Naturally Light Higgs Doublet in Supersymmetric E_6 Grand Unified Theory

Ashoke Sen
Fermi National Accelerator Laboratory, Batavia, Illinois 60510
(Received 20 March 1985)

We show how some supersymmetric E_6 grand unified theories, which may arise as the low-energy limit of superstring theory, provide a natural solution of the fine-tuning problem.

PACS numbers: 12.10.En, 11.30.Pb, 14.80.Gt

Although supersymmetry may protect a large mass hierarchy in a theory against radiative corrections,[1] it does not explain why such a hierarchy is present in the first place. In particular, it does not explain why the weak-doublet Higgs field is so light compared to its color-triplet partner. In this paper I propose a simple mechanism for obtaining light Higgs doublets in supersymmetric E_6 grand unified theories. The model is motivated by the recent observation that supersymmetric E_6 grand unified theories may appear as the low-energy limit of superstring theories.[2-4]

The model that we shall consider contains chiral superfields $\phi(\mathbf{78})$, $H(\mathbf{27})$, and $\tilde{H}(\mathbf{27}^*)$, with the superpotential

$$W = M_1^3 \operatorname{Tr}_{27} \cos(\phi/M) + \alpha\phi H\tilde{H}, \tag{1}$$

where, for simplicity, we have ignored all the E_6 group indices. Here Tr_{27} denotes the trace in the **27** representation of E_6. Although the superpotential given in (1) seems *ad hoc,* its general features that we shall use may come naturally in the low-energy limit of superstring theories. For example, this model has supersymmetric minima at eigenvalues of ϕ which are integral multiples of some minimum mass πM. This is a general feature of theories in which the breaking of E_6 occurs as a result of a twisted gauge-field configuration in certain higher dimensions.[3,5] Also, I have omitted mass terms of the form $MH\tilde{H}$ from W, thereby assuming that in the absence of any background ϕ field, $H,\tilde{H}$ remain massless. Existence of massless fields belonging to $\mathbf{27}, \mathbf{27}^*$ representations is again a general feature of theories which appear as the low-energy limit of superstring theories.[3] As we shall see, it is only these two properties of the theory which will be relevant for generating large mass hierarchy in the theory.

In order to analyze the model given by the superpotential (1), let us first note that E_6 has $\mathrm{SU}(2) \otimes \mathrm{SU}(6)$ as one of its maximal subgroups, under which different representations of E_6 transform as

$$\mathbf{78} = (\mathbf{1},\mathbf{35}) \oplus (\mathbf{3},\mathbf{1}) \oplus (\mathbf{2},\mathbf{20}), \tag{2a}$$

$$\mathbf{27} = (\mathbf{2},\mathbf{6}^*) \oplus (\mathbf{1},\mathbf{15}). \tag{2b}$$

The condition for the potential obtained from (1) to have a supersymmetric minimum is

$$\frac{\partial W}{\partial \phi} = \frac{\partial W}{\partial H} = \frac{\partial W}{\partial \tilde{H}} = 0, \tag{3}$$

which may be satisfied, if, for example,

$$\langle H \rangle = \langle \tilde{H} \rangle = 0, \tag{4a}$$

$$\langle \phi \rangle_{(1,35)} = \pi M \begin{pmatrix} n_1 &&&&& \\ & n_1 &&&& \\ && n_1 &&& \\ &&& n_2 && \\ &&&& n_2 & \\ &&&&& -(3n_1+2n_2) \end{pmatrix} \tag{4b}$$

with all other elements zero,

$$\langle \phi \rangle_{(3,1)} = \pi M \begin{pmatrix} n_3 & 0 \\ 0 & -n_3 \end{pmatrix}, \tag{4c}$$

$$\langle \phi \rangle_{(2,20)} = 0, \tag{4d}$$

where the n_i's are integers.[6] This is the most general minimum consistent with an unbroken symmetry group that contains $\mathrm{SU}(3) \otimes \mathrm{SU}(2) \otimes \mathrm{U}(1)$ as its subgroup. The fields belonging to the $(\mathbf{2},\mathbf{6}^*)$ representation may now be split into a color antitriplet $(\mathbf{3}^*)$, a weak doublet $(\mathbf{2})$, and a color and weak SU(2) singlet $(\mathbf{1})$ part, each part also carrying an index $\uparrow$ or $\downarrow$ showing the SU(2) quantum numbers. The masses of various fields are given by

$$\begin{aligned} M_{3^*\uparrow} &= \alpha\pi M(-n_1+n_3), \\ M_{3^*\downarrow} &= \alpha\pi M(-n_1-n_3), \\ M_{2\uparrow} &= \alpha\pi M(-n_2+n_3), \\ M_{2\downarrow} &= \alpha\pi M(-n_2-n_3), \\ M_{1\uparrow} &= \alpha\pi M(3n_1+2n_2+n_3), \\ M_{1\downarrow} &= \alpha\pi M(3n_1+2n_2-n_3). \end{aligned} \tag{5}$$

Similarly, the fields belonging to the $(\mathbf{1},\mathbf{15})$ representation of $\mathrm{SU}(2) \otimes \mathrm{SU}(6)$ have a color antitriplet component $(\mathbf{3}^{*\prime})$, a color triplet component

(3′), a weak doublet part (2′), a component which transforms as weak doublet and color triplet (2′, 3′), and a component (1′) which is a singlet under the color SU(3) and weak SU(2) group. The masses of these particles may again be read directly from their quantum numbers, and are as follows:

$$M_{3^{*\prime}}=\alpha\pi M 2n_1,\quad M_{(2',3')}=\alpha\pi M(n_2+n_1),$$
$$M_{3'}=\alpha\pi M(-2n_1-2n_2), \tag{6}$$
$$M_{2'}=(-3n_1-n_2)\alpha\pi M,\quad M_{1'}=\alpha\pi M(2n_2).$$

Let us now consider a particular minimum where

$$n_3=n_2=-3n_1. \tag{7}$$

Then

$$M_{2\uparrow}=M_{1\downarrow}=M_{2'}=0, \tag{8}$$

whereas all the other fields acquire masses of order M. Thus from the **27** representation, we get a massless chiral superfield which is a singlet of $SU(3)^c\otimes SU(2)^w$, and two massless chiral fields which transform as doublets of $SU(2)^w$. The **27*** representation gives rise to massless chiral superfields belonging to complex conjugate representations. The vacuum expectation values (vev) of these light fields break the remaining gauge group to $SU(3)^c\otimes U(1)^{em}$ at some scale below the grand-unified-theory scale. I expect that after the effect of supersymmetry breaking is taken into account, the $2\downarrow$ and $2'$ fields will acquire vev of order m_w, whereas the $1\downarrow$ component should acquire a vev order of 10^{10} GeV, at least in the case of four generations of quark-lepton fields, for reasons to be discussed shortly.[7]

The quark fields in one generation may be taken to belong to a single **27** representation of E_6, with a coupling to the Higgs field of the form

$$\lambda QQH. \tag{9}$$

To see how the quark fields get mass, we may decompose the **27** representation in terms of its transformation properties under the $SU(2)\otimes SU(6)$ subgroup. Thus, for example, the field Q may be taken to consist of the fields $Q_{6^*,i}$ and $Q_{15'}$, where i is the SU(2) index. Equation (9) may then be written as

$$\lambda\{a\epsilon_{ij}Q_{6^*,i}Q_{6^*,j}H_{15}+b\epsilon_{ij}Q_{6^*,i}Q_{15}H_{6^*,j}+cQ_{15}Q_{15}H_{15}\}, \tag{10}$$

where a, b, and c are constants. We may now further decompose the fields in terms of their SU(5) quantum numbers, under which **6*** decomposes as a **5*** and a singlet, whereas **15** decomposes as a **5** and a **10**. The vev of the $1\downarrow$ component coming from $H_{6^*,\downarrow}$ then produces a mass term of the form $Q_{5^*,\uparrow}Q_5$. We assume this mass to be at least of order 10^{10} GeV, in order that there are not too many light fields in the theory, and that the gauge coupling constants do not blow up to infinity before reaching the grand unification scale. We are then left with light fields $Q_{5^*,\downarrow}$ and Q_{10} belonging to the **5*** and **10** representations of SU(5), as well as two light fields $Q_{1,\uparrow}$ and $Q_{1,\downarrow}$ belonging to the singlet representation. [Although SU(5) is not a good symmetry of the theory at low energy, the light quark-lepton fields belong to full multiplets of SU(5), and hence it is more convenient to describe the low-mass fields in terms of their SU(5) transformation properties.] The vev of the $2\uparrow$ component (of order m_w), which comes from $H_{6^*,\uparrow}$, then produces a mass term of the form $Q_{5^*,\downarrow}Q_{10}$, giving mass to the d quark and the electron, whereas the vev of the $2'$ component, which comes from the H_{15}, produces a mass term of the form $Q_{10}Q_{10}$, giving mass to the u quark.

The vev of the $2'$ component, however, also gives mass to the neutrino contained in $Q_{5^*,\downarrow}$ by coupling it to the light singlet field $Q_{1,\uparrow}$. Another way to see the existence of neutrino mass is to note that the unbroken subgroup below the grand-unified theory scale is

$$SU(3)\otimes SU(2)_L\otimes SU(2)_R\otimes U(1)\otimes U(1),$$

so that light right-handed neutrinos arise naturally in this model. Although this is not obvious by looking at the decomposition of various fields according to their transformation properties under the $SU(2)\otimes SU(6)$ subgroup, this becomes clear if we look at the decomposition of ϕ according to irreducible representations of the $SU(3)\otimes SU(3)\otimes SU(3)$ subgroup of E_6. Under this decomposition, the vev of ϕ may be written as

$$\langle\phi\rangle=2\pi M\left[\begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix}\oplus\begin{pmatrix}n_1&0&0\\0&n_1&0\\0&0&-2n_1\end{pmatrix}\oplus\begin{pmatrix}n_1&0&0\\0&n_1&0\\0&0&-2n_1\end{pmatrix}\right], \tag{11}$$

where the three matrices refer to the generators of the three SU(3) subgroups. The vev of the $1\downarrow$ field will break the symmetry to

$$SU(3)^c\otimes SU(2)_L\otimes SU(2)_R\otimes U(1),$$

which, in turn, breaks down to $SU(3)^c \otimes U(1)^{em}$ by the vev of the $\mathbf{2}\uparrow$ and $\mathbf{2}'$ fields.

Besides solving the problem of having a massive neutrino, we also have to find a way of breaking the symmetry between the $SU(2)_L$ and $SU(2)_R$ groups. Both these problems, however, may be solved if we assume that there exists another Higgs field belonging to the adjoint representation of the theory, whose vev breaks the $SU(2)_R$ group at the grand-unified theory scale, thereby also opening up the possibility of having a large mass for the right-handed neutrino. This keeps the left-handed neutrino almost massless. This new Higgs field, however, must not couple to the $H,\tilde{H}$ fields, so that the mass hierarchy is preserved.

Note added.—After completion of this work I found some work by Witten[8] and Breit, Ovrut, and Segre[9] who have also noted the existence of naturally massless Higgs doublets in superstring theories. I wish to thank H. Tye and M. Dine for drawing my attention to these papers. The possibility of the existence of an intermediate-scale symmetry breaking has also been discussed by Dine *et al.*[10]

[1]E. Witten, Nucl. Phys. **B188**, 513 (1981); S. Dimopoulos and H. Georgi, Nucl. Phys. **B193**, 150 (1981); N. Sakai, Z. Phys. C **11**, 153 (1981); R. K. Kaul, Phys. Lett. **109B**, 19 (1982).

[2]M. B. Green and J. H. Schwarz, Phys. Lett. **149B**, 117 (1984), and California Institute of Technology, Report No. Calt-68-1194 (unpublished); D. J. Gross, J. Harvey, E. Martinec, and R. Rohm, Phys. Rev. Lett. **54**, 502 (1985). See also, P. G. O. Freund, Phys. Lett. **151B**, 387 (1985); J. Thierry-Mieg, to be published.

[3]P. Candelas, G. T. Horowitz, A. Strominger, and E. Witten, University of California, Santa Barbara, Report No. NSF-ITP-84-170 (unpublished).

[4]Original proposal for considering E_6 as a possible unifying gauge group was made by F. Gürsey, P. Ramond, and P. Sikivie, Phys. Lett. **60B**, 177 (1976); F. Gürsey and P. Sikivie, Phys. Rev. Lett. **36**, 775 (1976); P. Ramon, Nucl. Phys. **B110**, 214 (1976).

[5]The symmetry breaking in these models is obtained by giving a vev to

$$P\exp\left(i\oint_\gamma A_m dy^m\right) \equiv \exp(i\phi/M')$$

while keeping the E_6 field strength to be zero. Here y_m are the coordinates of the internal manifold, and γ is a noncontractible loop. In the manifolds considered in Ref. 3, although γ is noncontractible, if we travel along γ a certain number (n) of times, we get a contractible loop. Thus $[\exp(i\phi/M')]^n = 1$, which shows that ϕ is quantized. Similar mechanism for gauge symmetry breaking has also been considered by Y. Hosotani, Phys. Lett. **126B**, 309 (1983), and **129B**, 193 (1984).

[6]Actually, the value of ϕ where all the n_i's are half-integral also gives a supersymmetric minimum.

[7]This may occur naturally in theories with a light singlet field coupled to $H,\tilde{H}$. If supersymmetry is broken in the hidden sector at a scale of order 10^{10} GeV, then after the inclusion of radiative corrections, some light components of $H,\tilde{H}$ may acquire vev of order 10^{10} GeV. For a discussion of the instability of mass hierarchy in the presence of a light singlet field, see J. Polchinski and L. Susskind, Phys. Rev. D **26**, 3661 (1982); H. P. Nilles, M. Srednicki, and D. Wyler, Phys. Lett. **124B**, 337 (1982); A. B. Lahanas, Phys. Lett. **124B**, 341 (1982); M. Dine, *Lattice Gauge Theories, Supersymmetry, and Grand Unification, Proceedings of the Sixth Johns Hopkins Workshop on Current Problems in Particle Theory, Florence, 1982* (John Hopkins University, Baltimore, 1982).

[8]E. Witten, "Symmetry Breaking Patterns in Superstring Models" (to be published).

[9]J. D. Breit, B. A. Ovrut, and G. C. Segre, University of Pennsylvania Report No. UPR-0279T (unpublished).

[10]M. Dine, V. Kaplunovsky, M. Mangano, C. Nappi, and N. Seiberg, "Superstring Model Building" (to be published).