New Gapless Modes in the Fractional Quantum Hall Effect of Multicomponent Fermions

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We derive the dispersion relation for a new gapless Goldstone mode in the fractional quantum Hall effect of a multicomponent electron gas. We expect these modes to be important for some of the observed filling factors (e.g., $\nu=1, \frac{1}{3}, \frac{1}{5}$) in multivalley semiconductors.

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Predictions for the ground state of the fractional quantum Hall effect (FQHE) and the associated elementary excitations for a wide range of filling factors ν, of the form $\nu=p/q$, have been almost exclusively confined to single-valley semiconductors.[1-3] In such systems recent calculations show the existence of an energy gap in the excitation spectrum which explains the fractional quantum Hall effect.[1,4] Interest in a multicomponent extension of these predictions has been largely applied to spin-up and spin-down electrons in the lowest Landau level.[5-7] Of course, the Zeeman splitting will always destroy the perfect symmetry between the two components.[5]

In this Letter we consider a multivalley (or equivalently multicomponent) semiconductor with *full* symmetry between the valleys. For a system which favors a valley polarized ground state (VPGS), we present a closed-form expression for the corresponding *gapless* Goldstone modes (GM) and discuss their important possible implications. We also present numerical studies for the two-dimensional multicomponent plasma (2DMCP) and for the two-dimensional multicomponent charge-density wave (2DMCDW) which support a preference for a VPGS at $\nu=\frac{1}{3}$ and $\frac{1}{5}$ in the inversion layer of Si.

The multivalley Hamiltonian in the lowest Landau level is, in its second quantized form,[8]

$$H=(1/2L^2)\sum_{q_1} v(q_1)\sum_{\alpha,\beta}[\rho_\alpha(q_1)\rho_\beta(-q_1)-\exp(-q_1^2/2)\rho_\alpha(0)]+H_{\mathrm{SB}}, \tag{1}$$

where $v(q_1)=e^2 2\pi/elq_1$, $l=(c\hbar/eH)^{1/2}$ (all lengths are scaled by l), and L^2 is the area of the inversion layer. $\rho_\alpha(q_1)$ is the Fourier transform of the density operator $\psi_\alpha^\dagger(r)\psi_\alpha(r)$, i.e.,

$$\rho_\alpha(q_1)=\int d^2r\, e^{iq_1\cdot r}\psi_\alpha^\dagger(r)\psi_\alpha(r), \tag{2}$$

where α is the valley index of the field operators $\psi_\alpha^\dagger(r), \psi_\alpha(r)$ in the lowest Landau level.[9] Finally H_{SB} is a symmetry-breaking term given by

$$H_{\mathrm{SB}}=\lim_{\gamma\to 0}\int d^2r\,(\gamma/2)[\psi_1^\dagger(r)\psi_1(r)-\psi_2{}^\dagger(r)\psi_2(r)]. \tag{3}$$

(Physically we think of γ as an *infinitesimal* strain which breaks the symmetry between the two valleys.) We next define the propagator $K_{\alpha\beta}$ given by

$$K_{\alpha\beta}(r-r',t)=i\langle 0[\psi_\alpha^\dagger(r,t)\psi_\beta(r,t)\psi_\beta^\dagger(r',0)\psi_\alpha(r',0)\theta^>(t)-\psi_\beta^\dagger(r',0)\psi_\alpha(r',0)\psi_\alpha^\dagger(r,t)\psi_\beta(r,t)\theta^>(t)]0\rangle \qquad (\alpha\neq\beta) \tag{4}$$

$[\theta^>(t)=1$ for $t>0$ and $\theta^>(t)=0$ for $t<0$; $\theta^<(t)=1-\theta^>(t)]$ and proceed to show that it contains the gapless GM discussed above and to derive their dispersion relation.

It is difficult to work with Eq. (4) in its present second quantized form since it does not explicitly exploit the analytical properties of the lowest Landau level. We note, however, that Eqs. (1)–(4) can be rigorously mapped to a spin system. Equation (4) can then be written *identically* as

$$K_{\alpha\beta}(r-r',t)=i\langle 0[S_-(r,t)|n\rangle\langle n|S_+(r',0)\theta^>(t)-S_+(r',0)|n\rangle\langle n|S_-(r,t)|\theta^>(t)]0\rangle, \tag{5a}$$

where

$$S_-(r,t)=\sum\nolimits_j\sigma_{-,j}(t)\delta(r-r_j(t))\text{ and }S_+=S_-^\dagger, \tag{5b}$$

with $\sigma_{\mp,j}=\frac{1}{2}(\sigma_{x,j}\mp i\sigma_{y,j})$ the usual Pauli matrices. Now the crucial point is that Eqs. (5) and (4) are equivalent *only* if the states $|n\rangle$ are *all* within the first Landau level. In addition the time dependence of $S_-(r,t)$ and $S_+(r,t)$, which is governed by H of Eq. (1), must also be projected to the lowest Landau level. This can be expeditiously enforced by the projection-operator technique.[10] We can then rewrite Eqs. (1)–(4) as

$$\overline{H}=\frac{1}{2L^2}\sum_{q_1}v(q_1)[I\overline{\rho}(q_1)I\overline{\rho}(-q_1)-\exp(-q_1^2/2)I\overline{\rho}(0)]+\lim_{\gamma\to 0}\gamma\sigma_z\overline{\rho}(0), \tag{6a}$$

$$\overline{\rho}(q_1)=\sum\nolimits_j\exp(iq_1\partial/\partial Z_j)\exp(iq_1^*Z_j/2), \tag{6b}$$

$$\overline{S}_-(q)=\sum\nolimits_j\sigma_{-,j}\exp(iq\,\partial/\partial Z_j)\exp(iq^*Z_j/2), \tag{6c}$$

and the Fourier transform of Eq. (5a) is

$$\overline{K}_{-,+}(q,t)=i\langle 0[\overline{S}_-(q,t)\overline{S}_+(-q,0)\theta^>(t)-\overline{S}_+(-q,0)S_-(q,t)\theta^>(t)]0\rangle. \tag{6d}$$

In Eqs. (6a)–(6d) both Z_j and q are complex numbers,[10] i.e., $Z_j=x_j+iy_j$ and $q=q_x+iq_y$, and I is the identity operator in spin space. Taking several time derivatives of Eq. (6d) and using the commutations with $\overline{H}$ to evaluate these time derivatives, we get

$$\lim_{\gamma\to 0}K_{-,+}(q,\omega)=\lim_{\gamma\to 0}\left(\frac{\overline{F}_1(q)}{(\omega-\gamma)}-\frac{\overline{F}_2(q)}{(\omega-\gamma)^2}-\frac{i}{(\omega-\gamma)^2}\chi_{\overline{J}_-,\overline{J}_+}(q,\omega)\right), \tag{7a}$$

where

$$\overline{F}_1(q)=\langle 0[\overline{S}_-(q)\overline{S}_+(-q)-\overline{S}_+(-q)\overline{S}_-(q)]0\rangle, \tag{7b}$$

$$\overline{F}_2(q)=\langle 0[\overline{J}_-(q)\overline{S}_+(-q)-\overline{S}_+(-q)\overline{J}_-(q)]0\rangle, \tag{7c}$$

$$\chi_{\overline{J}_-,\overline{J}_+}(q,t)=\langle 0[\overline{J}_-(q,t)\overline{J}_+(-q)\theta^>(t)-\overline{J}_+(-q)\overline{J}_-(q,t)\theta^>(t)]0\rangle, \tag{7d}$$

and

$$\overline{J}_-(q,t)=[\overline{S}_-(q,t),\overline{H}]. \tag{7e}$$

The GM $\omega(q)$ can now be identified from the small-q behavior of $K_{-,+}(q,\omega)$,

$$\lim_{q\to 0}\lim_{\gamma\to 0}K_{-,+}(q,\omega)\approx\frac{A(q)}{\omega-\omega(q)-\gamma}. \tag{8}$$

Comparison with Eq. (7a) yields

$$\omega(q)=\lim_{\gamma\to 0}[\overline{F}_2(q)+i\chi_{\overline{J}_-,\overline{J}_+}(q,\gamma)]/\overline{F}_1(q), \tag{9a}$$

for the dispersion of the GM at small q.[11] We can evaluate Eqs. (7b) and (7c) in closed form for a fully VPGS. We get

$$\overline{F}_1(q)=-Ne^{-q^2/2}, \tag{9b}$$

and after a somewhat lengthy calculation,

$$\overline{F}_2(q)=(N/2L^2)e^{-q^2/2}\sum\nolimits_{q_1}v(q_1)\{[\exp((q^*q_1-q_1^*q)/2)-1]+[\exp(-(q^*q_1-q_1^*q)/2)-1]\}(S(q_1)-1), \tag{9c}$$

where $S(q_1)$ is the structure factor for the single-valley FQHE.

Equations (9a)–(9c) constitute our main results. Clearly at small q the GM goes like q^2 provided[12]

$$\lim_{q\to 0}\lim_{\gamma\to 0}\chi_{\bar{J}_-,\bar{J}_+}(q,\gamma)\approx q^{2+\epsilon}.$$

For finite q,[11] Eq. (9c) is evaluated numerically with the use for $S(q_1)$ of the Laughlin-proposed[1] ground state (or equivalently the two-dimensional one-component plasma, 2DOCP), and the results are displayed in Fig. 1; we return to these results shortly.

To observe experimentally the effect of such gapless excitations on the FQHE it remains to determine whether and when a VPGS is preferable over a nonpolarized ground state (NPGS). We clearly cannot cover all choices of polarizations, ground-state candidates, or filling factors ν. Suggestions for commensurate ground states of higher ratios of p/q, in a single-valley system,[2] are particularly difficult to extend to multivalleys. Here we examine only a few of the lower and best established filling-factor ratios ν. For the ground state of the FQHE we take two standard candidates, the 2DCDW[8] and the Laughlin[1] 2DOCP, and extend them to multivalley systems. The 2DMCDW are the exact extension of Yoshioka and Lee Hartree-Fock calculations[8] to the multivalley Hamiltonian of Eq. (1). In Table I the first row corresponds to the fully VPGS.[13] The second row corresponds to a NPGS.[14] The third row corresponds to the extension of the Laughlin[1] ground state ψ to a NPGS as suggested by Halperin,[5] i.e.,

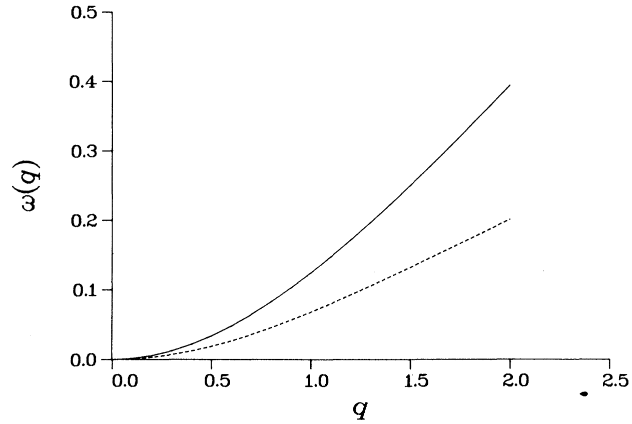

FIG. 1. The gapless Goldstone modes $\omega(q)$ for two different filling factors ν; $\nu=\frac{1}{3}$ for the solid curve and $\nu=\frac{1}{5}$ for the dashed curve. $\omega(q)$ is measured in units of $e^2/\epsilon l$ and q is scaled by l.

$$\psi=\prod_{i<j}(Z_i-Z_j)^m\prod_{l<k}(\tilde{Z}_l-\tilde{Z}_k)^m\prod_{i,k}(Z_i-\tilde{Z}_k)^n\prod_i e^{-|Z_i|^2/4}\prod_k e^{-|Z_k|^2/4}, \quad (10)$$

with m odd and n even, where Z_i and $\bar{Z}_k$ correspond to the positions of the particles in the two valleys. The state maps into a 2DMCP with a commensuration energy at $\nu=2/(m+n)$. We solve the 2DMCP, using the extension of the hypernetted-chain technique[15] (including bridge-function corrections for correlation effects at short distances[15]) to two components, for $m=3$, $n=2$ ($\nu=\frac{2}{5}$), and $m=5$, $n=2$ (or $\nu=\frac{2}{7}$). We also include the results without bridge functions in parentheses. To assess the accuracy of these new multivalley results we also list in row 4 the fully VPGS at $\nu=\frac{1}{3}$ and $\frac{1}{5}$. The starred numbers are the Monte Carlo results,[15] which as seen are basically identical to ours. Note that a Laughlin-type NPGS [Eq. (10)] cannot exist at $\nu=\frac{1}{3}$ or $\frac{1}{5}$.[16] From Table I we conclude that at $\nu=\frac{1}{3}$ and $\frac{1}{5}$ the fully VPGS of Laughlin is the lowest of the three. This conclusion is further confirmed by essentially exact Monte Carlo calculations of the 2DOCP.[15] At $\nu=\frac{2}{5}$ and $\frac{2}{7}$ Table I predicts a Laughlin-type NPGS. It is rewarding to note that similar conclusions were reached in a small-cluster calculation with no initial prejudice as to the form of the ground state.[7]

TABLE I. Energies per particle (in units of $e^2/\epsilon l$) for four different ground states of a two-component electron gas in the lowest Landau level at filling factors $\nu=\frac{1}{3}$, $\frac{1}{5}$, $\frac{2}{5}$, and $\frac{2}{7}$.

	$\nu=\frac{1}{3}$	$\nu=\frac{1}{5}$	$\nu=\frac{2}{5}$	$\nu=\frac{2}{7}$
Fully VPGS CDW	−0.3885	−0.3220	−0.4123	−0.3685
NPGS CDW	(−0.2204) −0.3823	(−0.1636) −0.3195	(−0.2491) −0.4013	(−0.2271) −0.3644
NPGS Laughlin state	Does not exist	Does not exist	(−0.434) −0.438	(−0.370) −0.372
Fully VPGS Laughlin state	−0.4100* (−0.406) −0.409	−0.3277* (−0.321) −0.327	Does not exist	Does not exist

ground state.[7]

We finally turn to the implication of the above results. First the polarized nature of the ground state at $\nu = \frac{1}{3}$ and $\frac{1}{5}$ implies the existence of gapless GM (of Fig. 1) in a multivalley semiconductor like Si at these filling ratios.[17] These excitations, according to Eq. (4), correspond to removal of an electron from one valley and addition of it to the other followed by complicated coherent scattering between them. It then follows, from Eq. (9), that a direct measurement of these excitations could shed important additional insight on the structure factor $S(q_1)$ for this fascinating quantum fluid. More important, however, is the obvious dissipation channel such GM provide.[18] If indeed the FQHE hinges on a gap in the excitation spectrum then we make the prediction that unlike in GaAs, the favorite filling factor $\nu = \frac{1}{3}$ should not show the usual strong features in the Hall conductance of a multivalley semiconductor. We expect such measurements to become available in the near future.

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[9]Here we only consider two components, i.e., $\alpha = 1, 2$. The extension to arbitrary α is not difficult.

[10]S. M. Girvin and Terrence Jach, Phys. Rev. B **29**, 5617 (1984).

[11]These results are, of course, rigorous only at small q; we will, however, take the liberty of extending them to finite q [see, e.g., A. Miller, D. Pines, and P. Nozières, Phys. Rev. **127**, 1452 (1962)].

[12]We can show that ϵ is indeed positive; details will be given elsewhere.

[13]These results are of course identical to Ref. 8.

[14]We in fact find the fully VPGS CDW to be always the lowest ground state. We remark, however, that this is not entirely a consequence of the exchange contributions. In a multicomponent system such a state is in close competition with a NPGS CDW, set on a square lattice, where the phases of the two components are chosen to form a uniform charge density (these are the results of row 2). Without such choice the fully VPGS CDW, as can be seen, is indeed the easy winner over the NPGS CDW. These numbers are given in parentheses.

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[16]This would require an odd n value in Eq. (10).

[17]Actually the 100 face of Si does not provide an absolutely perfect degeneracy between the valleys [see T. Ando, A. B. Fowler, and F. Stern, Rev. Mod. Phys. **54**, 437 (1982)]. The 110 would.

[18]We note that the dissipation channel couples to the disorder when the impurity potential scatters across intervalley or through two or more GM in intravalley scattering (i.e., higher order).