A Priori Definition of Maximal CP Nonconservation

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We define maximal CP nonconservation as occurring when a unique convention-independent parameter t, a quartic function of the Kobayashi-Maskawa matrix V, is maximized. The maximum value is much greater than the experimental upper limit, and so the observed CP nonconservation is much less than maximal. Maximal CP nonconservation corresponds to maximum mixing of the quark generations, just as maximal parity nonconservation corresponds to maximum mixing of the vector and axial-vector interactions.

Parity nonconservation was discovered in 1957. Soon after, several authors suggested that parity conservation might be violated "maximally." "Maximal" parity nonconservation means that the vector and axial-vector currents occur with equal normalizations and equal coupling constants in the fundamental Lagrangean of weak interactions. This suggestion proved to be correct. It is incorporated in the standard model by the fact that only the left-handed current couples to the W boson.

This success led to the suggestion that CP nonconservation, discovered in 1964, might also be maximal. In the Kobayashi-Maskawa (KM) framework,[1] CP nonconservation is associated with imaginary parts of elements of the KM matrix V. Maximal CP nonconservation might then correspond to some element, or some term in some element of V, being purely imaginary, or, in other words, to a term having a phase of $\pm \pi/2$. Several authors have considered this point of view.[2]

The situation for this notation of maximal CP nonconservation differs, however, from that for maximal parity nonconservation. Since a left-handed spinor field remains left handed under chiral transformations, the latter is an invariant concept. The former notion is not invariant, since the phases of elements of V can be changed by a change of the phases of the quark fields. Physical states in Hilbert space are rays; thus observables must remain unchanged under such rephasing of quark fields. Roos[2] and Gronau and Schechter[2] tried to avoid this difficulty by finding a parametrization (the Murnaghan construction[3]) in which a certain sum

of phases, the "invariant" phase, remains invariant under most (the similarity transformations) rephasings of the quark fields. Unfortunately, whether or not present data allow the invariant phase to be $\pm \pi/2$ depends not only on the convention of adoption of the Murnaghan construction, but, in addition, on a further convention: the order in which the matrices in the Murnaghan construction are multiplied. Thus the statement that the invariant phase is $\pm \pi/2$ is not convention independent.

One could adopt, as an alternate definition of maximal CP nonconservation, that choice of V which maximizes CP nonconservation in a specific process, for example, the choice of V which maximizes ϵ_K or ϵ_{B_d} (the ϵ parameter in the K^0-$\overline{K}^0$ system, where K^0 is $d\bar{s}$, or in the B_d^0-$\overline{B}_d^0$ system, where B^0 is $d\bar{b}$). Such a definition is process dependent, and is not analogous to the universal definition of maximal parity nonconservation.

We record the widely held view that whether or not CP nonconservation is maximal can only be decided when one knows the fundamental origin of CP nonconservation. We share the view that the KM framework is not a fundamental theory of CP nonconservation; rather the KM framework provides a description of CP nonconservation. Nonetheless an *a priori* definition of maximal CP nonconservation at the KM level may be useful.

A definition of maximal CP nonconservation should be convention independent and universal, i.e., process independent. We propose such a definition which uses a single parameter, t, defined below. When t vanishes,

CP is conserved in all processes. When t attains its maximum value (allowing an arbitrary three-generation KM matrix V) CP conservation is violated maximally. We find below that present data show that CP nonconservation is much less than maximal.

To give our definition of maximal CP nonconservation, we must first review the convention-independent formulation of CP nonconservation and of weak interactions generally.[4] The convention-independent functions of V which occur in weak-interaction rates are $|V_{ij}|^2$. Not all N^2 of these (for the case of N generations) are independent. The $2N-1$ independent conditions from the diagonal elements of the unitarity equations $V^\dagger V = 1$ and $VV^\dagger = 1$ reduce the number of independent quadratic parameters to $(N-1)^2$. These can be chosen to be

$$T^{i\alpha} = \mathrm{Tr}\, V^\dagger \lambda^i V \lambda^\alpha, \quad i,\alpha = 3, 8, 15, \ldots, N^2 - 1,$$

where λ^i or λ^a are Gell-Mann matrices. This is a complete set of convention-independent functions of V. Although these parameters are all real, they may implicitly require CP nonconservation. Nonetheless, we would like to find an imaginary parameter which explicitly requires CP nonconservation.

For this purpose, we consider quartic functions of V. In the three-generation case, CP nonconservation can be parametrized in terms of the nine convention-independent complex quantities

$$\Delta_{i\alpha} = V_{j\beta} V_{k\gamma} V_{j\gamma}^* V_{k\beta}^*,$$

with i,j,k and α,β,γ cyclic. One can equally well use the convention-independent quantities

$$T^{i\alpha j\beta} = \mathrm{Tr}\, V^\dagger \lambda^i V \lambda^\alpha V^\dagger \lambda^j V \lambda^\beta,$$

$$i,\alpha,j,\beta = 3, 8, 15, \ldots, N^2 - 1$$

for N generations. These obey $T^{i\alpha j\beta} = T^{j\beta i\alpha}$ and $T^{i\alpha j\beta *} = T^{j\alpha i\beta}$. For three generations, there are ten of these, namely, T^{3333}, T^{3338}, T^{3383}, T^{3388}, T^{3883}, T^{3838}, T^{8383}, T^{8838}, T^{8883}, and T^{8888}. All of these are real, except $T^{3388 *} = T^{3883}$. Thus there are ten parameters, nine real and one imaginary, associated with these quartic T's. For three generations, the relation between the Δ's and the traces is $\Delta_{i\alpha} = \mathrm{Tr}(V^\dagger \times \Lambda_j V \Lambda_\beta V^\dagger \Lambda_k V \Lambda_\gamma)$, i,j,k and α,β,γ cyclic, where the Λ's are projection operators in generation space; for example, $\Lambda_2 = \mathrm{diag}(0,1,0)$. The projection operators are sums of the diagonal matrices.[5] Using this relation, we showed that all the Δ's have the same imaginary part,

$$t \equiv \mathrm{Im}\,\Delta = \tfrac{1}{12} \mathrm{Im}\, T^{3388} = c_1 c_2 c_3 s_1^2 s_2 s_3 s_\delta,$$

using the KM parametrization. Thus there are also ten parameters, nine real and one imaginary, associated with the Δ's. The convention-independent parameter t controls all CP nonconservation in the KM framework.

When t vanishes, CP is conserved in all processes.[6]

We propose using t as the parameter which characterizes maximal CP nonconservation. We define maximal CP nonconservation as occurring when t assumes its maximum value, given any KM matrix V. This definition of maximal CP nonconservation is universal, and, like the usual definition of maximal parity nonconservation, it is an *a priori* definition, independent of the experimental situation. Experimental information is not used to formulate this definition of maximal CP nonconservation, but rather to determine whether or not maximal CP nonconservation is realized in nature.

We now calculate the maximum value of t and the form of V at the maximum. Since t is convention independent, we can use any parametrization of V to calculate its maximum value. Using the KM parametrization, we find

$$t = c_1 c_2 c_3 s_1^2 s_2 s_3 s_\delta.$$

The maximum value of t occurs at $c_1 = 1/\sqrt{3}$, $c_2 = 1/\sqrt{2}$, $c_3 = 1/\sqrt{2}$, and $s_\delta = 1$. The value is

$$T_{\max} = 1/6\sqrt{3}.$$

This value is much greater than the observed upper limit

$$t_{\mathrm{obs}} \lesssim 3 \times 10^{-4}.$$

Thus the observed CP nonconservation is much less than maximal. The KM matrix for the maximal case is

$$V_{\max} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & -1 & -1 \\ 1 & -x & -x^2 \\ 1 & -x^2 & -x \end{pmatrix},$$

where $x = e^{2\pi i/3}$. This matrix, which was discussed by Wolfenstein[7] in the context of a model with three neutrinos, corresponds to maximum mixing of the d-quark weak eigenstates in terms of the mass eigenstates. Thus maximum mixing of the quark generations corresponds to maximal CP nonconservation. We find this result of our *a priori* criterion for maximal CP nonconservation satisfying: Maximum mixing of the quark generations is analogous for CP nonconservation to maximum mixing of V and A for parity nonconservation.

It remains to assess the significance of models such as that of Gronau and Schechter. Our view is that such models are interesting, but that they should not be called models with maximal CP nonconservation. There is no convention-independent separation between the mixing angles and the phase in the KM matrix V. Whether or not a phase is $\pm\pi/2$ depends on the conventions used to parametrize V, and therefore the fact that the phase can have such a value in some parametrization does not have physical significance.

Finally, we emphasize that the fact that present data on weak interaction rates constrain $|V_{ij}|$ so severely that the observed CP-nonconservation parameter ϵ_K can only be fitted with the CP-nonconserving phase set to its maximum value $\pm \pi/2$ should be regarded as showing that the KM model with three generations is on the edge of being ruled out by experiment, rather than being regarded as evidence for "maximal" CP nonconservation.

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Note added.—After our article was submitted for publication we saw the article and talk by C. Jarlskog, Phys. Rev. Lett. **55**, 1039 (1985), the Institute of Theoretical Physics, University of Stockholm Report No. 4, July 1985, to appear in the Proceedings of the Fifth International Conference on Physics in Collision, Autun, France, July 1985. Jarlskog also discusses maximal CP nonconservation using invariants quartic in V and, as we do, reaches the conclusion that CP conservation is not violated maximally in nature. Jarlskog's analysis differs from ours in two ways: (1) Jarlskog uses $\det C$, where $iC = [M,M']$ and M (M') is the mass matrix for up (down) quarks normalized so that the largest eigenvalue is 1, as a measure of CP nonconservation. Her C differs from our t by factors involving quark masses. (Her J is the same as our t.) (2) Jarlskog considers possible maximal CP nonconservation in the nine processes associated with the nine fundamental four-quark transitions in the three-generation KM model and finds that maximal CP nonconservation cannot occur simultaneously in all nine cases. We consider a unique process-independent definition of maximal CP nonconservation. Jarlskog points out that the equality of the imaginary parts of our nine Δ's, which we demonstrated using the trace formalism, can also be shown using unitarity.

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[2]D. Hochberg and R. G. Sachs, Phys. Rev. D **27**, 606 (1983); B. Stech, Phys. Lett. **130B**, 189 (1983); L. Wolfenstein, Phys. Lett. **144B**, 425 (1984); M. Gronau and J. Schechter, Phys. Rev. Lett. **54**, 385, 1209(E) (1985); M. Ross, University of Helsinki Report No. Hu-TFT-84-38, 1984 (unpublished).

[3]The Murnaghan construction of a matrix in SU(N) is

$$V = D \prod_{i<j} V^{(ij)},$$

where D is a diagonal SU(N) matrix and $V^{(ij)}$ is an SU(2) matrix of the form

$$V^{(ij)} = \begin{pmatrix} c_{ij} & s_{ij} e^{i\phi_{ij}} \\ -s_{ij} e^{-\phi_{ij}} & c_{ij} \end{pmatrix}$$

acting on generations i and j. The form of V depends on the order in which the $V^{(ij)}$ are multiplied. Throughout this article, we abbreviate $\cos\theta_i$ or $\sin\theta_i$ by c_i or s_i. Certain sums of the ϕ's are invariant under similarity rephasings of the KM matrix. In particular,

$$\Phi = \phi_{12} + \phi_{23} + \phi_{31},$$

which Gronau and Schechter call the "invariant phase," has this property for the case of three generations.

[4]O. W. Greenberg, Phys. Rev. D **32**, 1841 (1985); D.-d. Wu, Phys. Rev. D (to be published).

[5]For the three-generation case, the relation is

$$\begin{pmatrix} \Lambda_1 \\ \Lambda_2 \\ \Lambda_3 \end{pmatrix} = \frac{1}{6} \begin{pmatrix} 2 & 3 & 1 \\ 2 & -3 & 1 \\ 2 & 0 & -2 \end{pmatrix} \begin{pmatrix} 1 \\ \lambda^3 \\ \sqrt{3}\lambda^8 \end{pmatrix}.$$

[6]Both first- and second-order CP nonconservation can be expressed in terms of the quartic Δ's or T's: first-(second-) order CP nonconservation is linear (quadratic) in these quantities.

[7]L. Wolfenstein, Phys. Rev. D **18**, 958 (1978).