$$

where $s^{1/2}$ is the total meson-nucleon energy and $m$ ($M$) is the meson (nucleon) mass.

The combination of two scalars or two vectors produces a matrix $K$ with zero determinant; thus, we consider $t$ to consist of a scalar and a vector amplitude. There are, however, several choices for the form of $t$ depending on whether the operator $P$ is in both terms (case 1), in the scalar only (case 2), in the vector only (case 3), or in neither (case 4). The forms $\gamma_\mu P \beta^\mu t_v$ and $\gamma_\mu \beta^\mu P t_v$ produce identical results. The elements of $K$ for case 1 are given by

$$K_{11}^1 = \frac{1}{4M(E+M)}[(E+M)^2 - k^2 + \tfrac{1}{4}q^2], \quad K_{12}^1 = \frac{1}{4M(E+M)}\left(\frac{E_m}{m}\right)[(E+M)^2 + k^2 - \tfrac{1}{4}q^1] + \frac{k^2}{2mM},$$

$$K_{21}^1 = \frac{ikq}{4M(E+M)}, \quad K_{22}^1 = \frac{-ikq}{4M(E+M)}\left(\frac{E_m}{m}\right) - \frac{ikq}{4mM},$$

where $E$ ($E_m$) is the c.m. energy of the nucleon (meson), $q = 2K \sin\theta/2$, and $k = K \cos\theta/2$, where $\theta$ is the c.m. scattering angle and $K$ is the c.m. momentum. For case 2, $K_{11}^2 = K_{11}^1$, $K_{12}^2 - 2K_{12}^1$, $K_{21}^2 = K_{21}^1$, and $K_{22}^2 = 2K_{22}^1$. For case 3, $K_{11}^3 = 2cK_{11}^1$, $K_{12}^3 = K_{12}^1$, $K_{21}^3 = 2cK_{21}^1$, and $K_{22}^3 = K_{22}^1$, where $c = (1 + q^2/4m^2)$. For case 4, $K_{11}^4 = K_{11}^3$, $K_{12}^4 = 2K_{12}^1$, $K_{21}^4 = K_{21}^3$, and $K_{22}^4 = 2K_{22}^1$. Note that $t_s$ and $t_v$ depend on both $f$ and $g$, thus, even in the impulse approximation, the scalar and vector potentials contain contributions from $f$ and $g$. The usual first-order nonrelativistic calculation only contains contributions from $f$.[12]

The invariant amplitudes for each of the four cases are used to construct optical potentials for use in Eq. (3). The optical potentials for spin-0 targets are given by

$$U_{s,v} = \sum_{i=p,n} \int \frac{dq^3}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{r}} t_{s,v}^i(q) \rho_{s,v}^i(q), \tag{8}$$

where $\rho_{s,v}(q)$ are the Fourier transforms of the relativistic Hartree densities of Horowitz and Serot.[13] Details of the construction of $U_{s,v}$ are given by Clark *et al.*[14] The KDP equation for meson-nucleus scattering may now be written as

$$[i\beta^\mu \partial_\mu - A_\mu \beta^\mu - U_j - m]\phi = 0, \tag{9}$$

where $j = 1$, 2, 3, and 4 for the four cases used. The electromagnetic potential $A_\mu$ has been added by minimal substitution. We take $A_\mu$ as the static Coulomb potential obtained from the empirical charge distribution. In addition, the spacelike components of $U_v$ do not contribute for spin-0 targets.

The KDP elastic cross sections are obtained by solving the second-order equation obtained for the first component of the KDP wave function.[14] For conserved current interactions, such as the electromagnetic interaction, this second-order equation is identical to the KG equation for electromagnetic interactions. Here, however, a different second-order equation results for each case. They are

$$[(E - U_c - U_v)(E - U_c) - m(m + U_s) + \nabla^2]\phi_1 = 0 \tag{10}$$

for case 1;

$$[(E - U_c - U_v)^2 - m(m + U_s) + \nabla^2]\phi_1 = 0 \tag{11}$$

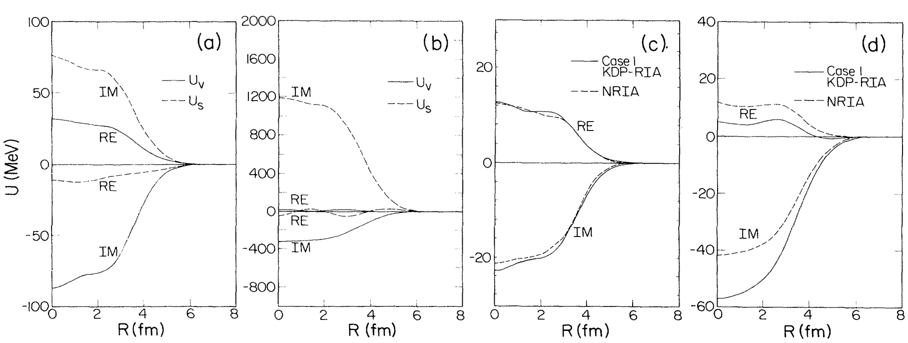


FIG. 1. Calculated KDP-RIA scalar and vector potentials for case 1. (a) The potentials for $K^+$-$^{40}$Ca at 800 MeV/$c$; (b) those for $\pi^+$-$^{40}$Ca at 800 MeV/$c$. The effective central potentials for the cases shown in (a) and (b) are given in (c) and (d) (solid curves). The NRIA potentials are the dashed curves.

for case 2;

$$[(E-U_c-U_v)(E-U_c)-(m+U_s)^2+\nabla^2-\mathbf{U}_D\cdot\nabla]\phi_1=0 \tag{12}$$

for case 3; and

$$[(E-U_c-U_v)^2-(m+U_s)^2+\nabla^2-\mathbf{U}_D\cdot\nabla]\phi_1=0 \tag{13}$$

for case 4. Here

$$\mathbf{U}_D=(m+U_s)^{-1}\nabla U_s. \tag{14}$$

The nonlocal Darwin term may be replaced by an equivalent local term through a wave function transformation, just as in the second-order Dirac equation. We could have used the KG equation to develop RIA potentials. In fact, if we write ($\hbar=c=1$)

$$[\Box+m^2+\Sigma]\phi_{\mathrm{KG}}=0, \tag{15}$$

and take

$$t=t_s+(p_\mu\gamma^\mu/m)t_v, \tag{16}$$

then we obtain results identical to those of case 1 except for the presence of the cross term $U_cU_v$ in the KDP-RIA.

As a first application of the formalism we considered elastic scattering of 800-MeV/$c$ beams of $K^+$, $\pi^+$, and $\pi^-$ from a $^{40}$Ca target. The Martin amplitudes[15] were used for the $K^+$ and the E100 solutions from Arndt[16] for $\pi^\pm$ in constructing the invariant $t$ matrices. The elastic cross sections were obtained by solving the appropriate Klein-Gordon equation.[17] We considered all four cases and found that cases 3 and 4 gave poorer agreement with $\pi^\pm$ data than did cases 1 and 2. At this energy the optical potentials for cases 1 and 2 are almost identical for a given reaction and it is not possible to choose between them on the basis of experiment. This is not true for $\pi^\pm$ scattering at lower energies where case 1 produces substantially better agreement with experiment.[14] The results for $K^+$ were very similar for all four cases.

Figures 1(a) and 1(b) show the case-1 optical potentials for $K^+$-$^{40}$Ca and $\pi^+$-$^{40}$Ca at 800 MeV/$c$. The scalar and vector potentials are large and tend to cancel just as in the $pA$ RIA. The $\pi^-$-$^{40}$Ca potentials are almost identical to the $\pi^+$-$^{40}$Ca potentials. We write Eqs. (10)–(13) as

$$\left\{\frac{1}{2E}[\nabla^2+U_c^2-2EU_c+E^2-m^2]-U\right\}\phi_1=0, \tag{17}$$

which allows us to define an effective central potential $U$. Figures 1(c) and 1(d) show this potential for the same two cases. These potentials arise from cancellations between large scalar and vector terms, just as in the nucleon-nucleus RIA. The corresponding nonrelativistic impulse-approximation potentials are also shown.[12] The kaon effective potentials resemble the nuclear densities; however, the real pion potentials have very unusual shapes for both cases. The real effective central pion potentials for cases 1 and 2 agree beyond 4 fm where they both have a small pocket of attraction but they are very different for $r<4$ fm.

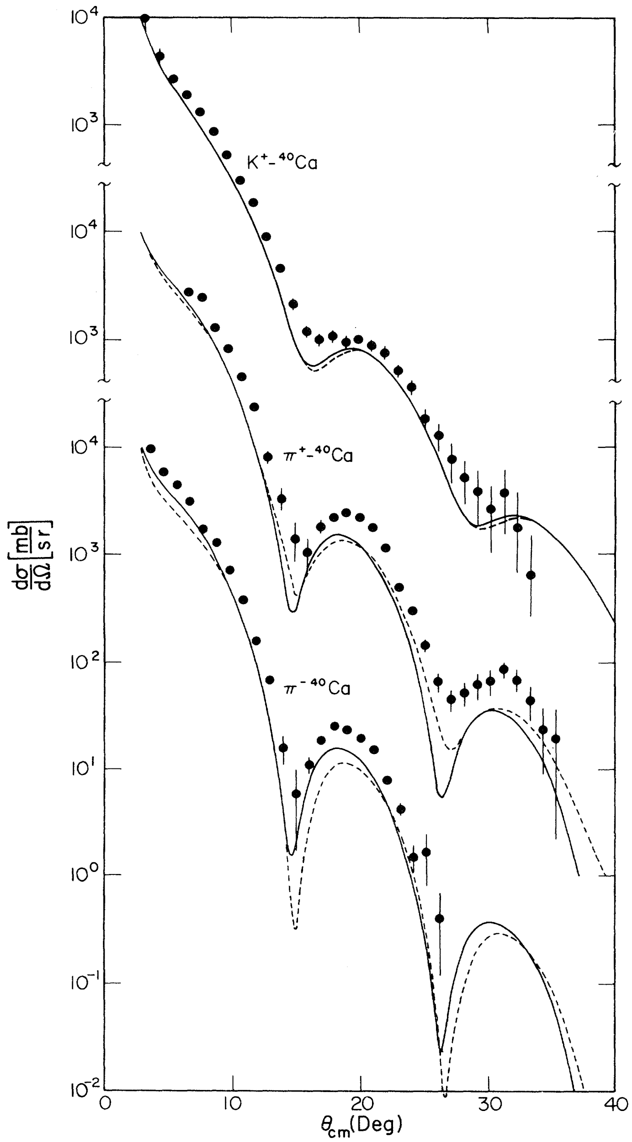


FIG. 2. Calculated KDP-RIA cross sections (solid curves) for case-1 potentials for $K^+$-$^{40}$Ca and $\pi^\pm$-$^{40}$Ca at 800 MeV/$c$. The data are from Refs. 18 and 19. The NRIA calculations are given by the dashed curve.

That the cross sections agree for the two cases is a consequence of the strong absorption. The imaginary effective potentials are essentially the same in the two cases.

Figure 2 shows the calculated $K^+$-$^{40}$Ca cross section for case-1 potentials, along with the measured cross sections from Marlow *et al.*[18] For the kaon all four choices produce essentially identical cross sections. The calculated cross sections for $\pi^\pm$ for cases 1 and 2 are also very similar; however, cases 3 and 4 are in much poorer agreement with experiment. Also shown in Fig. 2 are the $\pi^\pm$ case-1 results along with the measured cross sections from Marlow *et al.*[19] The dashed curves in Fig. 2 are nonrelativistic impulse-approximation (NRIA) calculations using the Schrödinger equation with relativistic kinematics[12] and the same input amplitudes and densities (vector densities only). The transformation of the two-body meson-nucleon amplitudes to the c.m. frame is accomplished in an analogous manner to the *pA* NRIA calculation.[20] The $K^+$-nucleon spin-dependent amplitude $g$ is very small at this energy. Because of this one would expect KDP and NRIA calculations to be quite similar, and this is the case. For the $\pi^\pm$ nucleon amplitude, $g$ is not small, and the differences between KDP and NRIA are larger, especially at small angles for the $\pi^-$ case. In addition, the presence of the cross term $U_c U_v$ in the KDP appears to improve the agreement with experiment at small angles.

We have presented a new treatment of meson-nucleus scattering using the KDP formalism. This formalism has the advantage of preserving the Lorentz character of the interaction, and allows the construction of RIA optical potentials. We are currently applying the KDP formulation to the scattering of spin-1 probes.[21] We have recently learned that the formalism introduced here is being considered for descriptions of inelastic meson-nucleus scattering.[22]

We acknowledge the support of the National Science Foundation through Grant No. Phy-8306268, the U.S. Department of Energy, and the Robert A. Welch Foundation.

---

[a]Present address: Department of Physics, Florida State University, Tallahasee, Fla. 32306.

[1]J. A. McNeil, J. Shepard, and S. J. Wallace, Phys. Rev. Lett. **50**, 1439 (1983).

[2]J. Shepard, J. A. McNeil, and S. J. Wallace, Phys. Rev. Lett. **50**, 1443 (1983).

[3]B. C. Clark, S.Hama, R. L. Mercer, L. Ray, and B. D. Serot, Phys. Rev. Lett. **50**, 1644 (1983); B. C. Clark, S. Hama, R. L. Mercer, L. Ray, G. W. Hoffmann, and B. D. Serot, Phys. Rev. C **28**, 1421 (1983).

[4]R. D. Amado, J. Piekarewicz, D. A. Sparrow, and J. A. McNeil, Phys. Rev. C **28**, 2180 (1983).

[5]G. R. Kälbermann, B. C. Clark, S. Hama, and R. L. Mercer, Bull. Am. Phys. Soc. **29**, 1030 (1984); B. C. Clark, unpublished.

[6]N. Kemmer, Proc. Roy. Soc. London, Ser. A **173**, 91 (1939).

[7]R. J. Duffin, Phys. Rev. **54**, 1114 (1938).

[8]G. Petiau, Acad. Roy. Belg. Cl. Sci. Mom. Collect. 8 **16**, No. 2 (1936).

[9]E. Fishbach, M. M. Nieto, and C. K. Scott, J. Math. Phys. (N.Y.) **14**, 1760 (1973), and Phys. Rev. D **7**, 207 (1973).

[10]R. F. Guertin and T. L. Wilson, Phys. Rev. D **15**, 1518 (1977).

[11]For a historical review of the KDP formalism see R. A. Krajcik and M. M. Neito, Am. J. Phys. **45**, 818 (1977).

[12]W. R. Coker, J. D. Lumpe, and L. Ray, Phys. Rev. C **31**, 1412 (1985). THe NRIA calculations here do not include the $A$-$1/A$ factor.

[13]C. J. Horowitz and B. D. Serot, Nucl. Phys. **A368**, 503 (1981).

[14]B. C. Clark, S. Hama, G. Kälbermann, R. L. Mercer, and L. Ray, to be published.

[15]B. R. Martin, Nucl. Phys. **B94**, 413 (1975).

[16]R. A. Arndt, Scattering Analysis Interactive Dial-in Program, and private communication.

[17]The calculations are done by partial-wave decomposition of the Klein-Gordon equation using program KGPRG; B. C. Clark, Ohio State University technical report, 1974 (unpublished). The program is available from the author.

[18]D. Marlow *et al.*, Phys. Rev. C **25**, 2619 (1982).

[19]D. Marlow *et al.*, Phys. Rev. C **30**, 1662 (1984).

[20]J. A. McNeil, L. Ray, and S. J. Wallace, Phys. Rev. C **27**, 2123 (1983).

[21]G. R. Kälbermann, B. C. Clark, R. E. Kozack, S. Hama, L. Ray, and R. L. Mercer, to be published.

[22]C.-Y. Cheung and J. R. Shepard, private communication; J. R. Shepard, in Proceedings of the International Conference on Hadronic Probes and Nuclear Interactions, Tempe, Arizona, March 1985 (American Institute of Physics, New York, to be published).