Hot Electrons and Energy Transport in Metals at Millikelvin Temperatures

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Using a new technique, we measure the energy-loss lifetime for electrons in a metal from 25 to 320 mK. Applying a small electric field, of the order of 1 μV/cm at 25 mK, causes carriers to heat above the temperature of the phonon bath. The temperature difference induced reflects the rate of energy transport between the electrons and lattice. The 1-ms energy-loss lifetime and 1-cm inelastic diffusion length we have measured at 25 mK are the largest ever obtained.

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At very low temperatures, the thermal coupling between electrons in a metal and their surroundings becomes extremely weak. A naive extrapolation of electron-phonon scattering rates measured above 1 K downward to 1 mK, and below, yields the prediction that electron inelastic lifetimes will range from minutes to hours. One expects that, as the temperature is lowered and electron-phonon coupling decreases, a point should be reached where other inelastic processes become important, processes playing a negligible role at higher temperatures. To date, however, there has been little investigation of energy transport in this domain.

A fundamental obstacle to such research has been the lack of a suitable probe of energy transport. Resistivity measurements, for example, show little temperature dependence even in the cleanest metal samples. Elastic scattering from residual impurities dominates weak temperature-dependent inelastic processes at very low temperatures. Inelastic lifetimes have recently been deduced from the temperature dependence of weak localization above roughly 100 mK. What is actually obtained, however, is a *phase-coherence* lifetime for the electrons, τ_ϕ. In general, τ_ϕ may differ from the *energy-loss* lifetime relevant to transport.[1,2]

With use of a new technique involving the generation of hot carriers, we are directly probing energy-loss lifetimes, τ_ϵ, for electrons in impure metal films at millikelvin temperatures. By application of a small electric field, of order 1 μV/cm at 25 mK, a measurable temperature difference between the electrons and the lattice is induced. This difference reflects the rate at which electrons lose the energy they absorb from the applied field, E.

Hot electrons are created when $E>0$ if energy transport between the carriers and metal lattice is weaker than that from the lattice to the surroundings. This occurs when $R_{ep}>R_K$, where R_{ep} is a phenomenological electron-lattice "thermal resistance"[3] and R_K is the Kapitza boundary resistance for (phonon-mediated) heat transport from the metal lattice to the substrate. In this situation, the electrons heat while the phonons stay cool.

In an applied field, the power absorbed by the electrons scales with the resistivity, ρ, of the metal. In the residual-resistivity regime, ρ is almost entirely controlled by the impurity-scattering rate, $\tau_{\rm imp}^{-1}$. For our metal films, with residual resistivity $\rho_0 \sim 0.3\times10^{-6}$ Ω cm, $\tau_{\rm imp}^{-1}$ exceeds the scattering rates for electron-phonon and electron-electron processes, τ_{ep}^{-1} and τ_{ee}^{-1}, by at least 5 orders of magnitude at 1 K. At lower temperatures this dominance becomes even stronger. The residual carrier mobility, μ, thus controls energy flow from the applied field to the electron gas in this temperature regime. Since μ results from temperature-independent elastic momentum scattering off of impurities, the resistivity remains (essentially) Ohmic, even as the electrons heat.

The consideration of processes controlling energy flow *out* of the electron gas, however, is a separate issue. Above 1 K, τ_{ep}^{-1} acts as the dominant *effective* source of inelastic scattering. Here "effective" is meant to imply efficiency in cooling the electrons relative to their surroundings. As discussed below, our work indicates that this continues to be the case down to 25 mK. Note that electron-electron collisions help to thermalize the electron gas internally, but are ineffective at cooling the electrons as a whole.

With sufficiently large electric fields, the system will cross over to a regime in which the electric field plays an important role in determining the electronic distribution function. The onset of this behavior occurs at a crossover field of order $E_{\rm co}=k_{\rm B}T/el_\epsilon(T_0)$, where T_0 is the lattice temperature (i.e., electron temperature at $E=0$) and $l_\epsilon(T)=[\mathscr{D}_{\rm el}\tau_\epsilon(T)]^{1/2}$ is the inelastic diffusion length for energy loss processes. $\mathscr{D}_{\rm el}=v_{\rm F}^2\tau_{\rm imp}/3$ is the elastic diffusion constant. Using τ_ϵ extrapolated downward from measurements at 1 K, we predict an extremely small crossover field at 25 mK of order 1 μV/cm.

As the electrons heat, the probability of phonon emission increases, ultimately allowing the attainment of a steady state. For $E >> E_{co}$, the field alone determines the electron temperature and energy-loss lifetime—little memory of the lattice temperature remains. When this holds, $k_B T^* \sim eEl_\epsilon(T^*)$. If the energy-loss lifetime obeys a power law $\tau_\epsilon(T)^{-1} = \alpha T^p$, then $T^* \sim E^{2/(2+p)}$. This was first argued by Anderson, Abrahams, and Ramakrishnan[4] and later independently discovered and more fully discussed by Arai.[5] For electron–acoustic-phonon scattering, $p = 3$, which leads to a $T^* \sim E^{2/5}$ relation for $E >> E_{co}$. Electron heating, because of this $\frac{2}{5}$ power, is much more persistent at small fields than is conventional Joule heating ($\propto E^2$).

We perform the experiment using a pair of interdigitated thin Cu film resistors (Fig. 1, inset) mounted on a millidegree nuclear demagnetization cryostat. Electron temperature is measured directly, by observation of current fluctuations.[6] A dc voltage applied across one resistor imposes the bias field causing the electrons to heat.[7] Phonon temperature in the metal lattice is obtained by measuring noise from the second (unbiased) resistor, which is tightly coupled thermally to the first. The ~ 1000-Å Cu films are deposited onto a single-crystal sapphire substrate at ~ 20 Å/sec in $\sim 10^{-6}$ Torr from 99.999% Cu with an electron-beam evaporator. The films are patterned by photolithography and a lift-off process using an electron-beam–generated Cr mask. The resistors thus fabricated have dimensions 1000 Å × 10 μm × 6 cm, and resistance $R = 196.70\ \Omega$ at 4 K. To the sensitivity of our measurement (< 10 ppm), R is temperature and field independent over the range 15 mK–10 K and 0–0.3 T. The typical high-temperature resistance is shown in Fig. 1. Since the substrate thickness ($\sim 250\ \mu$m) and resistor separation ($\sim 40\ \mu$m) are much smaller than the phonon mean free path in the substrate at millikelvin temperatures, the thermal gradient between the resistors should be negligible.

For $R = 100\ \Omega$ and $T = 20$ mK, the current noise generated by the resistors is quite small, roughly 10^{-13} A in a 1-Hz bandwidth. To make 1% measurements, a current-sensing system with a noise floor of 10^{-15} A in a 1-Hz bandwidth is required. We use thin-film dc SQUIDs, having a *coupled* energy sensitivity of $30\hbar$, at the front end to achieve this goal.[8] Two complete systems allow measurement of electron and lattice temperature. Integration bandwidths of order 6 kHz allow 1% measurements in ~ 0.5 sec. The "high"-resistance films are matched to the low-input-impedance dc SQUIDs by superconducting transformers without significant degradation of the system's energy sensitivity. Analysis of this technique and more complete experimental details are presented elsewhere.[9,10]

Our experimental results are depicted in Fig. 2, which shows the dependence of electron temperature upon applied electric field for the biased resistor. The experiments begin at zero field with the electrons and lattice in equilibrium at T_0. The inset of Fig. 2 contrasts the behavior of the biased and unbiased sensors at $T_0 = 25$ mK. The biased resistor shows a crossover to a power law of 0.41, close to $\frac{2}{5}$. As T_0 is varied, the

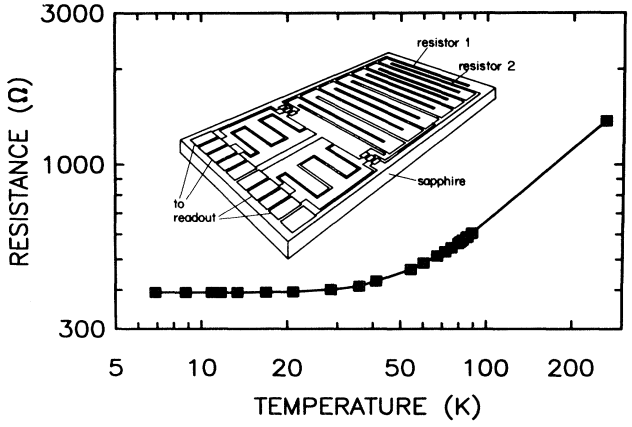

FIG. 1. Temperature dependence of the resistance of a thin-film resistor fabricated at the same time as the sample used in the experiment. Below ~ 50 K, impurity scattering dominates the phonon contribution to the resistivity. The inset depicts the resistor geometry.

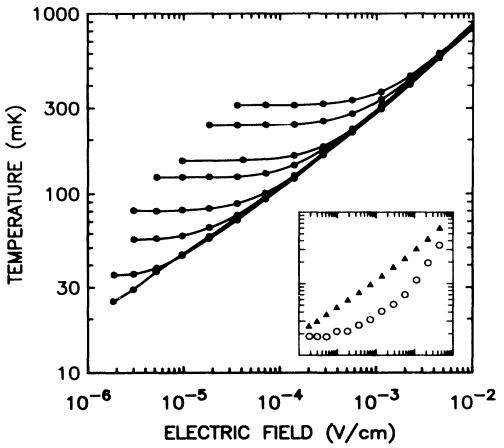

FIG. 2. Electron temperature vs applied electric field for the biased resistor. Once the crossover field is exceeded, electron temperature quickly loses memory of the lattice temperature T_0, and approaches a universal curve with slope of approximately $\frac{2}{5}$. The inset contrasts the behavior of the biased and unbiased resistors at 25 mK. The open circles depict the temperature rise in the unbiased resistor due to phonon-mediated heating while the triangles duplicate the 25-mK data from the large graph. Coordinates of the inset are identical to those of the larger graph.

crossover point (here defined as $T^*=2T_0$) progressively increases. Figure 3 shows the temperature dependence of this crossover field. A power-law dependence close to $T_0^{5/2}$ is observed for the biased sample. The exponent is insensitive to the precise definition of crossover, as expected. As seen in the inset of Fig. 2, the onset of *field*-induced electron heating in the biased resistor occurs much earlier than the phonon-mediated heating in the resistor which is not biased.

A more quantitative analysis of the data requires an understanding of electron heating in the regime near crossover, $E > E_{co}$. We present a simple model based on a comment of Anderson, Abrahams, and Ramakrishnan,[4] who note that $R_{ep}^{-1}(T^*) \sim C_{el}(T^*)/\tau_{in}(T^*)$. In the present context, we are careful to identify the inelastic lifetime, τ_{in}^{-1}, as the rate for *energy loss* from the electron gas, τ_ϵ^{-1}. $C_{el}(T^*)$ is the electronic heat capacity evaluated at T^*, the effective temperature of the electrons.[11] For small fields, $d\dot{Q}_{in}=2E\,dE/\rho_0$ and $d\dot{Q}_{out} \sim R_{ep}^{-1}(T')\,dT'$, where T' is the electron temperature. Energy balance requires $d\dot{Q}_{in}=d\dot{Q}_{out}$ in steady state. Integrating the expressions for $d\dot{Q}_{in}$ from zero field to E, and $d\dot{Q}_{out}$ from T_0 to T^*, gives

$$\ln\left[\left(\frac{T^*}{T_0}\right)^{2+p}-1\right]^{1/(2+p)} = \frac{2}{2+p}\ln\left(\frac{E}{T_0^{(2+p)/2}}\right)-\frac{1}{2+p}\ln\left(\frac{\alpha\gamma\rho_0}{(2+p)}\right). \quad (1)$$

In Fig. 4 we show data for all starting temperatures, which have been scaled with this model. If we assume $p=3$, appropriate for bulk electron–acoustic-phonon processes as indicated by Figs. 2 and 3, the agreement is excellent for all temperatures studied (25–320 mK). The last term of Eq. (1) is particularly significant. This intercept is the sole parameter determining the fit to the data and gives the product $\alpha\gamma\rho_0$ directly, where $\alpha=\tau_\epsilon^{-1}T^{-3}$, $\gamma=C_{el}/T$, and ρ_0 is the resistivity. Since electronic specific heats are well understood and the resistivity is measured, we can extract α, the strength of the energy relaxation process. We find $\alpha \sim 9\times10^7$ $\sec^{-1}$ K^{-3} over the range of this work. This is consistent with the values measured in clean bulk metals at much higher temperatures by cyclotron and other related resonance techniques.[12] The angular average of τ_{ep}^- that we sense is dominated by the fastest rates, occurring at the necks of the Fermi surface.

The inset of Fig. 2 indicates that phonon-mediated heating of the unbiased resistor becomes appreciable at the highest applied fields. We make several observations regarding its role in our experiment. First, it should be noted that in Eq. (1), which describes the onset of electron heating, the initial lattice temperature T_0 is quickly dominated by T^* once the carriers begin to heat. If $R_{ep} > R_K$ the electrons will begin to heat before the lattice heats appreciably. Electron heating will become important below a temperature of order T_{max}, at which $R_{ep} \sim R_K$. From our data we find $R_{ep}VT^4 \sim 5\times10^{-4}$ K^5 cm^3/W. Acoustic mismatch theory[13,14] predicts $R_K T^3 A \sim 20$ K^4 cm^2/W. Here V and A are the sample volume and surface area. For our 1000-Å Cu films we find $T_{max} \sim 600$ mK. T_{max} is inversely proportional to film thickness. As shown,

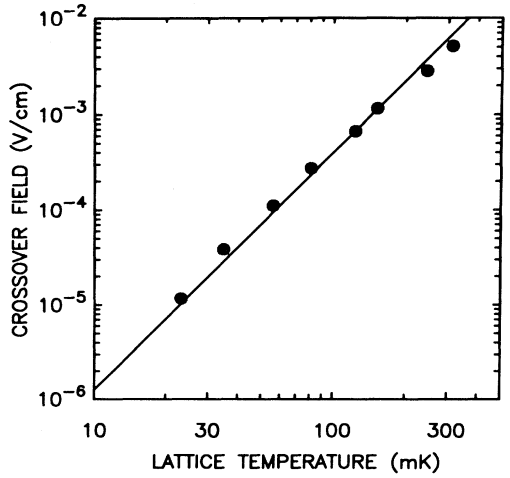

FIG. 3. Temperature dependence of the crossover field, defined as $T^*(E_{co})=2T_0$. A $\frac{5}{2}$-power law, shown by the solid line, is in good agreement with the data. A least-squares fit gives slope 2.3. This indication of a scattering rate exponent $p=3$ suggests that the electron–acoustic-phonon process dominates electron energy loss.

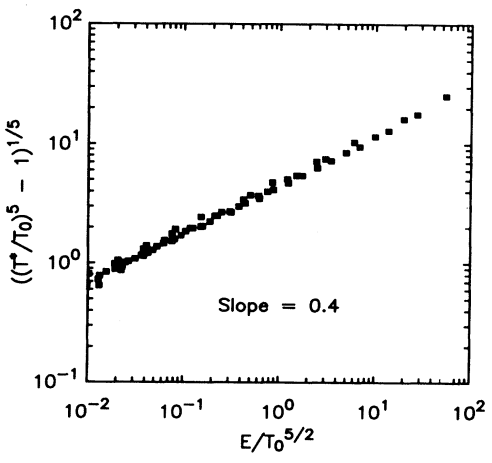

FIG. 4. Data scaled by the simple heating model [Eq. (1)]. Electron temperature is measured in kelvins and the electric field in volts per centimeter. The data fit a straight line with slope $\sim \frac{2}{5}$. The intercept, $b=\log_{10}(\alpha\gamma\rho_0/5)/5$, gives the ratio $\alpha\gamma\rho_0$. Here, $b=0.66$, which implies $\alpha\gamma\rho_0=30$ $\sec^{-1}$ K^{-5} J m^{-2} Ω.

the data follow a $\frac{2}{5}$-power law very closely for low lattice temperatures. For larger T_0 and for high fields we find a curvature away from the slope of $\frac{2}{5}$. The onset of this curvature is seen in Fig. 2 for fields > 1 mV/cm, where $T^* > 200$ mK.

We conclude that, for $T_0 < T_{\max}$, electron-heating will dominate phonon-heating effects as E is increased until the point where overall heating causes $R_K \sim R_{ep}$. Our conclusion is strengthened by additional experiments in which the sample was immersed in a bath of cold liquid ^{3}He while keeping all other thermal clamping to the cryostat the same. The phonon heating in this case is substantially reduced while the electron-heating phenomenon remains virtually unaffected. For $T_0 \sim 25$ mK, our cryostat and measurement system allow input power to be swept over more than 8 orders of magnitude. The practical limit upon the largest E we use is ultimately determined by the cooling power available from our apparatus subject to maintenance of temperature regulation of the resistor lattice.

The striking results of our investigation are the extremely long energy-loss lifetimes and inelastic diffision lengths we measure, much larger than quantities previously deduced from weak localization experiments. This difference arises from the fact that, using our electron heating technique, we measure $\tau_{in} \propto \tau_\epsilon$, whereas weak localization measures a $\tau_{in} \propto \tau_\phi$. The shortest inelastic lifetime will always determine τ_ϕ and destroy the coherence of the electron wave function. τ_ϵ, however, requires a coupling mechanism between the carriers and their surroundings—at low temperatures this evidently becomes exceedingly weak.

We have been able to induce and measure electron heating in impure metal films at low temperatures, using fluctuations as a probe of carrier temperature. Hot-electron effects are produced because heat flow into and out of the biased sensor proceed via different mechanisms. Elastic electron-impurity collisions determine $\dot{Q}_{in}(E)$, while the energy-loss rate $\dot{Q}_{out}(T^*(E))$ is constrained by less frequent electron–acoustic-phonon scattering events. The 1-ms energy-loss lifetime we have observed at 25 mK is several orders of magnitude longer than any electronic relaxation time ever measured. The corresponding inelastic diffusion length is $l_\epsilon \sim 1$ cm.[15] Future work will probe the region below 25 mK where, if $\tau_\epsilon T^3 = \text{const}$ continues to hold, energy-loss lifetimes can range from minutes to hours.

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