Leptons and "Horizontal" Neutral Scalar Bosons from Topological Particle Theory

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Besides hadrons and eight electroweak gauge bosons, topological particle theory predicts four generations of isodoublet Dirac leptons (e, μ, τ, λ plus neutrinos) and eight generation-changing neutral scalar bosons $H_{\lambda\tau}$, $H_{\lambda\mu}$, $H_{\tau e}$, $H_{\mu e}$, and conjugates. Seven vector bosons, all H's, and λ acquire masses $>>$ 1 GeV from direct couplings to heavy elementary hadrons with masses $\gtrsim$ 1 TeV. We estimate that $m_\lambda \lesssim$ 1 TeV. H-mediated radiative corrections give masses to μ and τ in first order, and to e in second order. Lepton-generation numbers are absolutely conserved and neutrinos acquire small masses but do not mix.

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This Letter describes in algebraic terms, suitable for comparison with the standard model, the nonhadronic content of topological particle theory (TPT). Selection rules and the qualitative features of the observed lepton mass spectrum are explained and related to a predicted family of horizontal neutral scalar bosons. Certain rules will appear arbitrary in this language; topological justification will be given in a separate paper.

TPT is based on a graphical expansion of the S matrix rather than on fundamental local fields. Feynman graphs are embedded in two-dimensional surfaces that were already implicit in Harari-Rosner duality diagrams[1] and that have a twofold capacity[2]: (i) Amplitudes can be expanded according to the topological complexity of the corresponding surfaces, as in field-theoretic $1/N$ expansions.[3] The lowest-complexity level, which corresponds to elementary hadrons, yields a set of nonlinear bootstrap equations that determine "soft" (nonpolynomial) elementary vertex functions[4] from which higher levels of the topological expansion are to be calculated via Feynman-type rules[5] which satisfy S-matrix unitarity level by level. Each level is required to be finite. (ii) The various one- and two-dimensional submanifolds that build TPT surfaces can be independently oriented and the orientations interpreted as discrete particle properties—spin, isospin, etc.[6] The particle spectrum is thereby uniquely determined. Most orientations pass unchanged between outgoing and ingoing particles, corresponding to global U(1) symmetries and conserved quantum numbers. Larger symmetry groups, U(2^N) or SL(2,C), are obtained where all possible orientations of a set of submanifolds are equivalent, but symmetries larger than U(1), except for the Lorentz group, are eventually violated by higher-level components of the topological expansion.

A supermultiplet of elementary hadrons has been deduced at the zero-complexity level,[7] with a scale-setting fundamental mass m_0 that has been inferred to lie in the teraelectronvolt range.[8] Leading corrections generate partons of mass $<< m_0$ which dominate gigaelectronvolt-scale strong-interaction phenomena[8] but the concern of this Letter is with a different—nonhadronic—level of the topological expansion.

This new level arises because the photon as a massless vector boson coupled to a conserved charge appears to be essential for a comprehensive bootstrap theory, giving meaning to measurement of momentum and to macroscopic space-time. The photon can be incorportated in TPT as part of a multiplet of massless elementary nonhadrons by using only oriented manifolds already encountered in hadrons. We now turn to the quantum numbers of nonhadrons.

Any elementary TPT particle can be algebraically characterized by an ordered two-index structure $\Phi_{\beta\alpha}(p)$, all discrete quantum numbers being carried by a right index α (fermion or boson number + 1) and a left index β (fermion or boson number −1). The indices α and β may separately be either fermionic (f) or bosonic (b), allowing the combinations Φ_{ff} (gauge boson or meson), Φ_{bb} ["H" boson or hexon[8] (hadronic counterpart of the horizontal boson)], Φ_{fb} (lepton or antibaryon), and Φ_{bf} (antilepton or baryon).[9]

For *nonhadrons* a fermionic index depicts three (two-valued) topological orientations. One orientation is absolutely conserved and corresponds to electric charge 1 or 0, transforming as an isospinor. The other two orientations correspond to spin and chirality coupling to momentum as required by Lorentz invariance and together transforming as a Dirac four-spinor.

A nonhadron bosonic index is only four-valued because its isospin turns out to be fixed in the neutral direction. The bosonic analogs of fermionic spin and chirality are completely disconnected from momentum and do not transform under the Poincaré group. Topological consistency requires these orientations to be conserved, i.e., for each external nonhadron with a *left* bosonic index $G = 1, \ldots, 4$ there is another in the amplitude with the same *right* bosonic index G. This matching of indices implies four absolutely conserved (internal) quantum numbers L_G.

The photon lies within the $\Phi_{ff'}$ family, which generally contains isosinglet and isovector bosons, with

$L_G = 0$. For nonhadrons f and f' are restricted to opposite chiralities, and so only vectors occur (both right and left handed).[10] A similar restriction on the bosonic analog of chiral orientation reduces the number of nonhadronic $\Phi_{bb'}$ to eight $H_{GG'}$ neutral scalar bosons. We choose a convention where only H_{12}, H_{13}, H_{24}, H_{34}, and conjugates occur. $H_{GG'}$ has $L_G = -1$ and $L_{G'} = +1$.

L_G can be identified with lepton number for generation G: Φ_{fb} has no chiral restrictions and for nonhadrons describes four generations of Dirac neutrinos and electrons. It will turn out that $G = 1$, 2, 3 corresponds, respectively, to e, μ, τ while the charged lepton with $G = 4$ will be denoted by λ^-. We remark at this point that hadrons all have $L_G = 0$; hence proton decay and semileptonic transitions such as $K^0 \to e^+\mu^-$ or $e^-\mu^+$ are strictly forbidden. Lepton mixing and transitions such as $\mu^- \to e^- e^+ e^-$ are also forbidden by conservation of L_G.

Nonhadron couplings are topologically much more complex than those between hadrons and lack duality. Purely nonhadronic *tree-graph* amplitudes are given by the Feynman rules of a massless Lagrangean consisting of $\mathrm{U}(2)_R \otimes \mathrm{U}(2)_L$ gauge theory with four lepton generations plus H boson and lepton-H terms to be discussed below. Now, the topological expansion gives priority to radiative corrections from hadrons, summed to infinite order, over any nonhadronic loops. At energies will below m_0 the hadronic effects lead to an effective Lagrangean with most symmetries broken[11] and substantial masses given to many nonhadrons. The concomitant softening of nonhadronic vertices and propagators at the m_0 scale, together with the small value of the fine-structure constant, is presumed to preclude divergences and generally to assure numerical smallness for purely nonhadronic loops; the high multiplicity of elementary hadrons[12] normally guarantees their dominant role in corrections to nonhadronic tree graphs. An exception is the mass of leptons, as we now explain.

While vector hexons generate H boson masses which are expected to approach m_0 because of the high hexon multiplicity, chiral symmetry protects leptons from vector-generated mass. The restrictions on the lepton numbers carried by H bosons can be shown to also prevent the onset of lepton mass generation. However, the topological meaning of hadron bosonic indices allows certain (neutral) *scalar* hexons to couple to *charged* leptons with $G = 4$. The resulting lepton mass m_λ is *a priori* expected to be of order 1 TeV. A recent study of an anomalous neutral scalar particle θ—neither hadron nor nonhadron and predicted by TPT several years ago[13]—suggests a coupling to charged leptons with $G = 3, 4$ (τ and λ). The mass generated by this single state would be smaller than m_λ by 2 or 3 orders of magnitude but still might contribute most of the τ mass.

Once λ has acquired a mass, so will all other leptons through nonhadron loops; hadronic radiative corrections will just shift these masses because they couple only to flavor- and generation-conserving lepton currents. Figure 1(a) shows how one H-boson loop generates masses for μ and τ while the electron mass requires two loops [Fig. 1(b)]. Neutrinos inherit their masses from charged leptons of the same generation via W bosons, but these diagrams are suppressed by the required mixing between W_L and W_R (Fig. 2) that we shall parametrize by an angle θ_χ in the following estimates (θ_χ is calculable in principle). As soon as $m_{\nu_\lambda} \neq 0$, H-boson loops contribute in the same way as for charged leptons.

We now make a few plausible assumptions that will enable us to estimate the masses of all leptons in terms of m_λ: (i) Vector hexon corrections to lepton masses are approximately proportional to these masses. (ii) The mass splitting Δm between μ and τ is due to the particle θ. (iii) The lepton-H coupling constant in the effective nonhadronic Lagrangean

$$\mathscr{L}^{\text{eff}}_{H\text{-lepton}} = -h\bar{\psi}^i_G \psi^i_{G'} H_{G'G} \tag{1}$$

(spinor indices are suppressed while isospin and generation indices are to be summed over) is of the same order of magnitude as the gauge-coupling constant and hence electric charge:

$$\alpha' \equiv h^2/4\pi \approx \alpha \equiv g^2/4\pi \approx \alpha_{\text{e.m.}}. \tag{2}$$

This assumption is supported by the high degree of symmetry between fermionic and bosonic nonhadron indices and by the anticipated common dynamical origin of g and h.[13] (It has long been known[4] that g is of

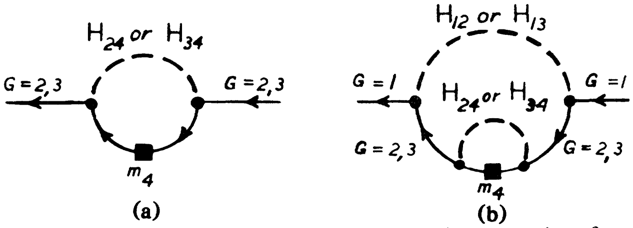

FIG. 1. Indirect lepton mass generation, starting from $m_4 = m_\lambda$.

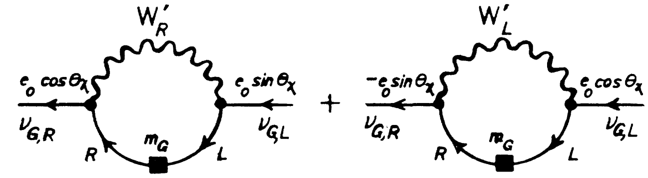

FIG. 2. Neutrino mass generation from charged lepton mass, via mixed W'_L, W'_R.

the same order of magnitude as TPT's dimensionless strong-interaction coupling constant.)

Evaluating the diagrams of Fig. 1 with a cutoff m_0 one obtains

$$m_\tau = m_\mu + \Delta m, \tag{3}$$

$$m_\mu \approx \frac{\alpha'}{4\pi} \ln\left(\frac{m_0^2}{m_H^2}\right) m_\lambda \equiv q m_\lambda, \tag{4}$$

$$m_e \approx \frac{\alpha'}{4\pi} \ln\left(\frac{m_0^2}{m_H^2}\right) (m_\mu + m_\tau) \approx q m_\tau. \tag{5}$$

Conversely, we may use the measured masses of e, μ, and τ to estimate m_λ and α' from (3)–(5) as a check on our expectations:

$$m_\lambda \approx m_\mu (m_\tau / m_e) \approx 400 \text{ GeV}, \tag{6}$$

$$\alpha' \approx \frac{4\pi}{\ln(m_0^2/m_H^2)} \frac{m_e}{m_\tau}; \tag{7}$$

indeed $\alpha' \approx \alpha_{\text{e.m.}}$ if $m_H \lesssim m_0$.

From Fig. 2 the neutrino masses are found to be

$$m_{\nu_\lambda} \approx \frac{3\alpha}{4\pi} \theta_\chi \ln\left(\frac{M_R^2}{M_L^2}\right) m_\lambda \equiv r m_\lambda, \tag{8}$$

$$m_{\nu_\tau} \approx r m_\tau + q m_{\nu_\lambda} \approx r m_\tau, \tag{9}$$

$$m_{\nu_\mu} \approx r m_\mu + q m_{\nu_\lambda} \approx 2 r m_\mu, \tag{10}$$

$$m_{\nu_e} \approx r m_e + q (m_{\nu_\mu} + m_{\nu_\tau}) \approx 2 r m_e. \tag{11}$$

θ_χ is the mixing angle between W_L and W_R, whose masses are M_L and M_R, respectively. From the present upper bound, $m_{\nu_e} < 50$ eV, we conclude that $r < 5 \times 10^{-5}$. With the further assumption that $M_R > 20 M_L$,[14] left-right mixing is restricted to $\theta_\chi < 2 \times 10^{-3}$. Neutrino masses may thus provide a crucial test for TPT.

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