Noise Scaling in Continuum Percolating Films

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(Received 25 March 1985)

Measurements of the scaling of $1/f$ noise magnitude versus resistance were made in metal films as the metal was removed by sandblasting. This procedure gives an approximate experimental realization of a Swiss-cheese continuum-percolation model, for which theory indicates some scaling properties very different from lattice percolation. The ratio of the resistance and noise exponents was in strong disagreement with lattice-percolation predictions and agreed approximately with simple continuum predictions.

PACS numbers: 72.70.+m, 05.40.+j, 05.70.Jk, 71.30.+h

The critical scaling of various transport properties on percolating clusters provides a probe of the structure of those clusters. Recently, attention has been drawn to the possibility that the scaling of the mean square fluctuations S_R in electrical resistance may provide information not obtainable from the scaling of R, the resistance itself.[1,2] In particular, since S_R is sensitive to a higher moment of the current density distribution than is R it may be among those transport properties[3–5] for which the universality of scaling found in lattice models and many continuum models breaks down, according to theory. In this Letter we report measurements of the scaling of S_R vs R in a simple experimental system which show unambiguously that the lattice models are inapplicable to continuum systems and which approximately confirm theoretical expectations for a simple continuum model.

Monte Carlo simulations[1,6] of percolating clusters on a lattice (consisting of identical resistors with independent fluctuations) yield resistance and noise critical exponents $\kappa = 1.12$ and $\beta_L = -0.973$, where the exponents are defined by $R \sim \xi^{-\beta_L} \sim \Delta^{\nu\beta_L}$ and $S_R/R^2 \sim \Delta^{-\kappa}$, with ξ the percolation coherence length and $p = p_c + \Delta$ the filling fraction. Halperin *et al.*[3] have recently suggested that the permeability and elasticity exponents for the Swiss-cheese model, in which round holes with a fixed radius and randomly placed centers are removed from a material, are larger than those for the standard lattice model, while the resistance exponent $\nu\beta_L$ should not differ significantly. These Swiss-cheese calculations were made with a nodes-links-blobs (NLB) model.[7] The agreement of the resistance exponent with the lattice value is also consistent with previous works,[4,5] which predict deviations only for singular distributions of single-link conductivities.

Whether a quantity scales the same in a continuum model as in lattice models depends on how sensitive it is to the behavior of the weakest or narrowest links in the network. In other words, it depends on what moment of the current density, strain, etc., is probed by that quantity. Although the NLB model is not especially accurate for β_L, which probes the second moment of the current density and thus depends on blobs as well as links, it is expected to become increasingly accurate for higher moments, since the scaling of the number of links is known exactly. Recent experiments have shown that the conclusion that β_L is unchanged in Swiss-cheese-like continuum percolation is correct.[8] Noise from local, independent sources (such as $1/f$ noise in metal films) probes the fourth moment of the current density, as opposed to R, which probes the second. The Swiss-cheese model would predict that κ would be greater than 1.12, by an amount that we shall calculate later.

We made resistance-versus-noise measurements on sandblasted metal films. A dry sandblasting process[9] was used to remove, at random, approximately 2-μm-diam regions from a uniform metal film, leaving behind a two-dimensional Swiss-cheese structure (see Fig. 1). This technique is well suited for the fabrica-

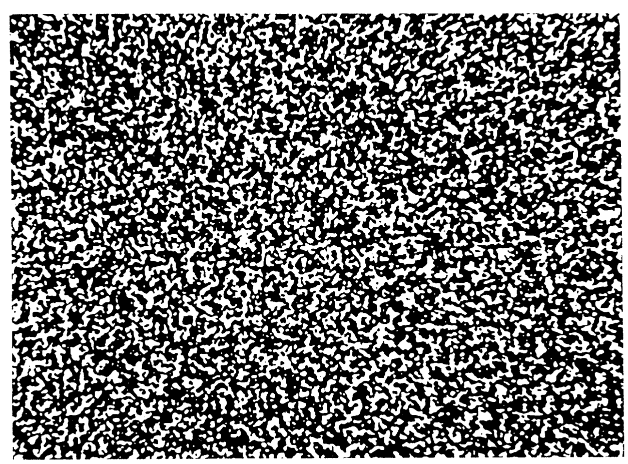

FIG. 1. Contrast-enhanced transmission-illuminated photomicrograph of typical sandblasted film. Dark areas correspond to the presence of metal. Because of the finite resolution of the microscope the exact amount of dark vs light depends on how the picture is developed, and thus it is not suited for determining $p - p_c$, although it gives a good idea of the shape of the conducting region.

tion of percolating networks, since a complete spectrum of p values can be realized sequentially on a single sample without altering the film's local resistivity or any chemical properties likely to affect the noise. Furthermore, the sandblasted films have no obvious reasons to have nonrandom constraints on their local hole distributions, unlike clumped evaporated films in which one expects local surface-tension effects to determine the neck distribution. However, the sandblasted films have two drawbacks, in that it is hard to get a good enough measure of the coverage to measure $\nu\beta_L$ directly and there is some possibility that nonuniformities in the blasting process could cause some unevenness in the coverage.

Indium films (1000–2000 Å), aluminum films (500–1000 Å, evaporated in 10^{-4} Torr air), and chromium films (500–1000 Å), all deposited on glass substrates, were found to have suitable sandblasting properties. We used 10-μm-diam aluminum oxide grit, with larger diameter (more massive) grit resulting in considerably larger holes. Square sample regions 1–3 mm per side were physically masked and blasted by sweeping the blasting nozzle over the mask window. Small sample regions and special precautions in the masking and blasting techniques were necessary to minimize nonuniformities in p. Two samples were prepared by blasting a long narrow channel 10 mm×2 mm with a broad circular spot, leaving a $1-p$ profile which was relatively uniform across the channel width, but roughly Gaussian along its length, the direction of current.

Since the details of the noise scaling (in continuum models) can depend on such factors as whether the edges have sharp corners, whether the noise comes from the entire area of the conducting film or mainly from its edges, and whether the sandblasting scoops out divots of metal down to the glass or thins metallic spots, we used films that have a range of properties to check the universality of the results. The partially oxidized Al and the In were known to be very noisy materials for which noise throughout the area was expected to be dominant. The Cr is a relatively quiet material for which edge effects could be more likely. As judged from scanning electron microscopy results, both Cr and Al samples exhibited all-or-nothing spots, with no significant film thinning, while the In seemed to be smeared about the sandblasting.

A given curve of S_R vs R gives data taken on a single sample, with each data point measured between successive blasting steps. A representative selection of plots is given in Fig. 2. The full noise-power spectrum had a typical $1/f$ form, with S_R measured over the 1–35-Hz bandwidth. The rms noise in the Al samples was tested for linearity and found to be linear in current up to values eight times those used in the exponent measurements (4 mA). Samples were fabricated with separate current and voltage leads, with problems due to contact noise usually being negligible. A lower limit of measurable values of S_R (upper limit for Δ) was set by the background Johnson noise, and was still in the scaling regime. The upper limit of the scaling regime, set by intrinsic finite-size effects, was usually accessible, though large statistical fluctuations dominated the data here as expected. Beyond this upper limit, where a rolloff in the noise is predicted,[1] samples usually went insulating or were unstable. No reliable data exist for this regime close to threshold, though a rolloff in the noise was observed in three samples.

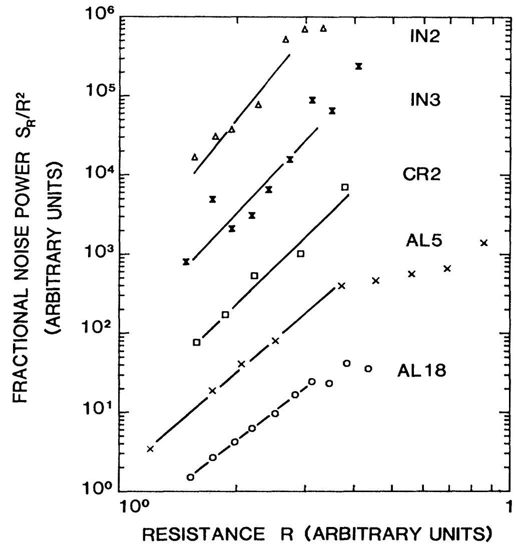

FIG. 2. The scaling of S_R/R^2 vs R for several samples.

Table I lists the exponents $Q = d(\ln S_R)/d(\ln R)$ determined for nine different samples, fabricated from three different metals on two types of glass substrates. For all samples the accessible part of the scaling regime spanned no more than 1.5 octaves in R values. We should emphasize that all these data were taken close to threshold, with only a small fraction of the total sandblasting giving the resistance range shown. The resistances were not in a convenient range for noise measurements until the transition was approached. Exponents were determined by a least-squares fit of a straight line to the logarithm of the data, excluding the clearly nongeneral finite-size behavior very close to threshold discussed below. A majority of the samples had exponents in the range 5.4–6.2, with values occasionally higher but never lower than this range.

The noise in two samples showed a tendency to scale more strongly in R for high R, very close to threshold, than for low R (see In2 and In3 of Fig. 2). This scatter in the exponent ratio, which becomes severe only very

TABLE I. Scaling exponents for the noise as a function of resistance in percolating sandblasted metal films.

Sample	Exponent(±): $d(\ln S_R)/d(\ln R)$	Correlation coefficient for straight-line fit
Al5	6.2(0.1)	0.998
Al7	6.2(0.4)	0.988
Al9[a]	6.2	. . .
Al12[b]	7.5(0.3)	0.999
Al18[c]	5.85(0.07)	0.999
In2	8.1(0.7)	0.976
	6.0(0.3)[d]	0.994
In3	7.3(1.1)	0.902
Cr2[c]	6.9(0.5)	0.987
Cr3	5.4(0.5)	0.933

[a]Two data points.
[b]Metal film evaporated on preblasted glass substrate.
[c]Prepared with approximately Gaussian variation for $1-p$.
[d]Computed for first four data points (see Fig. 2).

close to threshold, is presumably due to both the inevitable large fluctuations near threshold plus some systematic effects due to nonuniformities in p, to be discussed.

Although their two-dimensional Swiss-cheese model seems roughly appropriate for describing our samples, Halperin *et al.* do not calculate noise-scaling exponents. The procedure for doing so is somewhat different from that for calculating other exponents in that one must first consider how the current distributes itself to minimize the resistance, then use that distribution to obtain the noise scaling. Without going into great detail we may outline an approach that makes sense *a priori* and gives predictions similar to our results.

Following Halperin *et al.*[3] we calculate the noise exponent in an NLB model. We allow[3] for uniformly distributed neck widths, δ, with corresponding neck lengths proportional to $\delta^{1/2}$, giving resistances proportional to $\delta^{-1/2}$. The noise power from such a neck would scale as $\delta^{-5/2}$. The resistance then has almost no dependence on the small-δ necks while the noise depends strongly on them. (If the geometry is less regular than for an ideal Swiss-cheese model these scaling laws can be modified slightly.)

Above threshold, the current through the smallest necks falls off inversely with their resistance, $\delta^{1/2}$, and these do not give a divergent contribution to the noise. The key to determining the noise scaling is to find the scaling of δ_m, the size of the smallest necks through which almost as much current flows as would flow if the necks were of equal size, with Δ. The current, of course, is distributed so as to minimize the power dissipation. We may approximate the adjustment of the current from that for a simple lattice model by treating this adjustment as negligible above δ_m and as completely eliminating the current below δ_m. This approximation allows for an exact calculation of the dependence of the resistance on δ_m, within the NLB model, giving the δ_m which minimizes R as scaling as Δ^2. A different approximation, within the same model, would be to consider that the resistance of a particular link does not affect the current through that link until it becomes comparable to the node-to-node resistance with which it is in series. Considering only resistance from links in an NLB model, this would again give δ_m proportional to Δ^2, while using a corrected resistance scaling gives δ_m proportional to $\Delta^{-2\nu\beta_L}$ or $\Delta^{2.60}$. The difference between these two estimates gives a good idea of the uncertainty due to the approximation of the NLB model, and so we take δ_m proportional to $\Delta^{2.3\pm0.3}$.

In calculating some exponents, Halperin *et al.* use a δ_m proportional to Δ, determined by asking what the typical minimum δ is on a fixed backbone. We do not believe that such a procedure is appropriate for calculating the δ_m for the noise problem.[10]

The Swiss-cheese model, handled with the NLB picture, then predicts S_R scaling as the number of links (Δ^{-1}) multiplied by the coherence area ($\Delta^{-2\nu}$), which determines the number of independent chains averaged together, and by $\delta_m^{-3/2}$, which results from integrating the noise over neck widths. Then

$$Q=[1+2\nu+1.5\times(2.3\pm0.3)]/(-\nu\beta_L)$$

or $Q=5.47\pm0.35$. A Swiss-cheese model with δ_m proportional to Δ would give $Q=(1+2\nu+1.5)/(-\nu\beta_L)\approx3.97$. Simulations of the lattice model[1,6] give $Q=2.86$, while an NLB lattice model for the noise would give $Q=(1+2\nu)/(-\nu\beta_L)=2.82$.

These estimates must be corrected, however, to allow for possible effects of nonrandom nonuniformities in p caused by slight irregularities in the sandblasting. When variations δp in p are small over distances of order ξ, well-defined values for the resistivity and noise in terms of the local coarse-grained $p(x,y)$ are presumed to exist. For $\delta p/\Delta \ll 1$, the sample is essentially homogeneous, and $R\sim\Delta^{\nu\beta}$ and $S_R\sim\Delta^{2\nu\beta-\kappa}$ are valid expressions. For δp larger than Δ, most of the current can be confined to high-density channels, while most of the resistance and noise can occur in the lowest-density passes in those channels.

For variations with $\delta p/\Delta \gg 1$, we define $\delta p(x,y)=c_1x^n-c_2y^m$, where x is the direction of current and y is the orthogonal axis. For linear variations $n=1$, for quadratic variations $n=2$, etc. The integrals for R and S_R can be put in dimensionless form, assuming that c_1 and c_2 are each either zero or large enough so that the noise and resistance are dominated by regions whose boundaries are determined by the inhomogeneities. This yields $R \sim \Delta^{\nu\beta_L + (1/n) - (1/m)}$ and $S_R \sim \Delta^{2\nu\beta_L\kappa + (1/n) - (3/m)}$ where the n and m terms are included only if the corresponding coefficient is nonzero. Thus, for the expected value $\nu\beta_L \approx -1.3$, series-type nonuniformities ($c_2 = 0$) with small n and $\delta p/\Delta >> 1$ can lead to dramatic increases in the measured exponent ratio Q. If $\delta p/\Delta \sim 1$ anywhere in the scaling regime, the exponent will change with the ratio $\delta p/\Delta$, and increase as threshold is approached. Inclusion of parallel-type nonuniformities ($c_2 \neq 0$) results in smaller deviations which lower the exponent ratio toward the homogeneous value.

The values of Q in Table I, then, are likely to be higher than those for the corresponding uniform systems. The most plausible nonuniformities for our samples would be smooth random patchiness, with $n = m = 2$, giving an increase in Q of $-1/\nu\beta_L$ or about 0.77. In this case the nonuniformities have no effect on the resistance scaling, but do cause the effective active area to shrink as Δ goes to zero, increasing the normalized noise intensity. If we consider the homogeneous case to give the probable minimum Q, the patchy case to give a good likely estimate, and the smooth ($n = 2$) series variations to give a probable maximum estimate, the values of the three theoretical predictions become $6.2 \pm^{2.2}_{0.9}$ for our version of the Swiss-cheese model, $4.7 \pm^{1.1}_{0.8}$ for the Swiss-cheese model with δ_m taken from Halperin *et al.*, and $3.6 \pm^{0.5}_{0.8}$ for the lattice model.

All our data are clearly inconsistent with anything like the lattice model even if we make unreasonably large allowances for inhomogeneities. Thus we have confirmed that the continuum percolation problem is fundamentally different from the lattice problem for experimentally accessible exponents. All of our samples gave results consistent with our version of the Swiss-cheese model. Only two of the nine samples gave results consistent with δ_m proportional to Δ. Thus we also have evidence that the appropriate narrow-neck cutoff δ_m in the Swiss-cheese resistor scales as predicted from the power-minimization principle in an NLB model, i.e., approximately as Δ^2, not as Δ. However, definitive resolution of such relatively subtle questions would require more precise theory, probably using continuum simulations, as well as more detailed characterization of the distribution of the sizes and shapes of our conducting patches.[11]

We wish to thank A.-M. S. Tremblay, S. Feng, B. I. Halperin, R. Koch, and R. Laibowitz for very helpful conversations as well as for sharing their results with us prior to publication. This work was supported by National Science Foundation Grants No. DMR 83-16981 (through the Materials Research Laboratory) and No. DMR 83-04470.

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[11]R. Koch and R. Laibowitz have unpublished data on clumped evaporated gold films indicating $Q = 4$. Their films look very different from ours, and so the different results may further indicate the nonuniversality in the continuum problems.