Couplings and Scales in Superstring Models

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The domains of weak coupling of superstring theories are identified. Compactifications on Calabi-Yau spaces encounter a problem related to vacuum stability. Although the string theory may be weakly coupled and a semiclassical approximation may be valid, both theoretical and phenomenological arguments indicate that the nonlinear σ model on the string world sheet must be strongly coupled.

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Superstring theories[1] have no dimensionless parameters.[2] However, they do have a perturbation expansion. Classically, and in perturbation theory, string theories have a large set of physically inequivalent, degenerate vacua. These states are labeled by the vacuum expectation value (VEV) of the ten-dimensional "dilaton" field, ϕ, and also by VEV's of fields which describe the size and shape of possible compactified dimensions. These VEV's may serve as expansion parameters.[2] It is the purpose of this note to clarify the nature of the perturbation expansions, and to determine the regimes of weak coupling. We will see that there are two senses in which the theory may be weakly coupled. The VEV of the dilaton field determines whether or not the string theory itself may be treated semiclassically. The size of the internal manifold, measured in string units, determines whether or not the corresponding nonlinear σ model on the string world sheet is weakly coupled. It is logically possible to be in a semiclassical regime for the string theory while in a regime of strong coupling for the σ model. In such a situation, there is no energy range in which the theory looks like a ten-dimensional field theory. We will argue that the true (nonperturbative) ground states of string theory either lie at flat, ten-dimensional space and/or vanishing couplings, or they lie in a regime where the σ model, and possibly the full string theory itself, are strongly coupled.

The low-energy limit of string theory can be described by a field theory of massless particles. The Lagrangian of this theory, even classically, contains operators of arbitrarily high dimension obtained by the integrating out of massive modes of the string. This field theory must be viewed, of course, as a cutoff theory, with a cutoff of order the string scale, $M_S \sim \alpha'^{-1/2}$. (In particular, ghosts and other objects which may appear in this theory at scales of order M_S are to be ignored; they exist in a momentum regime where the field-theory description is inappropriate.)

The $d=10$ supersymmetric Lagrangian, with terms up to second order in derivatives, has been obtained by Bergshoeff *et al.*[3] for the Abelian case, and generalized to the non-Abelian case by Chapline and Manton[4] (an additional term required for the supersymmetry of the non-Abelian Lagrangian has been obtained by Dine, Rohm, and Witten[5]). With a rescaling of the fields in this Lagrangian by appropriate powers of κ and g_{10}, the ten-dimensional gravitational and gauge couplings, respectively, the bosonic terms are

$$\frac{1}{e}L_B^{(10)} = -\frac{1}{2\kappa^2}R - \frac{1}{4}\frac{\phi^{-3/4}}{g_{10}^2}F_{\mu\nu}^2 - \frac{3}{4}\frac{\kappa^2}{g_{10}^4}\phi^{-3/2}H_{\mu\nu\rho}^2 - \frac{9}{16}\frac{1}{\kappa^2}\left(\frac{\partial_\mu\phi}{\phi}\right)^2. \tag{1}$$

It is clear that by rescaling ϕ we may eliminate g_{10} from this Lagrangian, and measure all dimensionful quantities in units of $M_p^{(10)} = \kappa^{-1/4}$ (this definition is not conventional). The Lagrangian then contains only one length scale and no dimensionless parameters. Also, ϕ (or some power of it) acts as a coupling constant.

The metric here has been defined so that there are no powers of ϕ in front of the gravitational term. In particular, the graviton has a canonical kinetic term. The scale $M_p^{(10)}$ thus appears fundamental. With this convention, the scale M_S, which is the cutoff for the field theory, is ϕ dependent; perturbation theory in the different vacua labeled by ϕ appears to be cut off at different scales. To see this, it is convenient to work with a field, D, defined by $\phi = \exp[(\frac{8}{9})^{1/2}\kappa D]$. D has a canonical kinetic term; this is the field which creates properly normalized single-dilaton states. Both in the Veneziano model and in superstring theory, a zero-momentum D insertion is proportional to the free string action, with proportionality κa. A change in D just corresponds to a change in M_S (this is most easily seen in the functional integral formulation,[6] i.e., $\partial M_S^2/\partial D = a\kappa M_S^2$ or $M_S^2 = M_S^2(D=0)\,e^{a\kappa D}$. To determine the constant a for the various superstring theories, it is not necessary to perform an actual string computation; we need simply note certain well-known facts. For the heterotic string,[7] $\kappa = g_{10}/M_S$. (For notational simplicity, we define M_S so that the proportionality constant here is 1). For this to be consistent with the ϕ dependence of the gauge coupling implicit in Eq. (1), we require $M_S^2 = \phi^{3/4} M_p^{(10)2}$. For type-I strings, $\kappa = g_{10}^2 M_S^2$, so that $M_S^2 = \phi^{-3/4} M_p^{(10)2}$.

It is natural, instead, to view M_S as fundamental. In Eq. (1), we can rescale the metric, and thus rescale our units of length, so that M_S is ϕ independent. Consider, first, the heterotic string. If we take $g_{\mu\nu} \to \phi^{-3/4} g_{\mu\nu}$, and also rescale the fermion fields according to $\psi_\mu \to \phi^{-3/16}\psi_\mu$, $\chi \to \phi^{3/16}\chi$, and $\lambda \to \phi^{3/16}\lambda$, the ten-dimensional Lagrangian becomes

$$\frac{1}{e} L^{(10)} = \phi^{-3}\left[-\tfrac{1}{2}R - \tfrac{1}{4}F_{\mu\nu}^2 - \tfrac{3}{4}H_{\mu\nu\rho}^2 + \frac{9}{2}\left(\frac{\partial_\mu \phi}{\phi}\right)^2 + \text{fermionic terms}\right]. \tag{2}$$

Here we have set $M_S = 1$. While we have not indicated all of the terms in the Lagrangian explicitly, we must stress that with these field redefinitions, there are no powers of ϕ in the brackets. We see that the perturbation expansion is an expansion in ϕ^3. For not only does ϕ^{-3} sit in front of the whole Lagrangian, but M_S is the only scale appearing here, and it is the cutoff. One can show the same thing in the full string theory using, e.g., the functional integral formation. It is clear that an expansion in ϕ^3 in either the field theory or the string is an expansion in loops. The limit $\phi \to 0$ is the semiclassical limit of these string theories. Indeed, there is no power of ϕ in front of the counter terms added by Green and Schwarz[8] to cancel the anomalies, as they are a one-loop effect.

This rescaling also illuminates other features of the string theory and its low-energy limit. In particular, Witten[9] has noted that the classical equations of motion of string theory possess a scale symmetry under which ϕ and $g_{\mu\nu}$ are rescaled. In the old variables, $\phi \to \lambda\phi$, $g_{\mu\nu} \to \lambda^{-3/4} g_{\mu\nu}$. Under this transformation, the Lagrangian itself is multiplied by a constant. The new metric we have defined above is invariant under this transformation; only ϕ transforms. In the new Lagrangian, this symmetry is manifest. Witten's observation ensures that, even if we include some number of massive modes, or higher-dimension operators (obtained by integrating them out classically), the Lagrangian can still be cast in this form, i.e., with ϕ^{-3} out front.

It is intriguing that, with these redefinitions, the supersymmetry transformation laws in the limiting field theory look simpler; in particular, they do not contain explicit powers of ϕ. For example, $\delta A_\mu = \frac{1}{2}\bar{\epsilon}\Gamma_\mu \chi$. (Here ϵ has also been rescaled, $\epsilon \to \phi^{-3/16}\epsilon$.) Perhaps these variables hint at a more natural formulation of string theory.

For type-I strings, things are not so simple. As noted above, in this case, $M_S^2 = \phi^{-3/4} M_p^{(10)2}$. Rescaling the metric so that M_S is ϕ independent yields the Lagrangian

$$\frac{1}{e} L^{(10)} = -\tfrac{1}{2}\phi^3 R - \tfrac{1}{4}\phi^{3/2} F_{\mu\nu}^2 - \tfrac{3}{4} H_{\mu\nu\rho}^2 - \dots \tag{3}$$

This is not homogeneous in ϕ. Loosely speaking, loops of gauge particles go as $\phi^{-3/2}$, while gravitational loops go as ϕ^{-3} (reflecting the original relation $\kappa \sim g^2$). Also, weak coupling here corresponds to *large* ϕ. Clearly expansion in powers of ϕ does not correspond to expansion in loops. This is a reflection of the well-known fact that certain open string loop diagrams can be deformed into tree diagrams with closed string exchanges.

There are also fields whose VEV's label different sizes and shapes for the possible internal spaces. (We have in mind, for example, the compactifications of Candelas *et al.*,[10] and Witten[11] and Strominger and Witten,[11] to $M^4 \times K$, where K is a Calabi-Yau space.) We focus, again, on the heterotic string, and work in terms of the rescaled variables. Consider, first, dilations of the internal manifold. If we call $g_{ij} = X g_{ij}^0$, where g_{ij}^0 is some fixed, reference metric [such that $\int d^6x (g^0)^{1/2} = M_S^{-6}$], then $X^{1/2}$ is essentially the radius of the internal space measured in string units. A simple computation also shows that $X^{-1/2}$ is the unification scale, in string units: $X^{-1/2} = M_{\rm GUT}/M_S$.

The string action is $(1/4\pi\alpha')\int d^2\xi\, g_{IJ}\partial_\alpha X^I \partial^\alpha X^J + \dots$. Note that the metric appearing here is the rescaled metric of Eq. (2), not the canonical one, and so α' is independent of ϕ. We see from this expression that X^{-1} is the coupling constant of the nonlinear σ model on the world sheet. For large X (large manifolds or small $M_{\rm GUT}$, in string units), the σ model is weakly coupled; for small X (small manifolds), the σ model is strongly coupled. Note that it is perfectly possible to have a weakly coupled string theory, i.e., a theory in which complicated world-sheet topologies may be ignored, with a small internal manifold. The problem, in such a case, is to solve a strongly coupled, two-dimensional field theory; string theory, however, is semiclassical. The regime of large X corresponds to a situation where the unification scale is much less than M_S; the "Kaluza-Klein" states are lighter than the other string excitations. In the regime of small X (and small ϕ), the Kaluza-Klein excitations are as heavy as the other string excitations; there is no regime in which the theory can be described as a ten-dimensional field theory. Note that, for $\phi \ll 1$,

$M_p^{(10)} = M_S \phi^{-3/8}$ is larger than both $M_{\rm GUT}$ and the cutoff M_S and gravity is indeed semiclassical.

Thus we see that upon compactification, we have two expansion parameters. ϕ controls the size of quantum corrections. X determines the importance of higher-dimension operators in the effective ten-dimensional field theory (at either the classical or quantum level). For large X, the four-dimensional Lagrangian has the form

$$\frac{1}{e} L^{(4)} = Y(-\tfrac{1}{2}R - \tfrac{1}{4}F_{\mu\nu}^2 + \ldots), \tag{4}$$

where $Y = X^3\phi^{-3}$. Y and X are massless fields in four dimensions (in Refs. 5 and 9, X and Y were called T and S, respectively). Note that $Y^{-1} = g_{\rm GUT}^2$, the gauge coupling at the GUT scale. Also, $M_p^{(4)} = \kappa^{(4)^{-1}}$ and $M_{\rm GUT}$ are related to ϕ, X, and M_S through $M_P^{(4)^2} = YM_S^2$ and $M_S^2 = XM_{\rm GUT}^2$. From renormalization-group arguments,[12] we expect that Y is of order 1 at $M_{\rm GUT}$. Since $\phi^3 = Y^{-1}(M_S/M_{\rm GUT})^6$, if ϕ is to be small and the string perturbation expansion is to be reasonable, $M_{\rm GUT}$ cannot be significantly less than M_S. (The fact that $M_{\rm GUT}$ cannot be much less than $M_p^{(4)}$ has already been noted by Kaplunovsky using a somewhat different argument.[13] This means that X is necessarily greater than or of order 1. The σ model cannot be too weakly coupled.

To construct a realistic theory of compactification, we must first assume that we are in the semiclassical regime ($\phi \ll 1$); otherwise, the string is strongly coupled and we cannot hope to attack the problem at present. Reference 10 suggests a particular family of compact configurations—Calabi-Yau spaces. This suggestion is based on two independent arguments, both of which assume that X can be arbitrarily large:

(1) Space-time supersymmetry in four dimensions: For $X \gg 1$, it is sufficient to consider the low-dimension terms in the $d = 10$ Lagrangian and in the supersymmetry transformation laws. The equations for unbroken supersymmetry can be analyzed perturbatively in X^{-1}, and their solutions are the Calabi-Yau spaces.

(2) World-sheet conformal invariance for every X, required for consistency of string theory, dictates zero β function to every order in the coupling constant, X^{-1}. This again leads to Calabi-Yau spaces. A closely related argument shows that these manifolds are solutions to the classical equations of motion, to all orders in X^{-1}.

Since these two requirements are satisfied for *every* X, X, like ϕ, is massless and has no potential at tree level; this is almost certainly true in perturbation theory as well. For large X and Y, the analysis of Refs. 10 and 11 is justified, the minimal $d = 10$ Lagrangian is a good approximation, and all corrections, both classical and quantum, are small. However, as shown in Ref. 5, nonperturbative effects in the second E_8 (if it has an unbroken, non-Abelian subgroup) generate a potential for the fields X and Y, which tends to zero for large Y and/or large X—weak coupling and/or large manifolds. The true vacuum is thus flat ten-dimensional space and/or zero gauge coupling.

It must be stressed that this instability forces the system to the domain in which the approximations are more and more reliable. Thus, if string theory is to describe the real world, then either at small X, where our methods break down, there is a minimum of the potential with vanishing cosmological constant and broken SUSY, or there are such vacua among field configurations other than those of Ref. 10. One possible modification was suggested in Ref. 5: a nonvanishing background H field proportional to the covariantly constant three-form. In this case, there does exist a vacuum state in the lowest nontrivial order, with broken supersymmetry and vanishing cosmological constant. In this order, the field Y is determined in this state while the field X is not. This sounds very promising. However, $\langle H \rangle$ is the coefficient of a Wess-Zumino term in the σ model, and hence it is quantized.[14] As a result, several problems arise:

(1) The nonlinear σ model on the world sheet is no longer conformally invariant for every value of X. At best, there are some discrete points where this is true.[14] X is then of order 1, and the compact manifold is small.

(2) The gluino condensate and the scale of the other E_8 are of order $M_{\rm GUT}$, and there is thus no energy range where the field-theoretical analysis of Ref. 5 is valid. In hopes of obtaining at least a qualitative picture, we can pretend that $\langle H \rangle$ is small. It was shown in that paper that, in this case, the effects of gluino condensation, at low energies, could be described in terms of an effective field theory for the X, Y, and gauge fields. Ordinary matter fields can be included along the lines of Ref. 9. At tree level, all soft breaking terms for the ordinary fields vanish. At one loop in this (cutoff) theory, these masses are generated. Their precise values depend on the cutoff, which in turn depends on details of the string theory. However, if $\langle H \rangle$ and X are of order 1, these corrections are of order $M_P^{(4)}$ (up to powers of $g_{\rm GUT} = Y^{-1/2} \sim 1$) and *supersymmetry does not lead to a gauge hierarchy.*

In conclusion, we have identified the fields whose VEV's play the role of the string coupling (ϕ^3 for the heterotic string and $\phi^{-3/2}$ for type-I strings) and the σ-model coupling (X). For the string to be weakly coupled (to have a valid semiclassical approximation), ϕ must be small (large, for type-I strings). We have seen that since the unified gauge coupling cannot be too much less than 1, if ϕ is to be small, X must be less than or of order 1.

We were forced to small X by other considerations as well. For large X, the solutions of the classical theory are the field configurations on Calabi-Yau manifolds described in Refs. 10 and 11. However, nonperturbative effects lift the degeneracy among these vacua, and cause a runaway to flat ten-dimensional space and/or zero coupling.[5] This instability may be avoided if the other E_8 is broken to an Abelian subgroup [e.g., $U(1)^8$]. In this case, however, it is unclear how supersymmetry breaking is to arise. Alternatively, it is possible that the theory has a stable vacuum at $X \sim 1$ where our approximations break down, or that the compact manifold is not a Calabi-Yau space and exists at an isolated point in field space, not connected to flat M^{10}. Correspondingly, the σ model would only be conformally invariant for isolated values of X. If such a configuration exists, and in addition leaves an unbroken $N=1$ supersymmetry in four dimensions, it might also provide a solution to the hierarchy problem.

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