Correlations between Preequilibrium Nucleons

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A quantum-mechanical model for the emission of fast nucleons in low-energy nucleus collisions is presented. The role of a classical emission function is shown to be played by the time derivative of an impact-parameter–integrated, time-dependent, distorted-wave Wigner function. Correlations between the emitted particles are shown to be caused by the Pauli principle, by the impact-parameter dependence of the emission process, and by two-body collisions including the final-state interactions, as well as those due to the size of the emitting source.

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In the collision of two nuclei at energies near the Coulomb barrier, there is a component of light-particle emission which is nonstatistical in nature and will hopefully serve as a probe of the early stages of the reaction. Recently, the correlations between two protons have been measured[1] at these energies. For the emission of two photons from a source of finite dimensions,[2] the correlations between observed particles serves as a direct measurement of the size of the source. For high-energy nuclear collisions where a classical treatment is valid, the same general physics has been applied[3] to determine the spatial extent of the emitting region.

However, at low collision energies, the energy of the emitted particles is of the order of 20 MeV or less. This implies that the motion of the particles should be treated quantum mechanically. In this case, the relation of the observed correlations among the emitted particles to the size of the emitting source will be less direct. The purpose of this Letter is to examine the sources of correlations between promptly emitted particles in ion reactions. We derive general formulas for the inclusive cross sections for the emission of one and two particles. The derivation follows closely that of an external mean-field emission model that we have presented[4,5] previously. However, the derivation is given here in a general context without specific reference to the dynamics of the emission process.

In the quantum-mechanical treatment we identify the function that serves the role of a classical emission function as the time derivative of a distorted-wave, time-dependent, impact-parameter–integrated Wigner function. For the inclusive, two-particle emission spectrum we find several modifications to the simple classical model proposed in Ref. 3. In addition to correlations which arise from the spatial extent of the source, we find that there are correlations which are caused by (1) the Pauli exchange between the two particles, (2) the impact-parameter dependence of the emission process, and (3) residual interactions, either those present in the incident nuclei or those dynamically induced during the collision. Of the latter type of correlations, the final-state interactions are found to be important and enter differently in the quantum-mechanical treatment than in the classical problem. In addition, the relation between the correlations and the size of the emitting region is rendered less direct by the distortions of the outgoing final waves.

We first derive a general expression for the inclusive cross section for nucleon emission. We intentionally do not choose a specific model for the dynamics of the emission process but derive, under some general assumptions, the relation of the differential cross section to the Wigner function, a formal result which is not specifically dependent on the treatment of the dynamics. The first assumption we make is that there is a variable $\mathbf{R}$ that can be associated asymptotically with the separation of the centers of the two ions for which the classical limit may be taken. The process and the validity of taking a classical limit for $\mathbf{R}$ to yield a classical variable $\mathbf{R}(t)$ is discussed in Ref. 5. Under this general assumption the motion of the nucleons will be described by a time-dependent Schrödinger equation in which $\mathbf{R}(t)$ appears parametrically,

$$i(\partial/\partial t)\Psi_b(t)=H(\mathbf{R}(t))\Psi_b(t). \qquad (1)$$

where b is the impact parameter associated with $\mathbf{R}(t)$.

A technique which can be used to systematically solve Eq. (1) has been presented by Baranger and Zahed.[6] In their approach, we require a basis, generated by $h_0\phi_\nu^0 = E_\nu\phi_\nu^0$, and a time-dependent basis generated by $i(\partial/\partial t)\phi_\nu(t) = h(\mathbf{R}(t))\phi_\nu(t)$, with the boundary condition that $\phi_\nu(t) \to \phi_\nu^0$ as $t \to -\infty$. The many-body problem of Eq. (1) is then to be solved perturbatively around the Hamiltonian $H_0(\mathbf{R}(t))$ which is the A-body sum of the single-particle Hamiltonians, $h(\mathbf{R}(t))$. Typical diagrams that might enter an approximate solution of Eq. (1) are presented in Fig. 1 where the meaning of the diagrams is that of Ref. 6.

Following Ref. 5, a current operator and differential cross section can be defined if we require that $h(\mathbf{R}(t))$ approach h_0 in the limit $t \to +\infty$. Preequilibrium particles have been seen in time-dependent Hartree-Fock calculations,[7] but because the single-particle Hamiltonian in that theory does not satisfy this last assumption, our derivation cannot be applied there. The result for the differential cross section is

$$\frac{d^3\sigma}{d^3k} = (2\pi)^{-2} \lim_{t\to+\infty} \int b\, db \langle \Psi_b(t) | a^\dagger_{\mathbf{k}(-)} a_{\mathbf{k}(-)} | \Psi_b(t) \rangle, \tag{2}$$

where the operator $a^\dagger_{\mathbf{k}(-)}$ is the creation operator for the state $\phi^0_{\mathbf{k}(-)}$ (a scattering state from the set ϕ^0_ν). The derivation of this equation follows the logic of that given in Ref. 5, but requires only the assumptions outlined above. The result, Eq. (2), is independent of the specific treatment of the dynamics of the emission process.

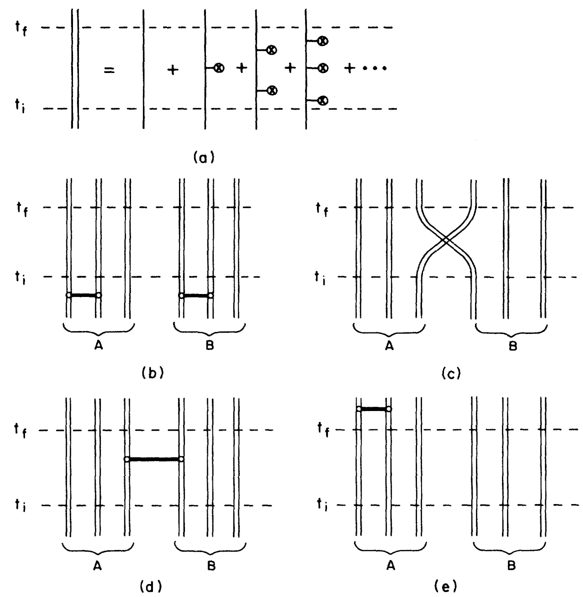

FIG. 1. (a) The summation of the interactions produces the time-dependent basis, $\phi_\nu(t)$. (b) The interaction of two nucleons in a nucleus before the collision which would build correlations into the incident nuclei in the usual time-independent perturbation theory. (c) The exchange of a nucleon between the target and the projectile. (d) A two-body collision between a nucleon in the target and one in the projectile. (e) The final-state interaction between the two detected particles. The time t_i represents the time at which the two nuclei begin to interact; t_f, the time when they cease to interact. The heavy dark line could represent either a phenomenological two-body interaction or the ladder sum of these interactions into a Bethe-Goldstone (here, time-dependent) reaction matrix.

The classical source function is defined[3] by

$$\sigma^{-1}\frac{d^3\sigma}{d^3k} = \int dt \int d^3R\, D(\mathbf{k},\mathbf{R},t), \tag{3}$$

where $D(\mathbf{k},\mathbf{R},t)$ represents the probability of a particle being emitted at a time t from a position $\mathbf{R}$ with a momentum $\mathbf{k}$. To rewrite Eq. (2) in a form that can be related to Eq. (3), we use the time-dependent and impact-parameter–dependent density matrix defined by $\langle r_2|\rho_b(t)|r_1\rangle = \langle \Psi_b(t)|\psi^\dagger(r_2)\psi(r_1)|\Psi_b(t)\rangle$, where $\psi(r_1)$ is the nucleon field operator. If we define a time-dependent and impact-parameter–dependent Wigner function by

$$W_b(\mathbf{R},\mathbf{p};t) = \int d^3r \exp(i\mathbf{p}\cdot\mathbf{r}) \langle \mathbf{R}+\mathbf{r}/2|\rho_b(t)|\mathbf{R}-\mathbf{r}/2\rangle, \tag{4}$$

and define a distorted-wave Wigner function by

$$\tilde{W}^*_{\mathbf{k}}(\mathbf{R},\mathbf{p}) = \int d^3r \exp(-i\mathbf{p}\cdot\mathbf{r}) \phi^0_{\mathbf{k}(-)}(\mathbf{R}+\mathbf{r}/2)\phi^{0*}_{\mathbf{k}(-)}(\mathbf{R}-\mathbf{r}/2), \tag{5}$$

the inclusive single-particle emission cross section can be written as

$$\frac{d^3\sigma}{d^3k} = (2\pi)^{-2}\int b\, db \int d^3R \int \frac{d^3p}{(2\pi)^3} \lim_{t\to+\infty} \int_{-\infty}^{t} dt\, \tilde{W}^*_{\mathbf{k}}(\mathbf{R},\mathbf{p}) \frac{d}{dt} W_b(\mathbf{R},\mathbf{p};t). \tag{6}$$

The lower limit, $t = -\infty$, does not contribute as a result of the orthogonality between the continuum states and the bound states of h_0. Comparison of Eqs. (3) and (6) yields

$$\sigma D(\mathbf{k},\mathbf{R},t) = (2\pi)^{-2}\int b\, db \int \frac{d^3p}{(2\pi)^3} \tilde{W}^*_{\mathbf{k}}(\mathbf{R},\mathbf{p}) \frac{d}{dt} W_b(\mathbf{R},\mathbf{p};t) = (2\pi)^{-2}\int b\, db\, D_b(\mathbf{k},\mathbf{R},t). \tag{7}$$

We thus see that under the general conditions of our derivation one can define a classical source function even though the particle motion is being treated quantum mechanically.

The Wigner function, $W_b(\mathbf{R},\mathbf{p};t)$, is characterized by a size which would represent the size of the emitting region. The particle spectrum is characterized by a different size since one must fold with the distorting Wigner function, $\tilde{W}^*_{\mathbf{k}}(\mathbf{R},\mathbf{p})$. In the limit that $|k| >> k_F$, with k_F the Fermi momtentum, one might approximate the scattered waves by plane waves, which yields

$$\sigma D(\mathbf{k},\mathbf{R},t) = (2\pi)^{-2}\int b\,db\,\frac{d}{dt}W_b(\mathbf{R},\mathbf{p};t). \tag{8}$$

This is an intuitive and appealing expression but is *not* valid for preequilibrium particles in low-energy reactions. In this case $|\mathbf{k}| \leq k_F$ and the lack of orthogonality between the plane waves and the bound-state wave functions would produce emission even when the impact parameter were so large as to imply that the ions do not interact.

In addition to the inclusive single-particle spectrum, the inclusive two-particle spectrum is of interest and will be given, under the same assumptions which were used in the derivation of Eq. (2), by

$$\frac{d^6\sigma}{d^3k\,d^3p} = (2\pi)^{-5}\lim_{t\to+\infty}\int b\,db\,\langle\Psi_b(t)|a^\dagger_{\mathbf{p}(-)}a^\dagger_{\mathbf{k}(-)}a_{\mathbf{k}(-)}a_{\mathbf{p}(-)}|\Psi_b(t)\rangle. \tag{9}$$

The question that arises is whether the correlations between the two particles are simply related to the size of the emitting source? In order to address this question, it is convenient to become more specific concerning the dynamics of the emission process. We will specialize at this point to the mean-field model of Refs. 4 and 5, in which case

$$\langle\Psi_b(t)|a^\dagger_{\mathbf{p}(-)}a^\dagger_{\mathbf{k}(-)}a_{\mathbf{k}(-)}a_{\mathbf{p}(-)}|\Psi_b(t)\rangle = \sum_{\nu=1}^{A}|C_{\nu\mathbf{k}}(b)|^2\sum_{\mu=1}^{A}|C_{\mu\mathbf{p}}(b)|^2 - \left|\sum_{\nu=1}^{A}C^*_{\nu\mathbf{k}}(b)C_{\nu\mathbf{p}}(b)\right|^2, \tag{10}$$

where $C_{\nu\mathbf{k}}(b)$ is given by

$$C_{\nu\mathbf{k}}(b) = \lim_{t\to+\infty}\int d^3r\,\phi^{0*}_{\mathbf{k}(-)}(\mathbf{r})\phi_\nu(\mathbf{r},t). \tag{11}$$

The inclusive two-particle emission spectrum becomes

$$\frac{d^6\sigma_0}{d^3k\,d^3p} = (2\pi)^{-5}\int b\,db\left\{D_b(\mathbf{R},\mathbf{k})D_b(\mathbf{R},\mathbf{p}) - \sum_{\nu=1}^{A}|C^*_{\nu\mathbf{k}}(b)C^*_{\nu\mathbf{p}}(b)|^2\right\}. \tag{12}$$

This expression implies two sources of correlations beyond those which are present in the product of two source functions. The first is the exchange term which is an expected result of the Pauli principle and the identity of the emitted particles. The second is a correlation which arises because the emission process is impact-parameter dependent. The expression is the integral over a single impact parameter of the product of two impact-parameter–dependent source functions, not the product of two source functions which have already been averaged over individual impact parameters. In addition, one could include the additional physics that is pictured in Fig. 1 in the solution of Eq. (1). One would then have additional correlations corresponding to the following: Fig. 1(b), correlations which were already present in the two nuclei; Fig. 1(c), the exchange of particles between the two nuclei; and Fig. 1(d), two-body collisions between a nucleon in the target and one in the projectile.

Of the additional correlations pictured in Fig. 1, the final-state interactions of Fig. 1(e) will be important for particles with a small relative velocity. This interaction becomes relatively easy to calculate if one utilizes the wave-packet formulation of Ref. 5. For times greater than t_f the Hamiltonian $h(\mathbf{R}(t))$ has become equal to h_0 and we are dealing with a time-independent problem. We also notice that for large times a wave packet made up of the states $\phi_{\mathbf{k}(-)}(t)$ evolves simply into a wave packet composed of plane-wave[8] states. Thus, the ladder sum of the two-nucleon interaction for times greater than t_f yields a *free* two-body scattering wave function. The two-particle inclusive cross section is then given by

$$\frac{d^6\sigma}{d^3k\,d^3p} = \int\frac{d^3k'}{(2\pi)^3}\frac{d^3p'}{(2\pi)^3}\frac{d^3k''}{(2\pi)^3}\frac{d^3p''}{(2\pi)^2}\psi^{*(-)}_{\mathbf{k},\mathbf{p}}(k'',\mathbf{p}'')\psi^{*(-)}_{\mathbf{k},\mathbf{p}}(\mathbf{k}',\mathbf{p}')$$
$$\times\langle\Psi_b(t)|a^\dagger_{\mathbf{p}''(-)}a^\dagger_{\mathbf{k}''(-)}a_{\mathbf{k}'(-)}a_{\mathbf{p}'(-)}|\Psi_b(t)\rangle, \tag{13}$$

where $\psi^{(-)}_{\mathbf{k},\mathbf{p}}(\mathbf{k}',\mathbf{p}')$ is the free nucleon-nucleon scattering wave function.

In Fig. 2 we plot the correlation function for two-neutron emission in the reaction $^{16}O + ^{93}Nb$ at $E_{lab} = 204$ MeV. The dynamic model used to produce this graph is the mean-field model of Ref. 5. The correlation function is de-

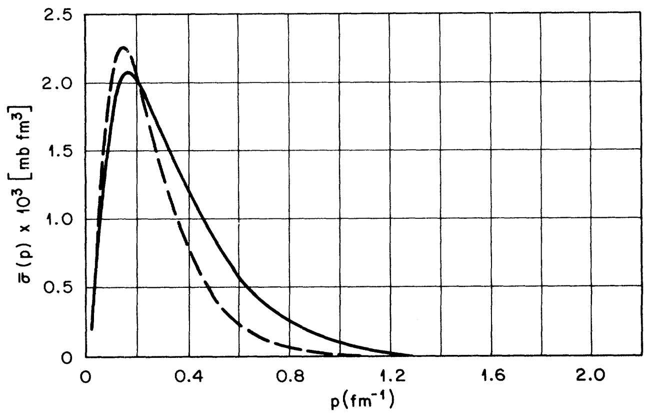

FIG. 2. Two-particle cross sections $\bar{\sigma}(p)$ without final-state interactions (solid curve) as a function of the relative momentum. The dashed curve shows the same quantity with the inclusion of the final-state interactions (divided by 100).

fined by

$$\bar{\sigma}(\mathbf{q}) = \int d^3Q \, \frac{d^6\sigma}{d^3k \, d^3p}, \tag{14}$$

with $\mathbf{Q} = \mathbf{k} + \mathbf{p}$ and $\mathbf{q} = (\mathbf{k} - \mathbf{p})/2$. In order to simplify the multidimensional integration in Eq. (13), we have used a local momentum approximation[8]

$$\psi_{\mathbf{k},\mathbf{p}}^{(-)}(\mathbf{k}', \mathbf{p}') \cong (2\pi)^6 \delta(\mathbf{k} - \mathbf{k}') \delta(\mathbf{p} - \mathbf{p}') f_J(-|\mathbf{q}|), \tag{15}$$

where $f_J(-|\mathbf{q}|)$ is the Jost function. We have used the separable Yukawa potential[9] to generate the Jost function. The correlations we see predicted here for two neutrons are of a similar character as those seen for protons.[1]

We have shown how the time derivative of a distorted-wave Wigner function, Eq. (7), serves the role of a classical source function for the emission of preequilibrium particles in heavy-ion collisions. The distortion factor implies that if one extracts a size for the emitting region from experimentally measured cross sections, the results will be a combination of the range associated with the emitting source and the range associated with the distortions. We have found several sources of correlations between promptly emitted particles. In addition to the correlations which arise from the size of the source and the distorting potential, there are Pauli correlations, correlations which arise from the impact-parameter dependence of the emission process, and dynamic correlations including the final-state interactions.

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