# Chaotic Dynamics in a Periodically Excited Air Jet

Marco Bonetti
*Von Karman Institute for Fluid Dynamics, B-1640 Rhode-Saint-Genese, Belgium*
and
Roland Meynart
*Faculté des Sciences Appliquées, Université Libre de Bruxelles, B-1050 Bruxelles, Belgium*
and
Jean Pierre Boon
*Faculté des Sciences, Université Libre de Bruxelles, B-1050 Bruxelles, Belgium*
and
Domenico Olivari
*Von Karman Institute for Fluid Dynamics, B-1640 Rhode-Saint-Genese, Belgium*
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We present measurements of the dynamic behavior of macroscopic structures observed in a periodically excited air jet. The transitional, weakly turbulent state is shown to be characterized by a strange attractor with low dimensionality ($\nu = 2.6$), whereas motion of the turbulent flow is not restricted to a low-dimensional attractor.

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Low-dimensional chaos in hydrodynamic systems has been evidenced experimentally for Rayleigh-Bénard convection[1,2] and Couette-Taylor flow.[3] Chaotic dynamics was identified by the fractal dimension of the attractors, which therefore appear to be of strange type.

Such most-studied systems have confined flows with steady structures.[4] However, the onset of chaos in Rayleigh-Bénard convection depends on the geometry of the system; e.g., in convection cells with small aspect ratios, routes to chaos may involve two or three independent modes,[4] whereas in larger cells, quasi-periodic regimes with up to five incommensurate frequencies have been observed before chaos would set in.[5] In this respect, free flows seem to exhibit more general behavior: In the transition to turbulence, macroscopic structures always break up into a great number of eddies with large variations in orientation, size, shape, and mean vorticity. Therefore, such flows are also expected to be governed by a large effective number of degrees of freedom.

We have studied the onset of chaos in a free flow, i.e., an isothermal air jet at low Reynolds number, modulated by a longitudinal (along the flow axis) sinusoidal excitation.[6] The excitation induces a periodic perturbation, with amplitude $2\pi f_0 A$, on the axial velocity. The control parameters are the amplitude, $A$, and the frequency, $f_0$, of the excitation, and the Reynolds number, $N_{\mathrm{Re}}$. The diameter of the nozzle ($D = 3$ mm) was kept constant throughout the experiments.

The unexcited jet was tested and found to be stable over an axial distance $\sim 150D$ with a well-defined Poiseuille profile at the exit (tube length $l = 80D$). Flow visualization of the excited jet (seeded with oil smoke) was performed with a stroboscoped laser-light sheet shone through the flow axis. Figure 1 shows a typical picture of the structures developed in the excit-

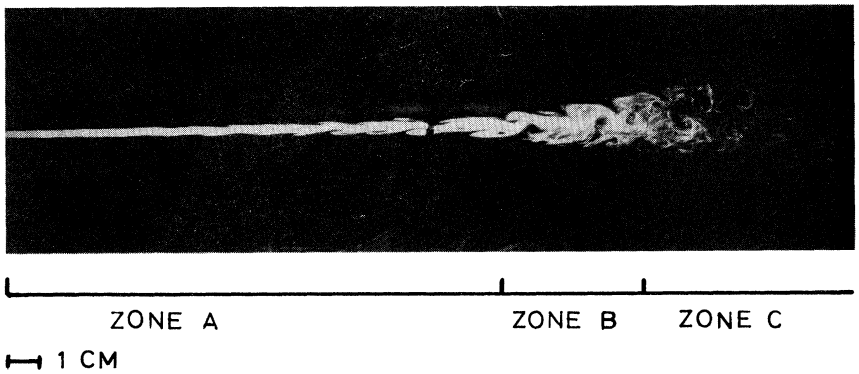


FIG. 1. Global structure of the excited jet (axially stroboscoped laser light sheet picture); $N_{\mathrm{Re}} = 500$; $f_0 = 30$ Hz; rms velocity ratio = 2%. Zone A, laminar regime; zone B, transitional regime; zone C, turbulent regime.

ed jet, with the following sequence: steady laminar and periodic laminar zone (A), transitional (weakly turbulent) zone (B), turbulent zone (C). So the excitation induces well-defined and reproducible spatial structures in the jet[7]; the vortical structures are stable in zone A and break up into an inhomogeneous pattern in zone C via a transitional oscillating regime (zone B). This sequence was observed to be insensitive to amplitude and frequency within the investigated range (20 Hz $\leq f_0 \leq$ 80 Hz).[8] The present work focuses on the onset of chaos as induced by the loss of stability of the initially stable structures.

Local velocity measurements were performed with a linearized hot-wire anemometer (frequency response up to 10 kHz)[9]; each measurement produced a time series of 20480 data points sampled at 3125 Hz. The results presented below for the transitional and turbulent regimes were obtained by adjusting the excitation amplitude $A$, with all other experimental parameters fixed ($D=3$ mm, probe position $L=50D$, $N_{Re}=500$, $f_0=40$ Hz). For the two regimes considered here, the values of the rms velocity relative to the mean exit velocity are 2.8% and 3.3%, respectively.

Typical velocity time series and power spectra are presented in Figs. 2 and 3. In the transitional regime [Fig. 2(a)], irregularities in the time series are indications of bursts of instability in the structures; correspondingly, the power spectrum [Fig. 2(b)] shows considerable broadband noise in addition to the sharp component $f_0$ and its harmonics. In the turbulent regime, the velocity variations become quite irregular [Fig. 3(a)], and the peak at $f_0$ is the only one to remain distinguishable in the power spectrum [Fig. 3(b)].

We used the Grassberger-Procaccia algorithm[10] to characterize the transitional and turbulent regimes. From the velocity time series $u(t)$, we computed the correlation integral $C(R)=\lim_{N\to\infty} N^{-2}\times$ (number of pairs of phase-space points whose distance $|\mathbf{x}_i-\mathbf{x}_j|$ is contained in a hypersphere of radius $R$ imbedded in a $d$-dimensional phase space), where the state vectors $\mathbf{x}_i$ are obtained as follows: Data points shifted with $\Delta t=p\tau$ ($1/\tau$ is the data aquisition frequency; $p$ is the shifting parameter) in the time series were used to construct the $d$-dimensional state vector $\mathbf{x}(t)=\{u(t),u(t+\Delta t),\ldots,u(t+(d-1)\Delta t)\}$; the imbedding dimension was varied from $d=2$ to 16. 500 points uniformly distributed along the trajectory were correlated with all the points on the trajectory to yield $500\times 20480$ correlations.

A typical plot ($p=2$) of the dimensionality analysis for the transitional regime is shown in Fig. 4; saturation of the slope $\nu$ of $\log(C(r))$ vs $\log(R)$ is obtained for imbedding dimension $d\geq 10$ yielding dimensionality $\nu_s=2.7$ [Fig. 4(a)].[11] Note that the saturation effect is well evidenced by the plot of the local slope versus $\log(R)$ for different values of $d$ [Fig. 4(b)]. A systematic analysis was performed by variation of the time-shifting parameter $p$ for the evaluation of the

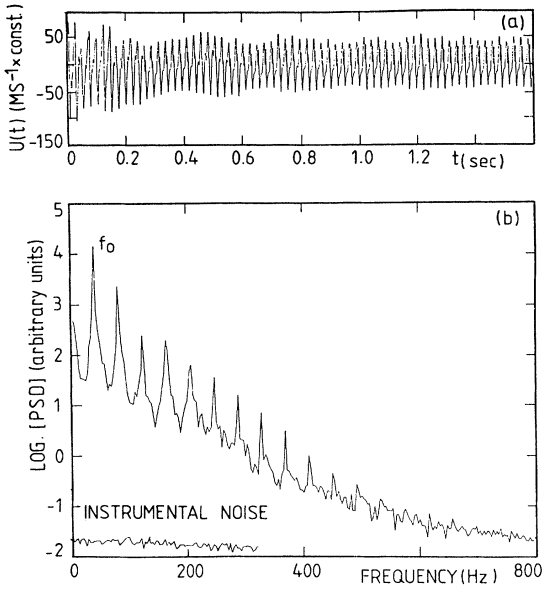


FIG. 2. Transitional regime. $N_{Re}=500$; $f_0=40$ Hz; rms velocity ratio = 2.8%. (a) Axial velocity time series (20480 data; one-fourth of record shown; vertical scale: const = const amplification factor). (b) Power spectrum (average over twenty fast Fourier transforms computed with 1024 points; acquisition frequency, 3125 Hz).

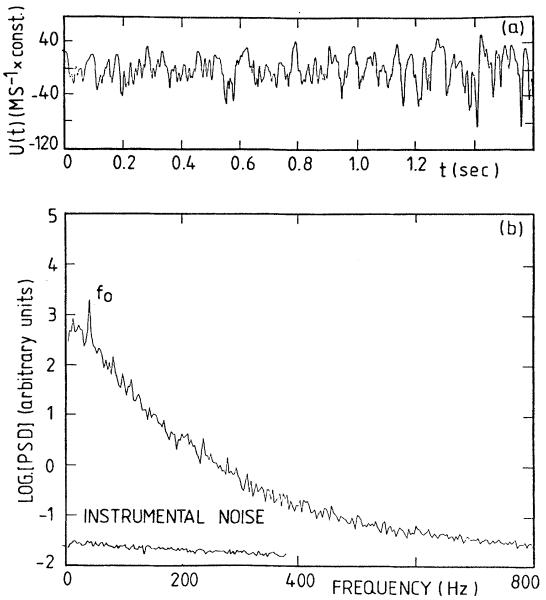


FIG. 3. Turbulent regime; same as Fig. 2 except rms velocity ratio = 3.3%.

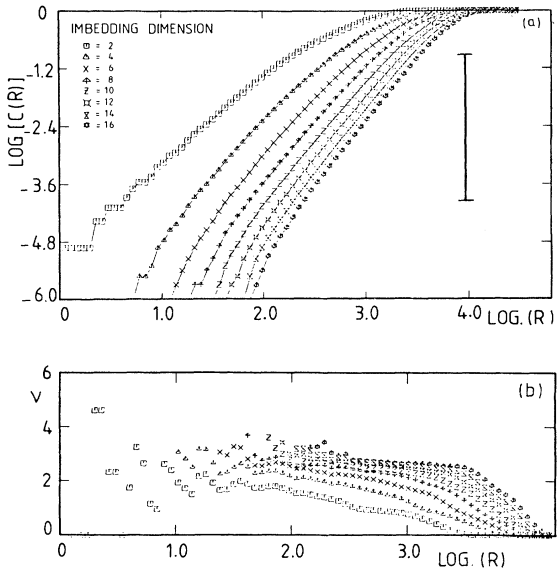


FIG. 4. (a) Dimensionality analysis for the transitional regime ($N_{Re}=500$; $f_0=40$ Hz; rms velocity ratio = 2.8%), $p=2$, $d=2$ to 16; vertical bar indicates domain of dimensionality evaluation. (b) Dimensionality vs $\log(R)$. Notice that for small $R$, the limited number of points yields poor average in the computation of $C(R)$, which results in a wide spread of the values of $\nu$.

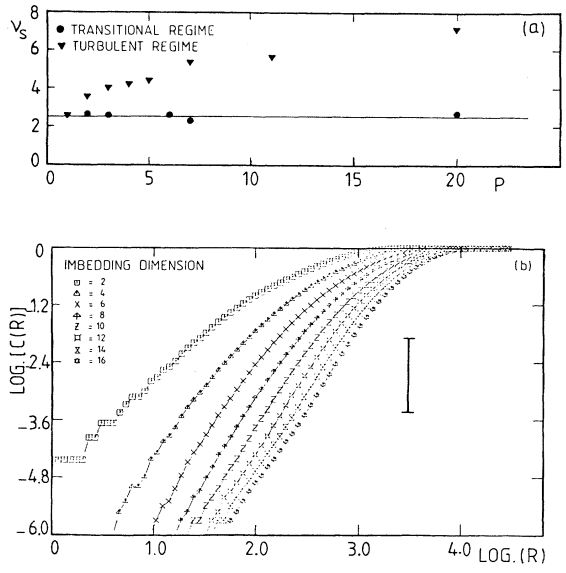


FIG. 5. (a) $p$ dependence of slope $\nu_s$ for transitional (dots) and turbulent (triangles) regimes. (b) Dimensional analysis for the turbulent regime ($N_{Re}=500$; $f_0=40$ Hz; rms velocity ratio = 3.3%), $p=2$, $d=2$ to 16; vertical bar, same as Fig. 4(a).

sensitivity of the dimensionality to different values of $p$. The results are given in Fig. 5(a). For the transitional regime, the dimensionality has average value 2.6 with a variance of 0.14. The analysis of the turbulent-regime data yields acceptable saturation of the slope $\nu$, in the range defined by the bar as shown in Fig. 5(b) for $p=2$ ($\nu_s=3.5$).[12] However, the dimensionality is found to be $p$ dependent as $\nu_s$ increases from $\sim 2.5$ for $p=1$ to $\sim 7$ for $p=20$, with imbedding dimension $d=16$ [Fig. 5(a)]. This observation could be interpreted as an indication of complex dynamics over a large range of time scales.[13] Furthermore, it follows that the saturation of the slope with increasing dimension cannot be taken in general as a sufficient condition to assert the dimensionality (or even the existence) of a strange attractor. This point has also been raised by Atten *et al.*[14] in a recent analysis of their study of electrohydrodynamic convection.[1]

As the analysis establishes unambiguously a fractal dimension for the transitional regime ($\nu_s=2.6$), the local structure of the attractor was investigated by constructing the three-dimensional phase portrait $u(t)$, $u(t+3\tau)$, $u(t+7\tau)$.[11] Figure 6 shows a projection of the phase portrait and three Poincaré sections, wherefrom some degree of stretching can be noticed; however, we did not detect, in any Poincaré section, evidence of a clearly folded structure.[15]

In summary, we have presented a study of the dynamical behavior of the macroscopic structures observed in a nonconfined excited system undergoing transition from laminar to turbulent. Dimensionality analysis established that the transitional regime exhibits chaotic dynamics characterized by a strange attractor with low dimensionality ($\nu_s=2.6$). In the turbulent regime, the dynamics could not be identified unambiguously, as motion is probably not restricted to

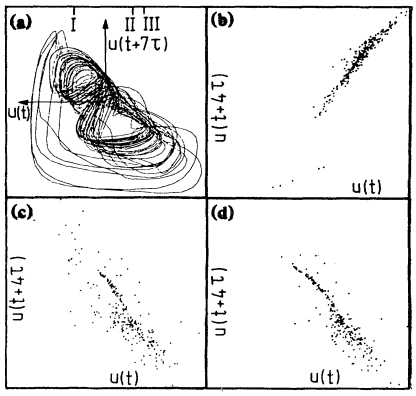


FIG. 6. Transitional regime. (a) Two-dimensional projection of phase portrait $(u(t), u(t+3\tau), u(t+7\tau))$ on plane $(u(t), +u(t+7\tau))$ (3000 points plotted out 20 480 data points). (b)–(d) Poincaré sections constructed by intersecting three-dimensional "positively" directed orbits with planes $(u(t+3\tau), u(t+7\tau))$ located at I, II, III in (a), and labeled $(u(t), u(t+4\tau))$ in (b)–(d).

a low-dimensional attractor. This point is presently under investigation.

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[1]B. Malraison, P. Atten, P. Bergé, and M. Dubois, J. Phys. (Paris) **L44**, L-897 (1983).

[2]M. Giglio, S. Musazzi, and U. Perini, Phys. Rev. Lett. **53**, 2402 (1984).

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[5]R. W. Walden, P. Kolodner, A. Passner, and C. M. Surko, Phys. Rev. Lett. **53**, 242 (1984).

[6]Experimental details will be reported elsewhere.

[7]The structures are periodic in time while spatially developing downstream. Figure 1 shows a picture of stroboscopically "frozen" structures.

[8]An increase in $f_0$ and/or $A$ shortens the axial distance at which the transition to turbulence takes place, but does not modify the sequence of the successive regimes; the same observation holds for an increase in $N_{Re}$ with fixed $f_0$ and $A$.

[9]As test measurements showed excellent agreement with LDV data [G. W. Rankin, K. Sridhar, M. Arulraja, and K. R. Kumar, J. Fluid Mech. **133**, 217 (1983)], we are confident that the flow was not perturbed by the probe.

[10]P. Grassberger and I. Procaccia, Phys. Rev. Lett. **50**, 346 (1983), and Physica **9D**, 189 (1983).

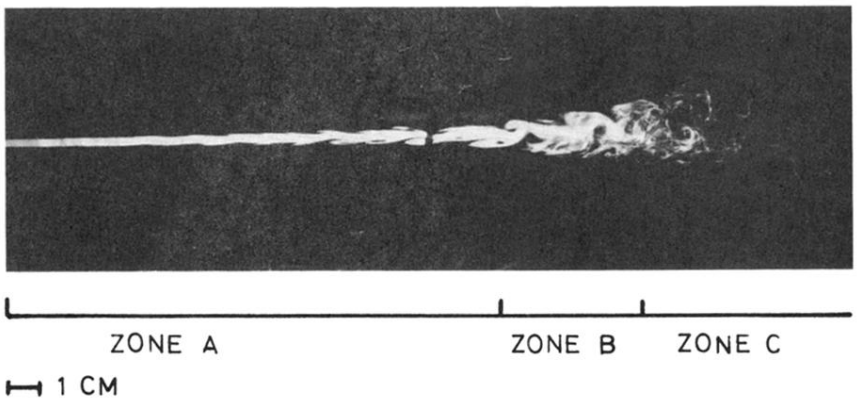
[11]We checked the periodic laminar regime ($N_{Re}=1000$; $L=16D$; $f_0=40$; rms velocity ratio = 2.5%); time series produced trajectories well confined on a limit cycle and $C(R)$ yielded a well-defined dimensionality $\nu_x=1.1$ (the excess value 0.1 could be due to weak incipient nonlinearities in the flow regime).

[12]This method of evaluation was preferred to a plot of the form of Fig. 4(b) which in this case yielded no unambiguous saturation value because of larger variations in the local slope (computed by central differences).

[13]However, operationally, this $p$ dependence may result from too low an upper bound of the imbedding dimension in performance of the dimensionality analysis ($d \geqslant 2\nu+1$); F. Takens, in *Dynamical Systems and Turbulence, Warwick 1980,* edited by D. A. Rand and L.-S. Young, Lecture Notes in Mathematics, Vol. 898 (Springer-Verlag, Berlin, 1981), p. 366. We also suspect a possible connection between this effect and the data acquisition frequency.

[14]P. Atten, J. G. Caputo, B. Malraison, and Y. Cagne, "Determination of Attractor Dimension for Various Flows," to be published.

[15]A larger number of intersection points should be necessary to ascertain the presence of folding. Attempts to construct significant return maps failed to produce typical curves with a maximum.

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